



Geophysics and natural risks: instruments and principles of data analysis

Microwave Tomography enhanced GPR

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Overview

Part I

- 🌐 Ground Penetrating Radar (GPR) vs Radar
 - 🌐 similarities and differences
 - 🌐 constitutive properties of materials

- 🌐 GPR measurement configurations

- 🌐 GPR data

Part II

- 🌐 Scattering equations

- 🌐 Inverse scattering problem

- 🌐 definition
- 🌐 non-linearity
- 🌐 ill-posedness

- 🌐 Microwave Tomography

Part III

- 🌐 Microwave tomography: a flexible tool for GPR imaging

- 🌐 Applicative examples



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🌐 Microwave tomography: a flexible tool for GPR imaging

🌐 Applicative examples



GPR vs Radar: similarities and differences

the term **GPR** is referred to a wide range of radar technologies designed for the detection and identification of buried metallic and dielectric artifacts or structures beneath the surface

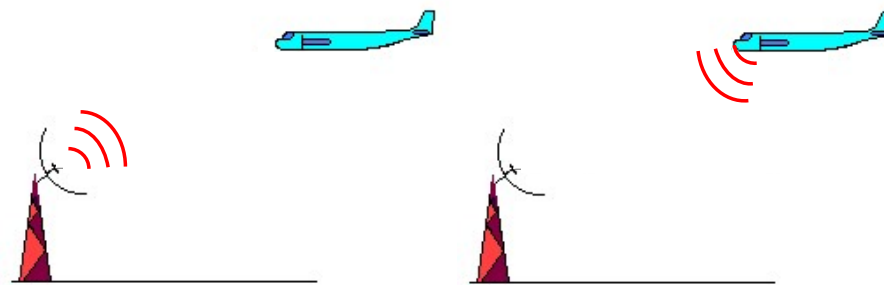
radar device + data processing



GPR vs Radar: similarities and differences

RADAR- RADIO DETECTION AND RANGING

devices to detect a target located in air by exploiting radio waves



- ❑ the transmitting antenna radiates a microwave signal
- ❑ if the transmitted signal strike on an obstacle (f.i, an airplane, a ship, a mountain) a portion of it is back-scattered towards the antenna and is gathered
- ❑ being known the wave propagation in air, it is possible to determine the target location by considering the round-trip travel time

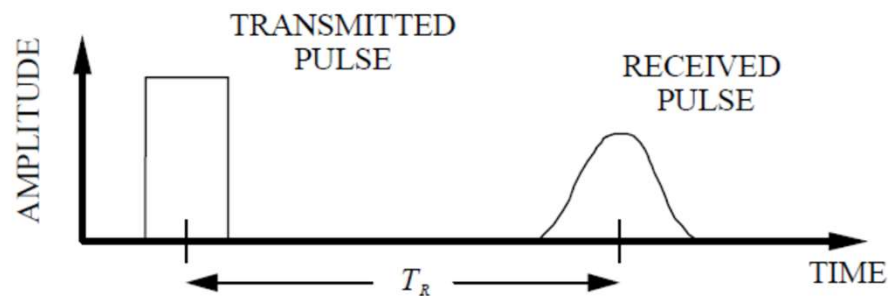
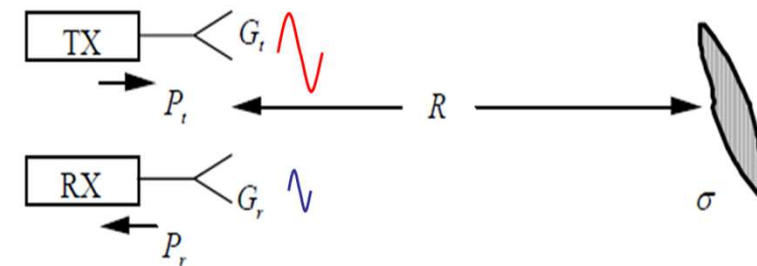
GPR vs Radar: similarities and differences

RADAR- RADIO DETECTION AND RANGING

devices to detect a target located in air by exploiting radio waves

- the system transmits an impulse by means of the Tx antennas
- the signal reflected by an e.m. discontinuity is received by the Rx antenna

The time it takes for a signal to travel from a given source to a given destination and back is called the roundtrip time (T_R) and depends on the distance R between target and antenna system

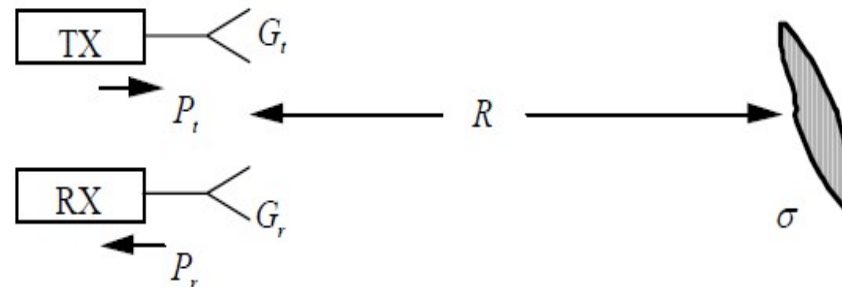


$$2R = v T_R$$

$v = c$ is the velocity of the signal in air

GPR vs Radar: similarities and differences

RADAR- Radio Detection And Ranging



σ = radar cross section (RCS) in square meters

P_t = transmitter power, watts

P_r = received power, watts

G_t = transmit antenna gain in the direction of the target (assumed to be the maximum)

G_r = receive antenna gain in the direction of the target (assumed to be the maximum)

$P_t G_t$ = effective radiated power (ERP)

From antenna theory: $G_r = \frac{4\pi A_{er}}{\lambda^2}$

$A_{er} = A_p \rho$ = effective area of the receive antenna

A_p = physical aperture area of the antenna

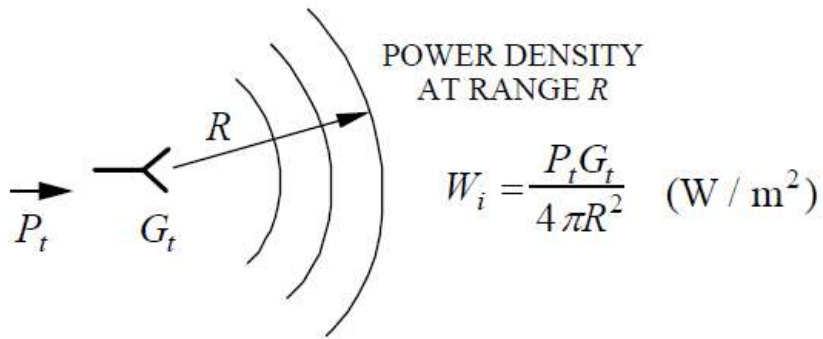
λ = wavelength ($= c / f$)

ρ = antenna efficiency

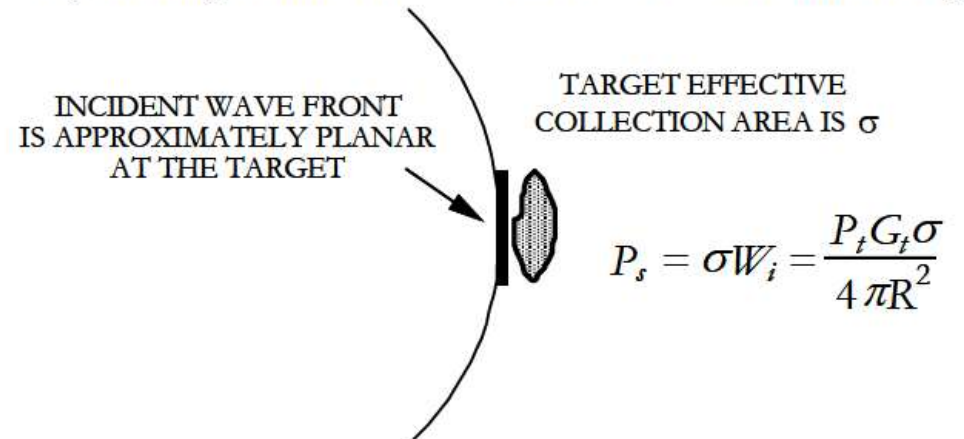
GPR vs Radar: similarities and differences

RADAR- Radio Detection And Ranging

Power density incident on the target



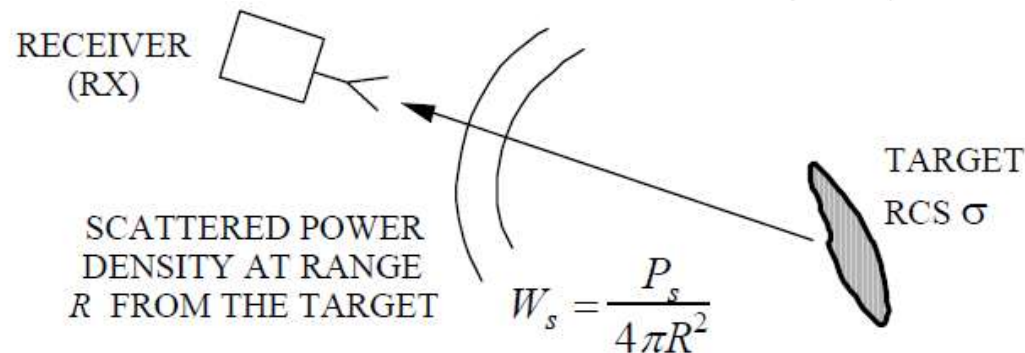
Power collected by the target and scattered back towards the radar is P_s



GPR vs Radar: similarities and differences

RADAR- Radio Detection And Ranging

The RCS gives the fraction of incident power that is scattered back toward the radar. Therefore, the scattered power density at the radar is obtained by dividing P_s by $4\pi R^2$.



The target scattered power collected by the receiving antenna is $W_s A_{er}$.

Thus the maximum target scattered power that is available to the radar is

$$P_r = \frac{P_t G_t \sigma A_{er}}{(4\pi R^2)^2} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}$$

This is the classic form of the radar range equation (RRE)

GPR vs Radar: similarities and differences

CHARACTERISTIC MAIN FEATURES OF A RADAR SYSTEM

□ sensitivity

capability of detecting weak targets

□ measurement

capability of accurately estimate the radar-target distance

□ resolution

capability of accurately localize the targets and distinguish two targets

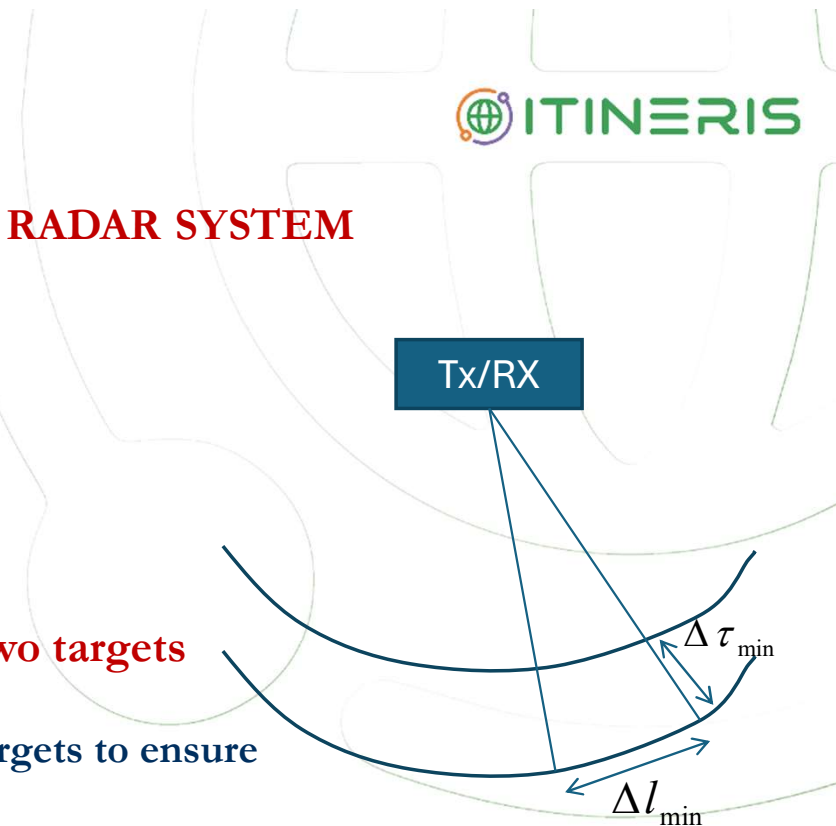
radial resolution: minimum radial distance required between two targets to ensure that both the targets are detectable

$$\Delta \tau_{\min} = \frac{v\tau}{2} = \frac{Wv}{4}$$

τ is transmitted wave duration
 v is the wave propagation velocity
 W is the amplitude of the signal at half of its maximum power

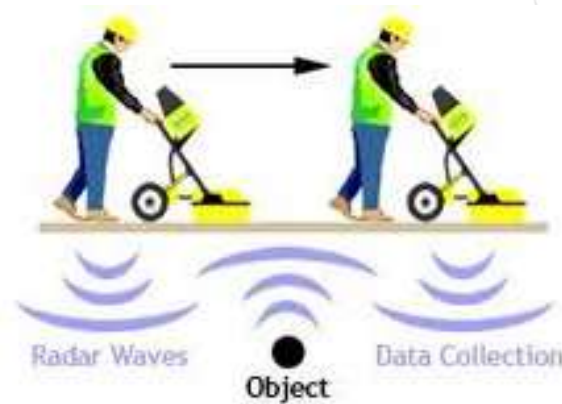
lateral resolution: minimum angular distance required between two targets to ensure that both the targets are detectable

$$\Delta l_{\min} = \sqrt{\frac{vrW}{2}}$$



GPR vs Radar: similarities and differences

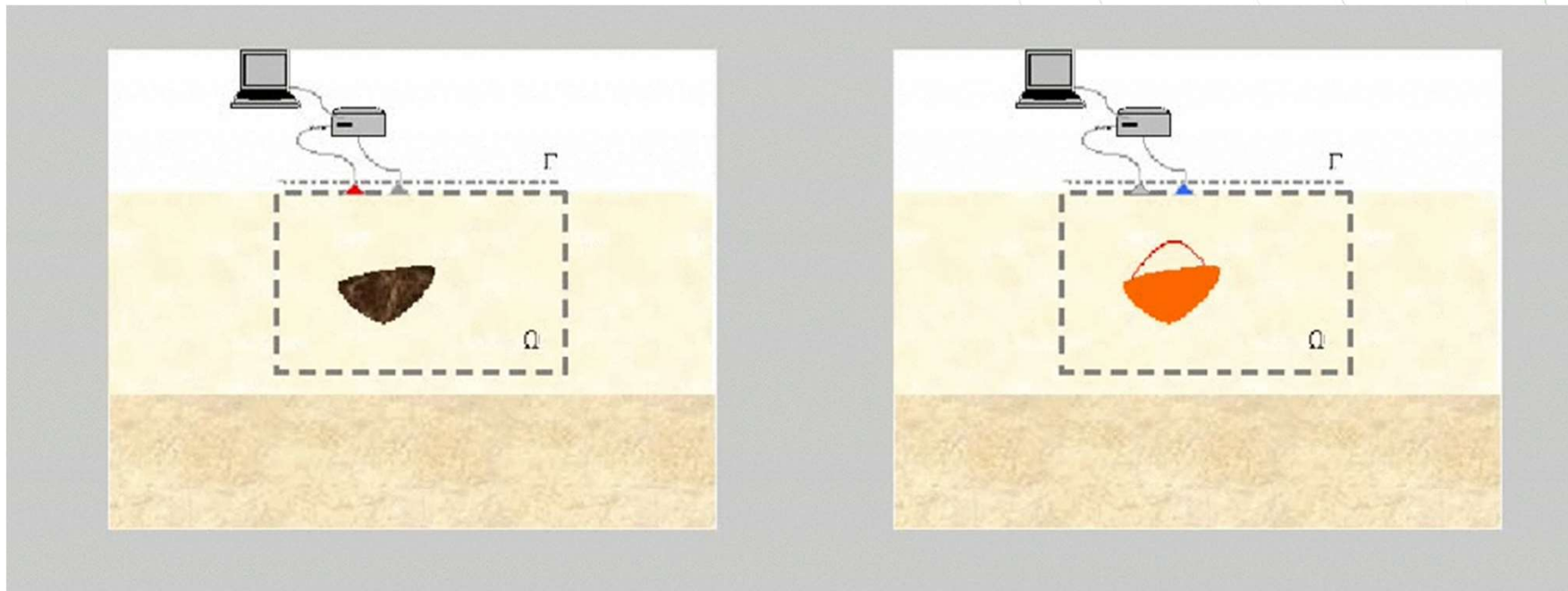
GROUND PENETRATING RADAR (GPR)



use the ability of microwaves to penetrate and interact with dielectric materials to image objects that are buried or hidden by an obstacle.

GPR vs Radar: similarities and differences

Ground Penetrating Radar (GPR)



- the antenna radiates a signal
- the signal interacts with the targets
- a back-scattered field arises
- the back-scattered field is gathered and processed

the signal propagation and its interaction with the targets take place in a material that is not air

GPR vs Radar: similarities and differences

Radar sensing

□ Maxwell's equations mathematically describe the physics of EM fields

In mathematical terms, EM fields and relationships are expressed as follows:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1.1)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (1.2)$$

$$\nabla \cdot \vec{D} = q \quad (1.3)$$

$$\nabla \cdot \vec{B} = 0 \quad (1.4)$$

where \vec{E} is the electric field strength vector (V/m); q is the electric charge density (C/m³); \vec{B} is the magnetic flux density vector (T); \vec{J} is the electric current density vector (A/m²); \vec{D} is the electric displacement vector (C/m²); t is time (s); and \vec{H} is the magnetic field intensity (A/m).

□ constitutive relationships describes a material's response to EM fields

$$\vec{J} = \tilde{\sigma} \vec{E} \quad (1.5)$$

$$\vec{D} = \tilde{\epsilon} \vec{E} \quad (1.6)$$

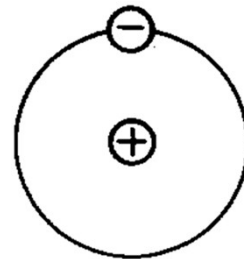
$$\vec{B} = \tilde{\mu} \vec{H} \quad (1.7)$$

fundamentals for describing radar signal

GPR vs Radar: constitutive properties of materials

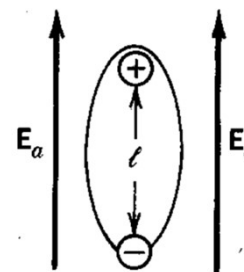
DIELECTRICS

Dielectrics are materials in which the dominant charges in the atoms and molecules are bound negative and positive charges that are held in place by atomic and molecular forces and cannot move around freely. Thus, ideal dielectrics contain no free charge, and their atoms and molecules are macroscopically neutral.



typical atom in absence of an applied electric field

When an external electric field is applied, the bound negative and positive charges do not move but their respective centroids can shift but creating electric dipoles.



typical atom under an applied electric field

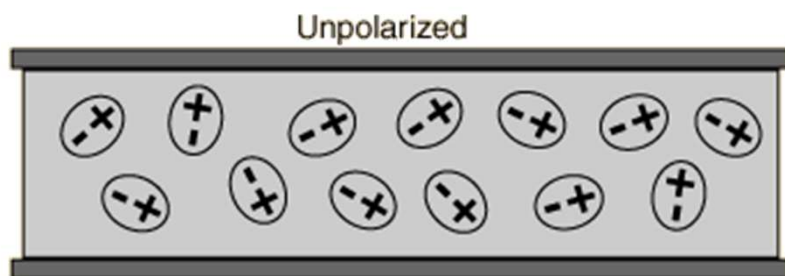
GPR vs Radar: constitutive properties of materials

DIELECTRICS

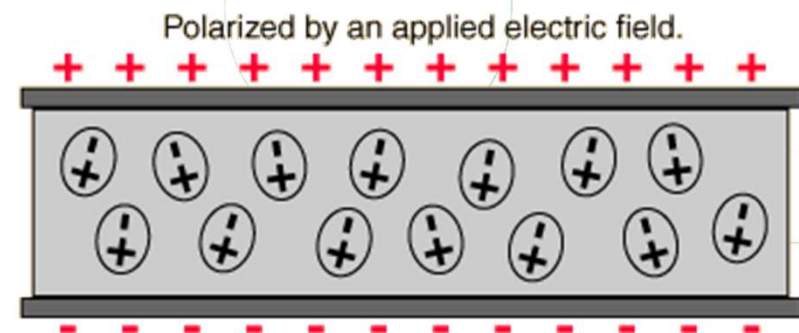
dielectric permittivity (ϵ): materials capabilities to store charge (energy)

it measures the material's particles capability of polarizing themselves due to the presence of an e.m. field, i.e. it characterizes the displacement of the material's constrained charges

homogeneous material



in the absence of the electric field, the bound charged particles are un-polarized and there is a net, overall, zero charge



as the electric field travels through the material, energy is transferred to the particles in form of charge concentration, so the particles become physically polarized, i.e., a dipole of moment is induced into the materials

GPR vs Radar: constitutive properties of materials

DIELECTRICS

dielectric permittivity (ϵ): materials capabilities to store charge (energy)

it measures the material's particles capability of polarizing themselves due to the presence of an e.m. field, i.e. it characterizes the displacement of the material's constrained charges

absolute electric permittivity of free space:

$$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ Farads/m}$$

absolute electric permittivity of medium:

$$\epsilon = \epsilon_0 \epsilon_r$$

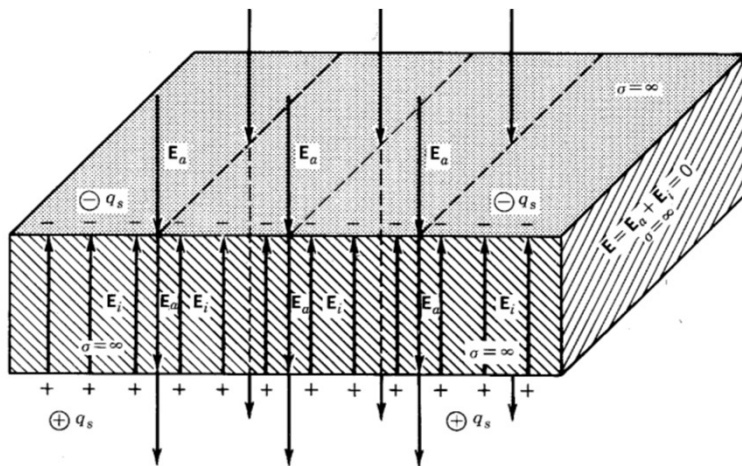
ϵ_r is relative permittivity, whose value is in the range 1-80 (non - dimensional quantity)

GPR vs Radar: constitutive properties of materials

CONDUCTORS

Conductors are materials whose prominent characteristic is the movement of electric charges and the generation of current. In other words, conductors are materials whose atomic outer shell electrons are not very tightly bound and can move from one atom to another. These are called free electrons, and they are very abundant in conductors.

In the absence of an external field, the free electrons move at different speeds in random directions, producing a zero net current across the surface of the conductor.



The free charges of a good conductor subjected to an electric field migrate very rapidly and are distributed as a surface charge density on the surface of the material.

For perfect conductors, the electric field inside them it is exactly equal to zero.

GPR vs Radar: constitutive properties of materials

CONDUCTORS

electric conductivity (σ): materials capabilities of being passed by electric charge

it measures material's ability to conduct an electric current, i.e. the material's free charges displacement due to the e.m. field

$$\sigma = \frac{J}{E} = \frac{1}{\rho} \text{ Siemens / m}$$

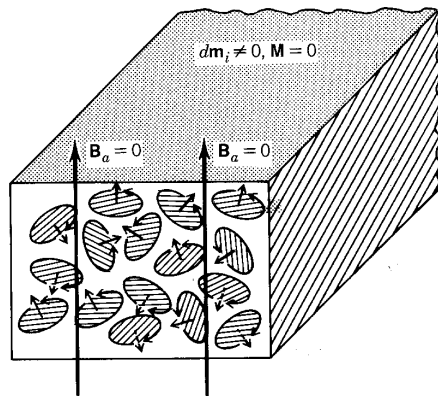
- ❑ J is the magnitude of the current density (Amperes/m²)
- ❑ E is the magnitude of the electric field (Volts/m)
- ❑ ρ is the electric resistivity (Ohm/m)

GPR vs Radar: constitutive properties of materials

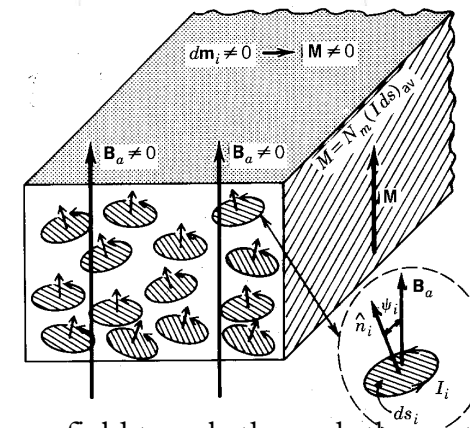
MAGNETIC MATERIALS

Magnetic materials are those that exhibit magnetic polarization when exposed to an applied magnetic field. The phenomenon of magnetization is represented by the alignment of the magnetic dipoles of the material with the applied magnetic field, like the alignment of the electric dipoles of dielectrics with the applied electric field.

A magnetic material is represented by several magnetic dipoles, which, in absence of an applied magnetic field, are oriented in a random fashion



in the absence of the em field, the magnetic dipole of the particles are un-polarized and there is a net, overall, zero magnetic charge



as the em field travels through the material, the magnetic dipole of the particles are oriented, and a magnetic moment is induced in the materials

GPR vs Radar: constitutive properties of materials

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absolute magnetic susceptibility of free space:

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henries/m}$$

absolute magnetic susceptibility of medium:

$$\mu = \mu_0 \mu_r$$

μ_r is relative magnetic susceptibility, being 1 for nonmagnetic materials (non - dimensional quantity)

GPR vs Radar: constitutive properties of materials

Since most of the materials are non-magnetic, permittivity and conductivity are the most important characteristic properties

The permittivity and conductivity values are controlled by the microscopic (atomic, molecular, and granular) behavior of the constituents of the materials and depend on the density, state (solid, liquid, gas), and water content of the materials, as well as the operating frequency.

GPR vs Radar: constitutive properties of materials

Since most of the materials are non-magnetic, permittivity and conductivity are the most important characteristic properties

- If the constitutive parameters are not a function of the applied field, the materials are called **linear**, otherwise they are **nonlinear**
- If the constitutive parameters are not function of the position, the materials are called **homogeneous**, otherwise they are **inhomogeneous**
- If the constitutive parameters vary as a function of the frequency, they are called **dispersive**, otherwise they are **non-dispersive**
- If constitutive parameters are a function of the direction of the applied field, the materials are called **anisotropic**, otherwise they are called **isotropic**

GPR vs Radar: constitutive properties of materials

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linear, homogeneous, isotropic materials ➡ ϵ, μ, σ are scalar

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typical range of dielectric characteristics of various materials measured at 100 MHz

Material	Conductivity, Sm^{-1}	Relative permeability
Air	0	1
Asphalt dry	$10^{-2} : 10^{-1}$	2-4
Asphalt wet	$10^{-3} : 10^{-1}$	6-12
Clay dry	$10^{-1} : 10^{-0}$	2-6
Clay wet	$10^{-1} : 10^{-0}$	5-40
Coal dry	$10^{-3} : 10^{-2}$	3.5
Coal wet	$10^{-3} : 10^{-1}$	8
Concrete dry	$10^{-3} : 10^{-2}$	4-10
Concrete wet	$10^{-2} : 10^{-1}$	10-20
Freshwater	$10^{-6} : 10^{-2}$	81
Freshwater ice	$10^{-4} : 10^{-3}$	4
Granite dry	$10^{-8} : 10^{-6}$	5
Granite wet	$10^{-3} : 10^{-2}$	7
Limestone dry	$10^{-8} : 10^{-6}$	7
Limestone wet	$10^{-2} : 10^{-1}$	8
Permafrost	$10^{-5} : 10^{-2}$	4-8
Rock salt dry	$10^{-4} : 10^{-2}$	4-7
Sand dry	$10^{-7} : 10^{-3}$	2-6
Sand wet	$10^{-3} : 10^{-2}$	10-30
Sandstone dry	$10^{-6} : 10^{-5}$	2-5
Sandstone wet	$10^{-4} : 10^{-2}$	5-10
Sea water	10^2	81
Sea-water ice	$10^{-2} : 10^{-1}$	4-8
Shale dry	$10^{-3} : 10^{-2}$	4-9
Shale saturated	$10^{-3} : 10^{-1}$	9-16
Snow firm	$10^{-6} : 10^{-5}$	6-12
Soil clay dry	$10^{-2} : 10^{-1}$	4-10
Soil clay wet	$10^{-3} : 10^{-0}$	10-30
Soil loamy dry	$10^{-4} : 10^{-3}$	4-10
Soil loamy wet	$10^{-2} : 10^{-1}$	10-30
Soil sandy dry	$10^{-4} : 10^{-2}$	4-10
Soil sandy wet	$10^{-2} : 10^{-1}$	10-30

GPR vs Radar: similarities and differences

SIGNAL PROPAGATION IN DIELECTRIC MATERIAL

the e.m. properties affect:

a) the propagation speed; b) the attenuation; c) the penetration depth

plane wave

$$E(z) = E_0 e^{-jkz}$$

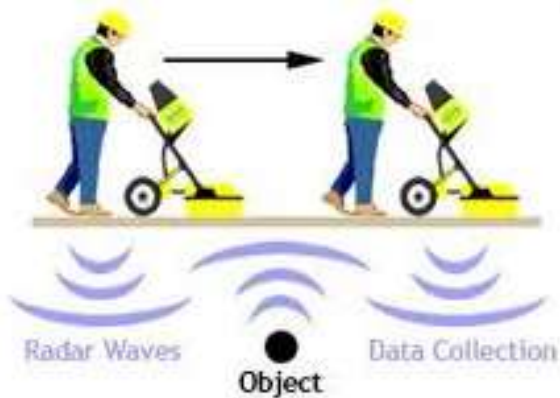
$$k = \omega \sqrt{\mu \epsilon_{eq}} = \omega \sqrt{\mu_0 \epsilon_0 \frac{\epsilon_{eq}}{\epsilon_0}} = \frac{\omega}{c} \sqrt{\epsilon_{eq,r}}$$

k describes the change in phase per unit length for each wave component

k may be considered as a constant of the medium at a fixed frequency - wave number

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \text{velocity of light in free space}$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} \quad \text{velocity of propagation in a not dispersive medium}$$



GPR vs Radar: similarities and differences

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plane wave

$$E(z) = E_0 e^{-jkz}$$

$$k = \omega \sqrt{\mu_0 (\epsilon' - j\epsilon'')} = \beta - j\alpha$$

$$E(z, t) = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)}$$

attenuation term

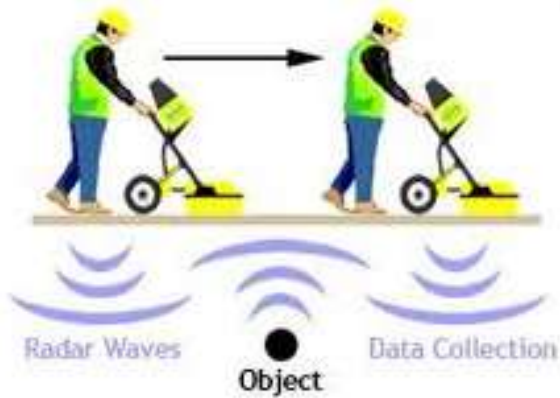
propagation term

$$\alpha = \omega \sqrt{\left[\frac{\mu\epsilon'}{2} \sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1 \right]}$$

attenuation constant

$$\beta = \omega \sqrt{\left[\frac{\mu\epsilon'}{2} \sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1 \right]}$$

propagation constant



GPR vs Radar: similarities and differences

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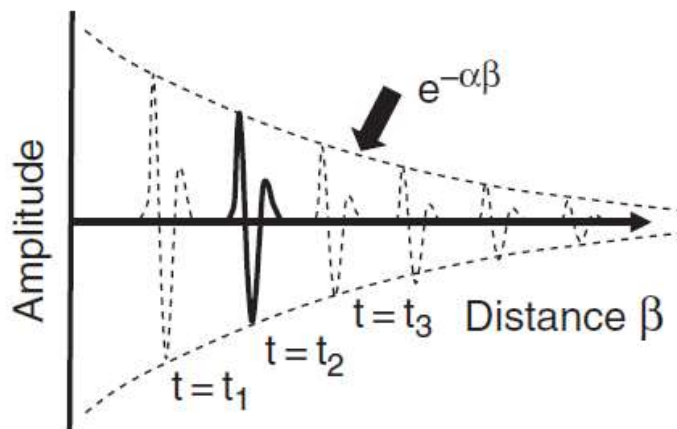
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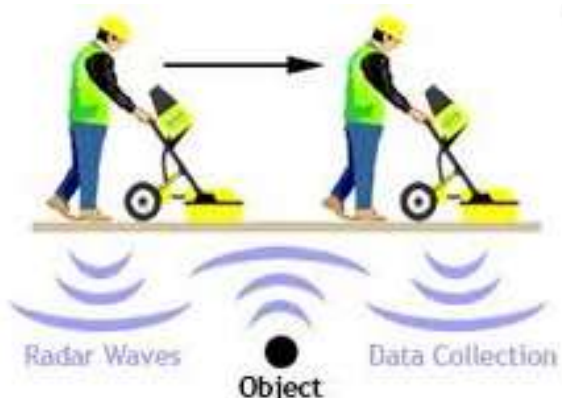
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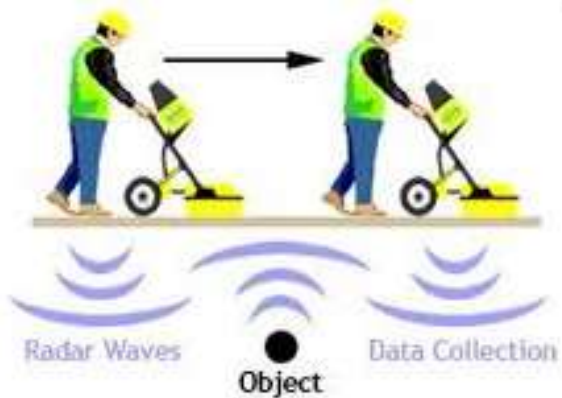
in low-loss environments, e.m. waves propagate at a finite velocity and decay in amplitude with minimal change in pulse shape



GPR vs Radar: similarities and differences

SIGNAL PROPAGATION IN DIELECTRIC MATERIAL

e.m. materials properties vs wave propagation



Material	Porosity (%)	Water Saturation (%)	Dielectric Constant	Electrical Conductivity (mS/m)	Velocity (m/ns)	Attenuation (Np/m)	Skin depth (m)
Air	-	-	1	0	0.300	0	∞
Water	-	-	81	1	0.033	0.021	47.7
Ice			3.7-4.0				
Dry Sand	30	0	4	0.1	0.150	0.009	106
Wet Sand	30	100	17.2 25	21.3 10	0.072 0.060	0.97 0.38	1.0 2.6
Dry Clay	30	0	4	10	0.150	0.94	1.1
Wet Clay	30	100	17.7 16	31.3 100	0.071 0.075	1.40 4.71	0.7 0.2
Average Soil	30	-	16	20	0.075	0.94	1.1

water is by far the most polarizable, naturally occurring material (in other words, it has a high permittivity with 80)

GPR vs Radar: similarities and differences

SIGNAL PROPAGATION IN DIELECTRIC MATERIAL

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plane wave

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attenuation term

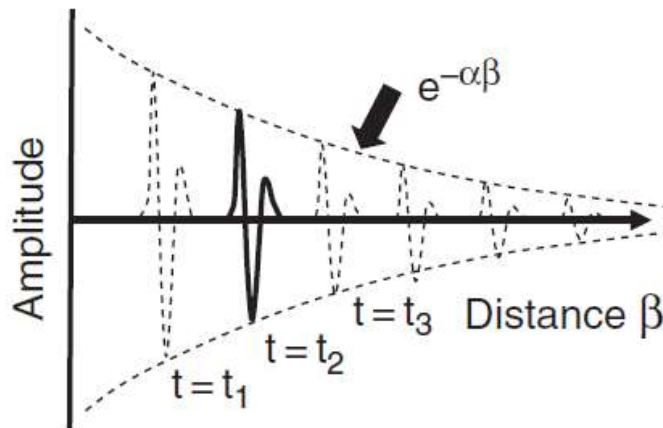
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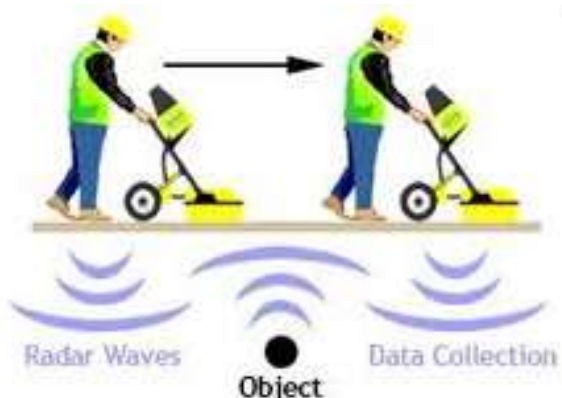
$$\beta = \omega \sqrt{\left[\frac{\mu \epsilon'}{2} \sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1 \right]}$$

propagation constant



skin depth: depth corresponding to a signal attenuation equal to $1/e$ ($e = 2,71828\dots$ being the Eureka's number)

$$\delta \approx 5.31 \frac{\sqrt{\epsilon_r}}{\sigma}$$

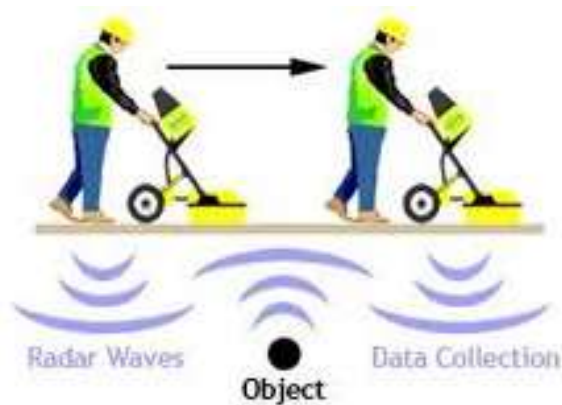


GPR vs Radar: similarities and differences



conventional radar

- the wave propagation is in air
- detection of far targets(km)
- the required resolution is of several meters



georadar

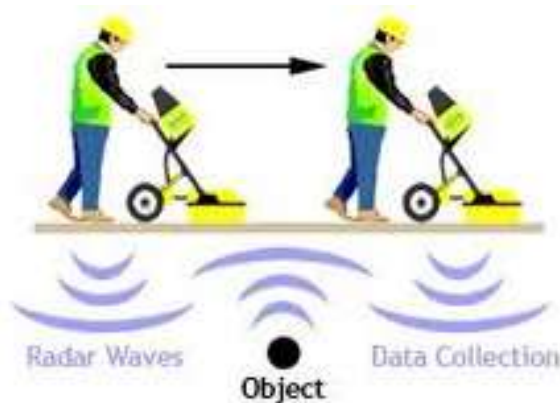
- the wave propagation is in lossy media
- detection of near targets (m or cm)
- the required resolution is of some centimeters or less

GPR vs Radar: similarities and differences



conventional radar

- the wave propagation is in air
 - the radiation pattern of the antenna is known – low attenuation
- detection of far targets(km)
 - low commutation time between Tx/Rx mode
- the required resolution is of several meters
 - relaxed design constrains (signal bandwidth)

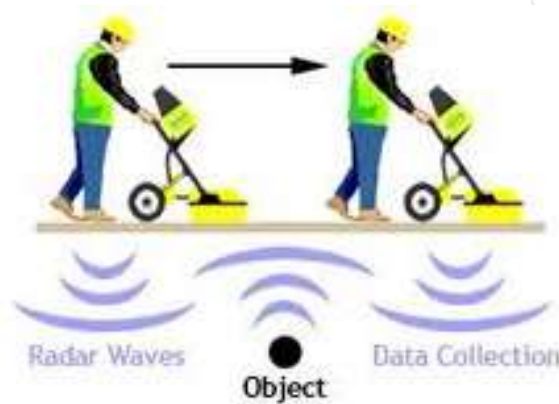


georadar

- the wave propagation is in lossy media
 - the radiation pattern of the antenna is unknown
 - high attenuation
- detection of near targets (m or cm)
 - high computation time /use of two antennas
- the required resolution is of some centimeters or less
 - not simple design constrains (ultra wideband systems)

GPR vs Radar: similarities and differences

GROUND PENETRATING RADAR (GPR)



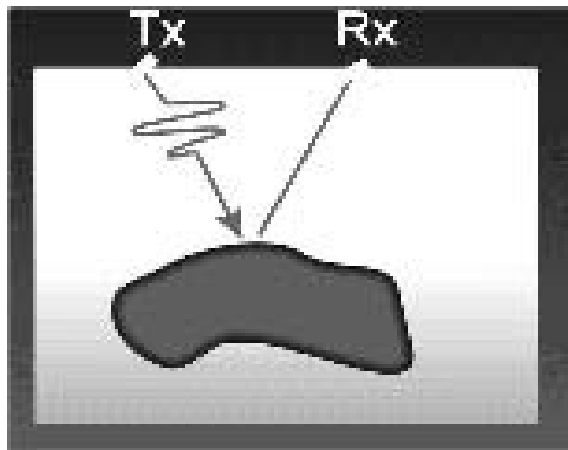
use the ability of microwaves to penetrate and interact with dielectric materials to image objects that are buried or hidden by an obstacle.

the capability of imaging buried / hidden targets depends on the difference arising among the electromagnetic features of the target and those of the surrounding medium

GPR vs Radar: similarities and differences

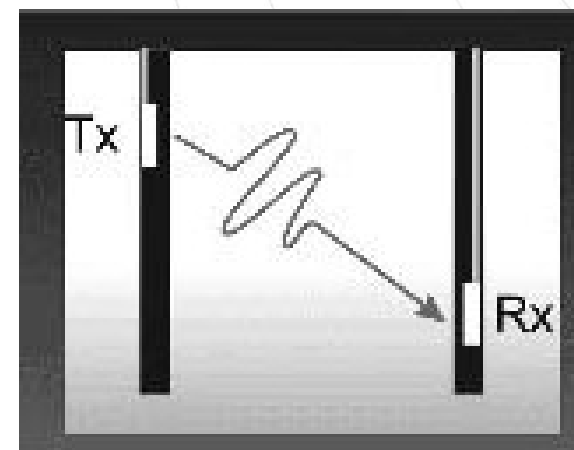
GPR measurement configurations

reflection modality



the Tx and Rx antennas are in air

transmission modality

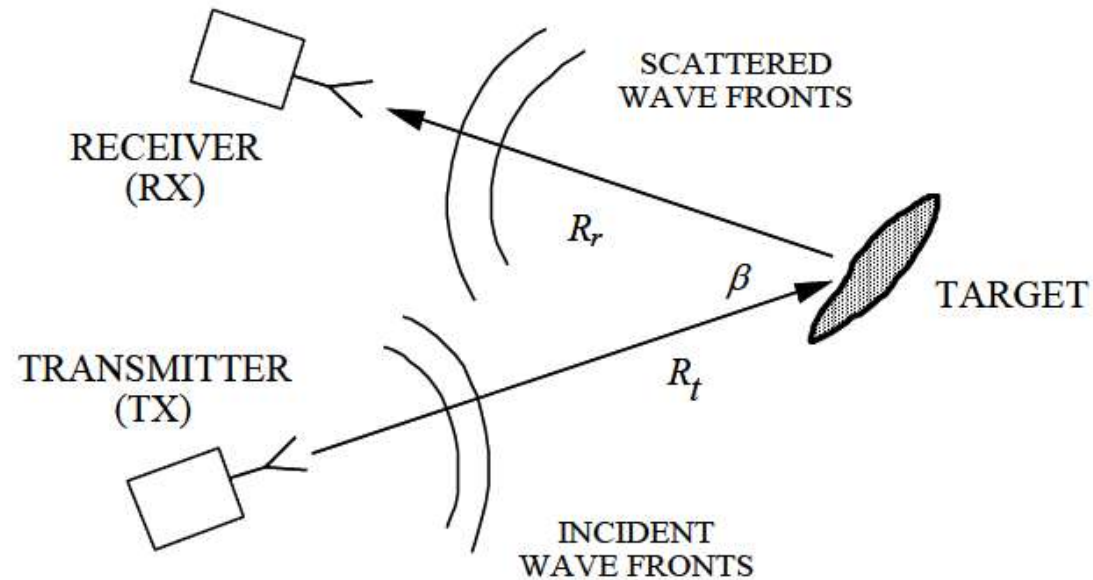


the Tx and Rx antennas are in the probed soil (holes are need)

single-source, single-receiver reflection surveys are the most common, although multiple-source, multiple-receiver configurations are occasionally used for some specialized applications and have received considerable attention in recent years

GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS



Bistatic: Tx and Rx antennas are at different locations from the target point of view

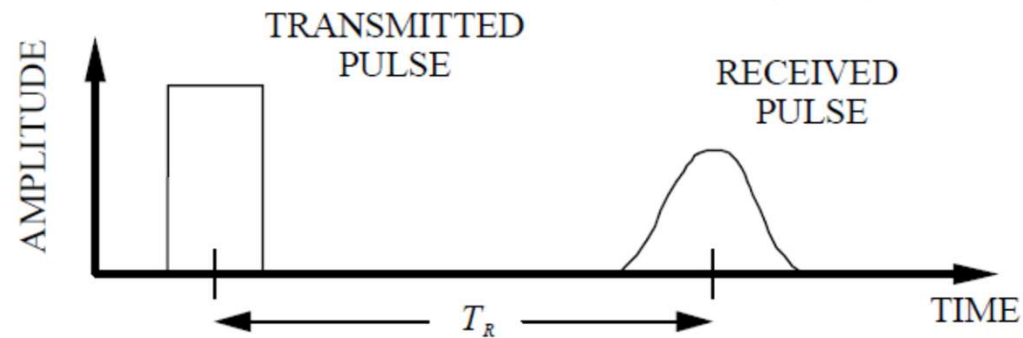
Monostatic: Tx and Rx antennas are co-located from the target point of view - the same antenna acts as Tx and Rx

Quasi-monostatic: Tx and Rx antennas are slightly separated but still appear at the same location

GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS

the round-trip travel time (T_R) depends on the distance occurring between the targets and the antennas system



bistatic: $R_t + R_r = v T_R$

monostatic: $2R = v T_R$ ($R_t = R_r = R$)

$T_R = \frac{2R}{v}$, $v = \frac{c}{\sqrt{\epsilon_r}} \text{ ms}^{-1}$

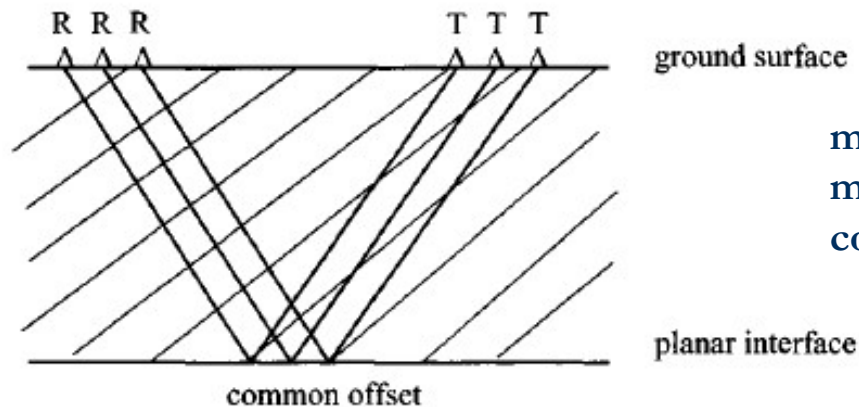
v is the velocity of the signal into the investigated medium

linear, homogeneous, isotropic materials

GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS

COMMON OFFSET



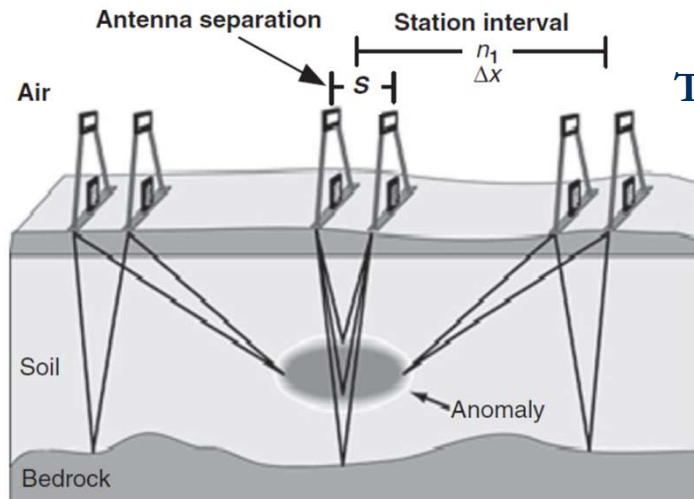
moving transmitting antenna
moving receiving antenna
constant offset

- ❑ common-offset surveys deploy a single transmitter and receiver with a fixed offset or spacing between the units at each measurement location
- ❑ the terminology for such a survey is single-fold common offset
- ❑ the transmitting and receiving antennas have specific polarization character for the field generation and detection
- ❑ the antennas are deployed in a fixed geometry (i.e., separation, s , and orientation) and measurements are made at regular station intervals (Δx)

GPR vs Radar: similarities and differences

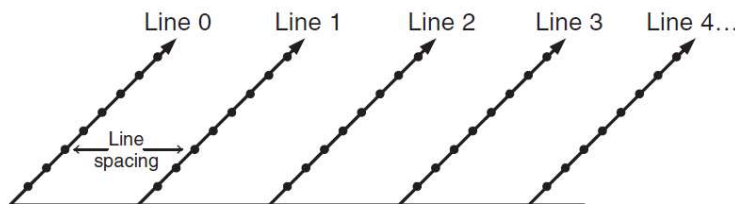
GPR MEASUREMENT CONFIGURATIONS

COMMON OFFSET



The parameters defining a common-offset survey are:

- ❑ GPR center frequency
- ❑ the recording time window
- ❑ the time sampling interval
- ❑ the station spacing
- ❑ the antenna spacing
- ❑ line spacing
- ❑ the antenna orientation
- ❑ the central frequency



- frequency selection is synonymous with defining both pulse width and bandwidth
- application exploration depths and resolution requirements determine bandwidth and also dictate temporal and spatial sampling intervals
- most often, attenuation is an issue; so frequencies are kept as low as possible to maximize penetration even if resolution is compromised

GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS

COMMON OFFSET

standard setup adopted in commercial GPR systems



GPR equipped by not shielded and low frequency antenna



GPR system equipped by shielded antennas working at different central frequency

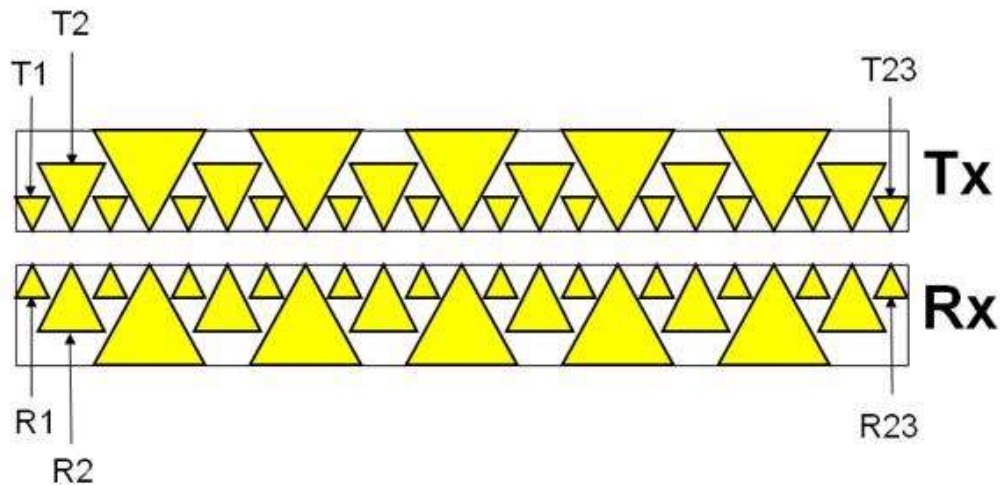


GPR system equipped by shielded and high frequency antennas

GPR vs Radar: similarities and differences

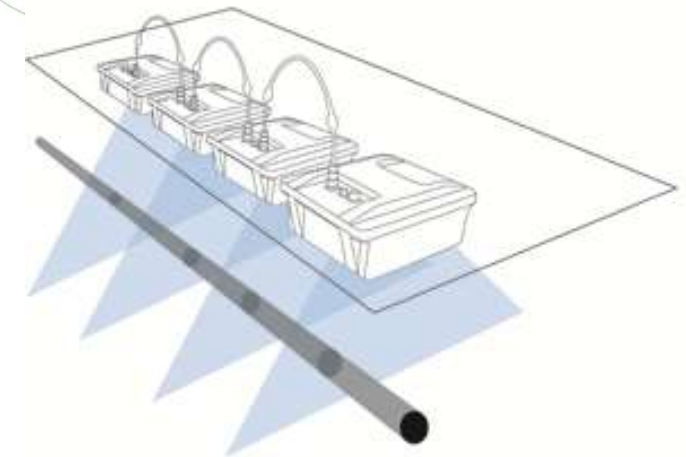
GPR MEASUREMENT CONFIGURATIONS

array



COMMON OFFSET

N transmitting antenna
 N receiving antenna
constant offset



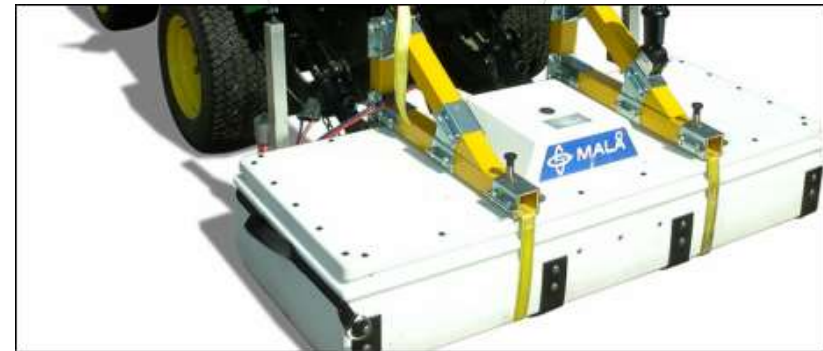
- ❑ N traces are collected simultaneously – reduction of the measurement time
- ❑ for each transmitting antenna the back-scattered field is measured by the corresponding receiving antenna - multi-monostatic data

GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS

COMMON OFFSET

array



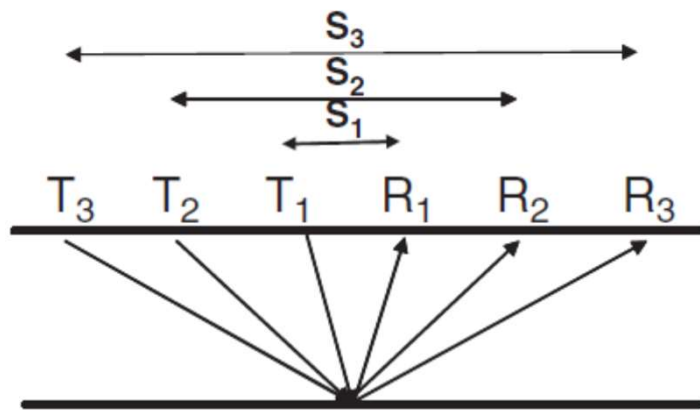
- ❑ measurement configuration adopted for soil investigations (geophysics/archeology) and for road monitoring
- ❑ last generation of GPR systems exploit this measurement configuration

GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS

COMMON MIDPOINT (CMP)

or wide-angle reflection and refraction (WARR)



moving transmitting antenna
moving receiving antenna
increasing offset

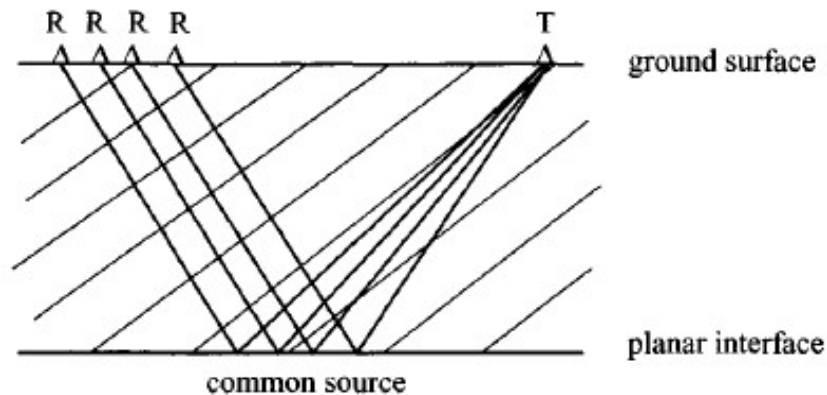
equivalent to seismic refraction and wide-angle reflection

common midpoint soundings are primarily used estimate the radar signal velocity versus depth by varying the antenna spacing and measuring the change in the two-way travel time

Multi-offset measurements can be made at each station, resulting in a multiple reflection survey. Two advantages are that CMP stacking can improve the signal-to-noise ratio and that a full velocity cross section can be derived. However, they are seldom performed because they are time consuming and more complex to analyze, and most of the cost-effective benefit is obtained with well-designed single-fold surveys.

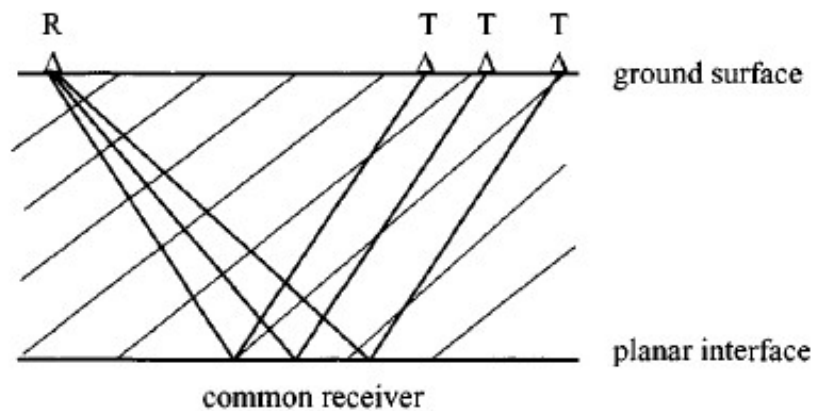
GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS



fixed transmitting antenna
moving receiving antenna

this setup allows observation diversity



fixed receiving antenna
moving transmitting antenna

this setup allows illumination diversity

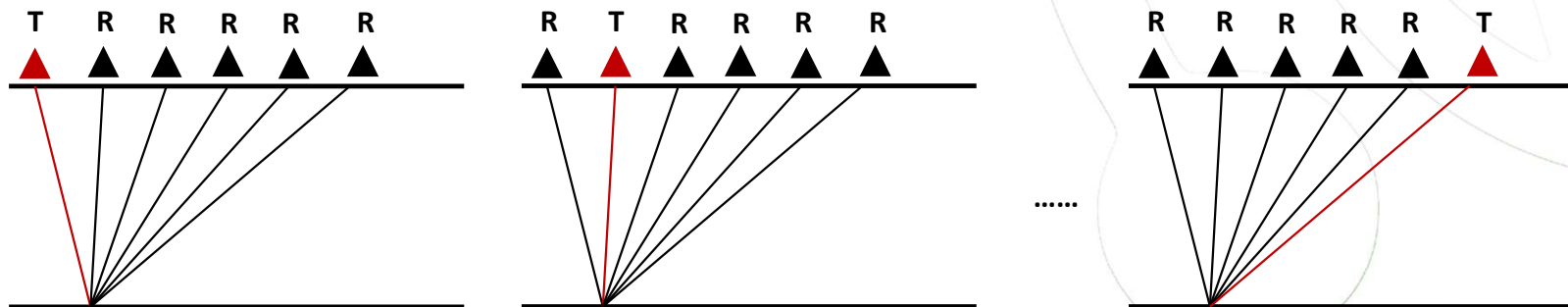
COMMON SOURCE

COMMON RECEIVER

GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS

MULTIPLE INPUT MULTIPLE OUTPUT (MIMO)



- ❑ physical or synthesized array of transmitting and receiving antennas
- ❑ when an antenna acts as transmitter the other antennas measure the backscattered field
- ❑ for each position of the array, multi-view / multi-static data are collected
- ❑ illumination diversity and observation diversity are exploited

last generation of GPR systems currently under development

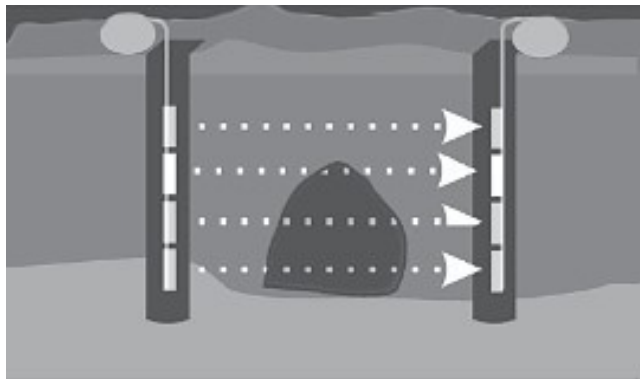
GPR vs Radar: similarities and differences

GPR MEASUREMENT CONFIGURATIONS

transmission measurements (**borehole**) are less common
(used for engineering and environmental studies)

BOREHOLL

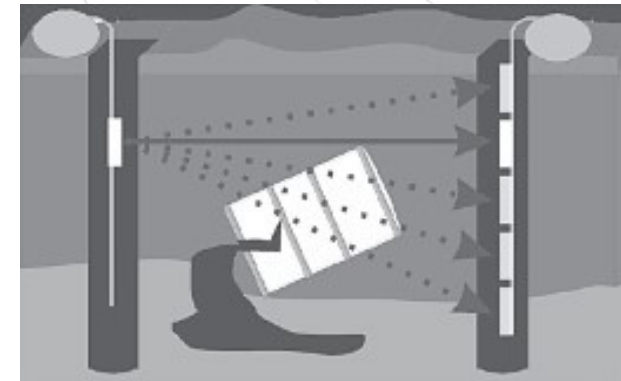
zero-offset profiling (ZOP)



Tx and Rx antennas are moved from station-to-station in a synchronized way

quick and easy survey to locate velocity anomalies or attenuation (shadow) zones - in a uniform environment, the received signal should be the same at each location

multi-offset gather (MOG)



for each position of the Tx antenna the signal is received in many points

geometry information as auxiliary data to ensure correct interpretation
deviations in geometry will introduce errors into values of v and α derived from GPR time and amplitude observations

GPR vs Radar: similarities and differences

GPR DATA

GPR data obviously depend on the target to be detected

targets can be classified according to their geometry

- ❑ **planar and rough interfaces**

transition between two materials having different EM features

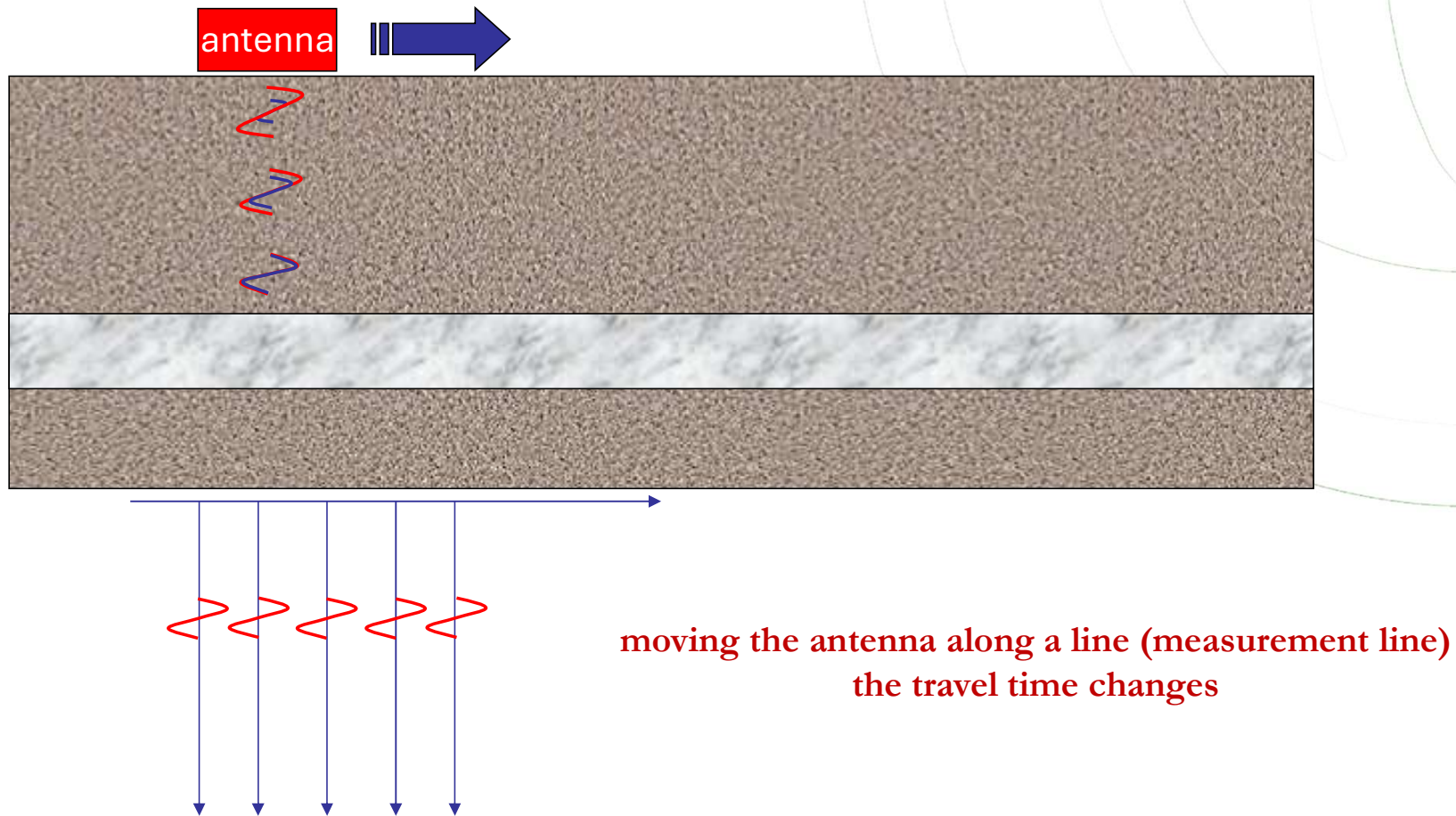
- ❑ **long, thin, localized spherical or cuboidal objects**

any kind of EM variation with a restricted spatial extension

in simple terms, a GPR system measures the time it takes for the signal to travel back and forth, and this time is modulated by the shape of the target.

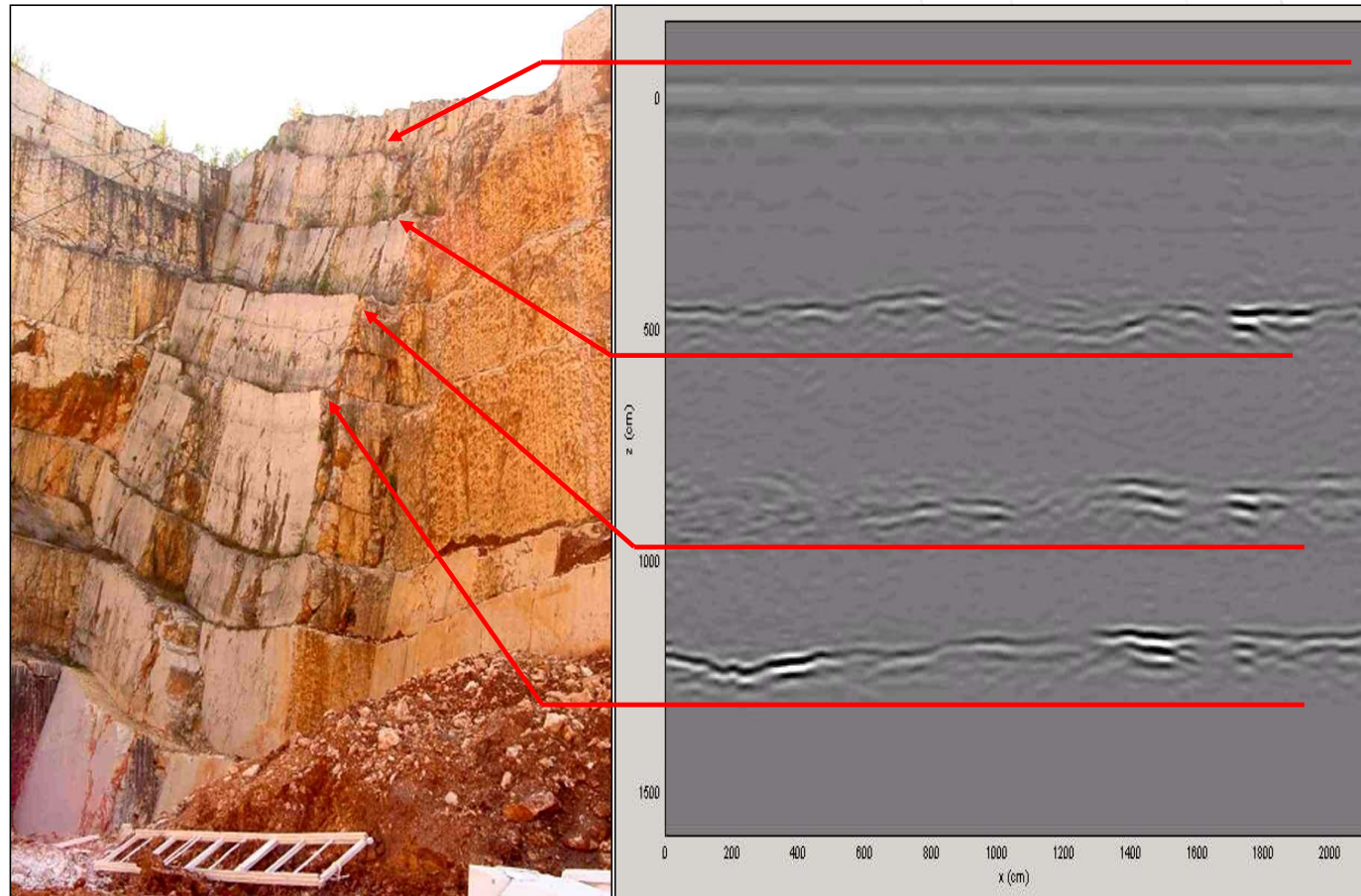
GPR vs Radar: similarities and differences

GPR DATA: RESPONSE OF A PLANAR INTERFACE



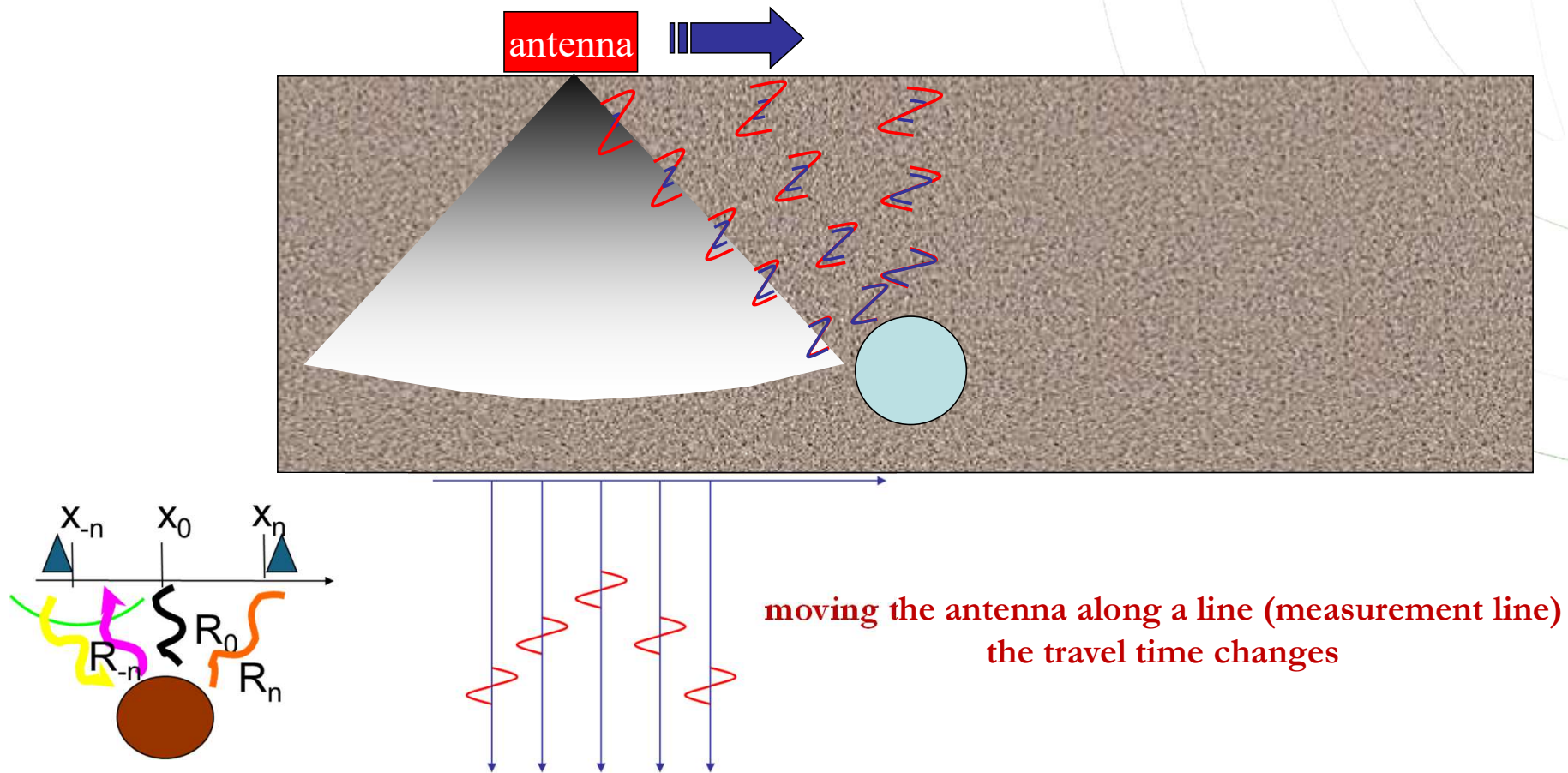
GPR vs Radar: similarities and differences

GPR DATA: RESPONSE OF A PLANAR INTERFACE

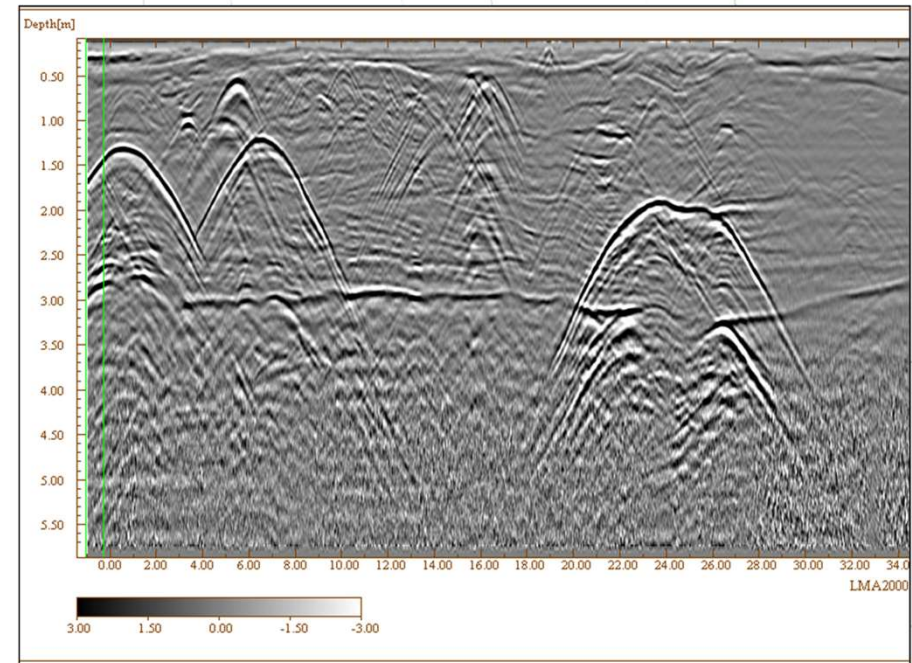
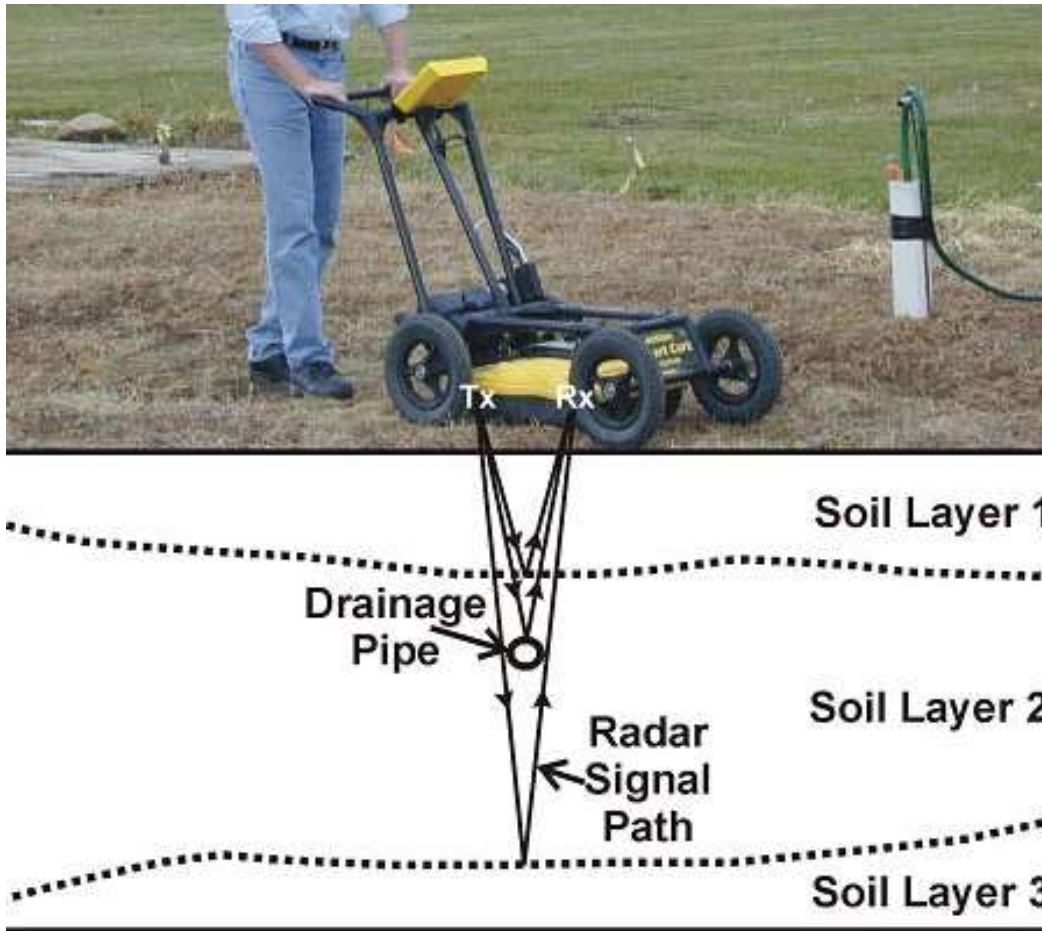


GPR vs Radar: similarities and differences

GPR DATA: RESPONSE OF A LOCALIZED TARGET



GPR vs Radar: similarities and differences



a radargram accounts for signals reflected by all electromagnetic discontinuities in the scenario

Overview

Part I

- 🌐 Ground Penetrating Radar (GPR) vs Radar
 - 🌐 similarities and differences
 - 🌐 constitutive properties of materials
- 🌐 GPR measurement configurations
- 🌐 GPR data

Part II

- 🌐 Scattering equations
- 🌐 Inverse scattering problem
 - 🌐 definition
 - 🌐 non-linearity
 - 🌐 ill-posedness

🌐 Microwave Tomography

Part III

- 🌐 Microwave tomography: a flexible tool for GPR imaging
- 🌐 Applicative examples



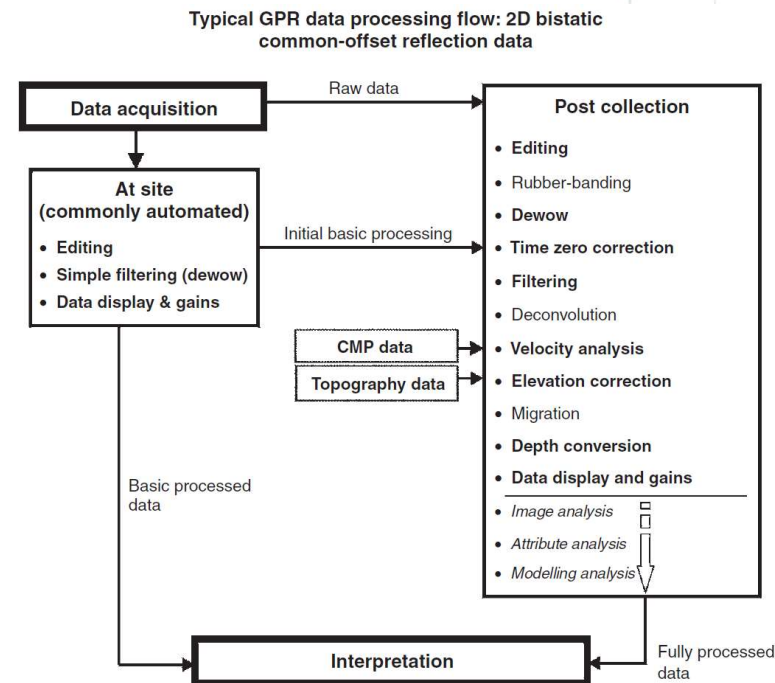
GPR data processing

radar device + data processing



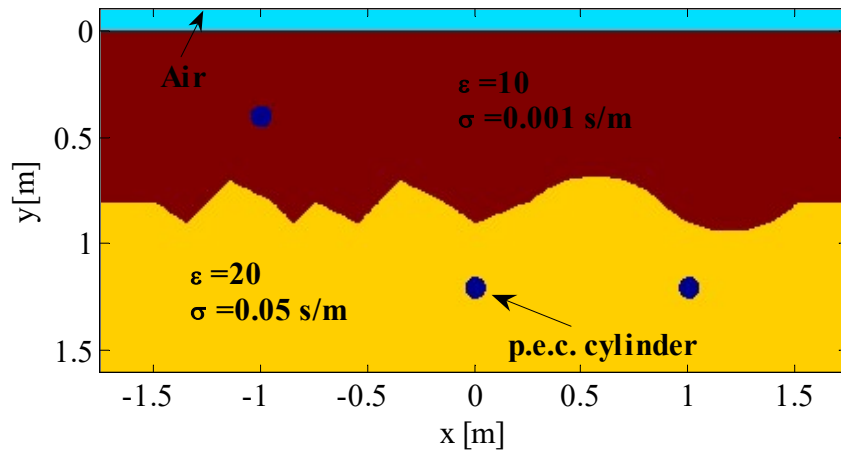
AIM: accurate and easily interpretable image of the inner structures of the surveyed region

GPR data processing

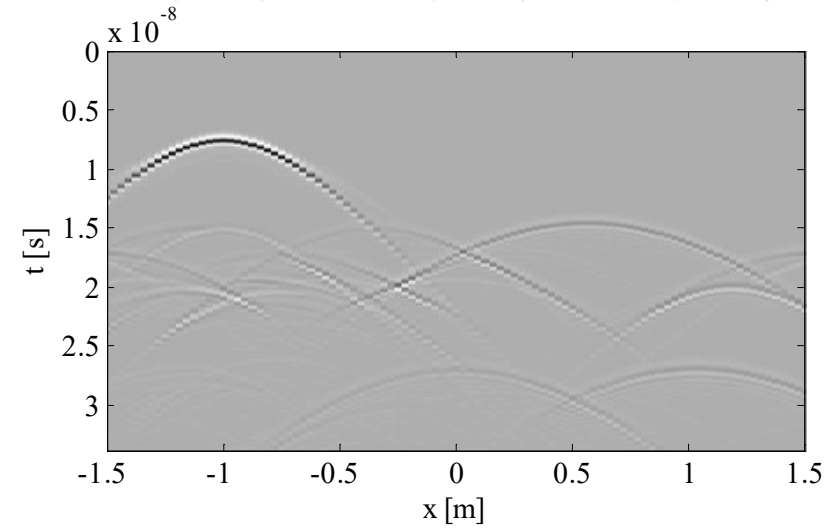


- **Processing steps in bold are essential for decent interpretations (e.g., post-collection filtering and depth conversion).**
- **Sophisticated and complex steps (such as deconvolution, migration, radar tomography) are optional, and their use must be balanced against the needs and costs of the project.**
- **Analysis tools such as image/pattern recognition, modeling, and attribute analysis are included, but in practice they are more an interpretive help than a specific processing feature.**

GPR data processing



reference scenario



radargram

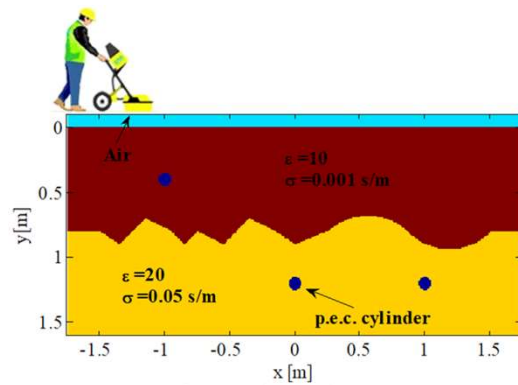
the image of targets does not correspond to their geometrical representation

the radargram interpretation may be a not easy task

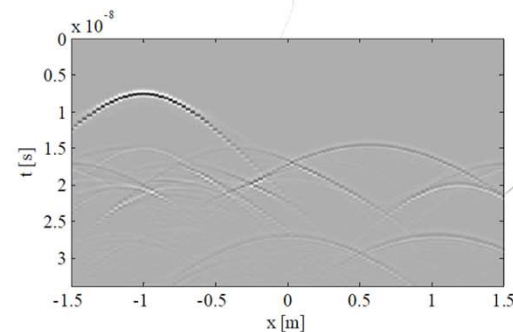
AIM: accurate and easily interpretable image of the inner structures of the surveyed region

GPR data processing

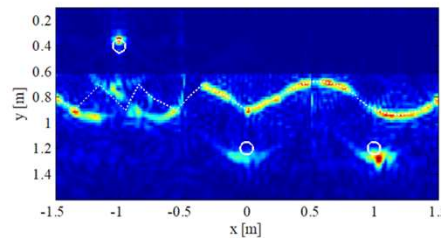
- **Coded image** of the scenario under test - **low Interpretability** of the raw radar images
- **Filtering procedures** to reduce noise, clutter and the direct coupling signal between the antennas
- **Imaging strategies** providing clear representations of the target with **acceptable** computational burden



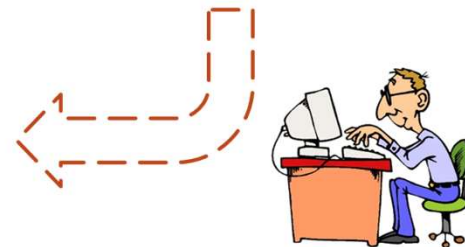
Scenario under test



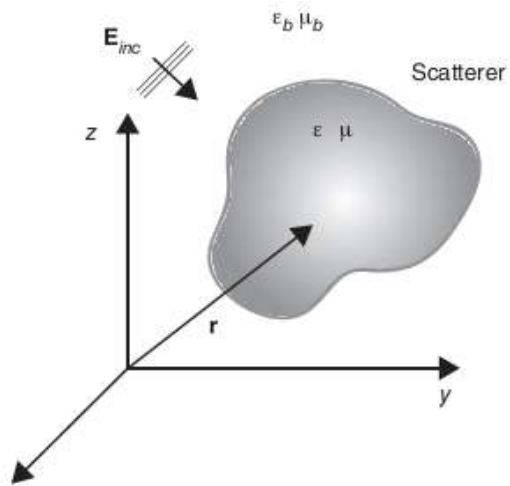
Filtered Radargram



Focused image

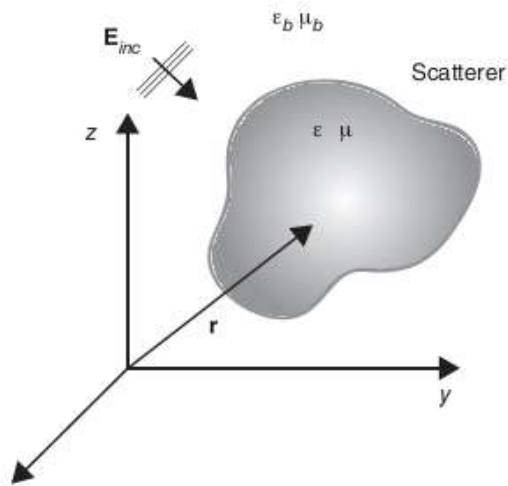


Scattering equations



when an object is present in the propagation medium, the wave produced by the source interacts with it and the field distribution is affected by the presence of the scatterer

Scattering equations



when an object is present in the propagation medium, the wave produced by the source interacts with it and the field distribution is affected by the presence of the scatterer

Maxwell's equations in frequency domain

$$\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega \mathbf{B}(\mathbf{r}),$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = j\omega \mathbf{D}(\mathbf{r}) + \mathbf{J}(\mathbf{r}),$$

$$\nabla \cdot \mathbf{D}(\mathbf{r}) = \rho(\mathbf{r}),$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}) = 0,$$

$$\mathbf{D}(\mathbf{r}) = \bar{\epsilon}(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r})$$

$$\mathbf{B}(\mathbf{r}) = \bar{\mu}(\mathbf{r}) \cdot \mathbf{H}(\mathbf{r})$$

$$\mathbf{J}(\mathbf{r}) = \mathbf{J}_0(\mathbf{r}) + \mathbf{J}_i(\mathbf{r})$$

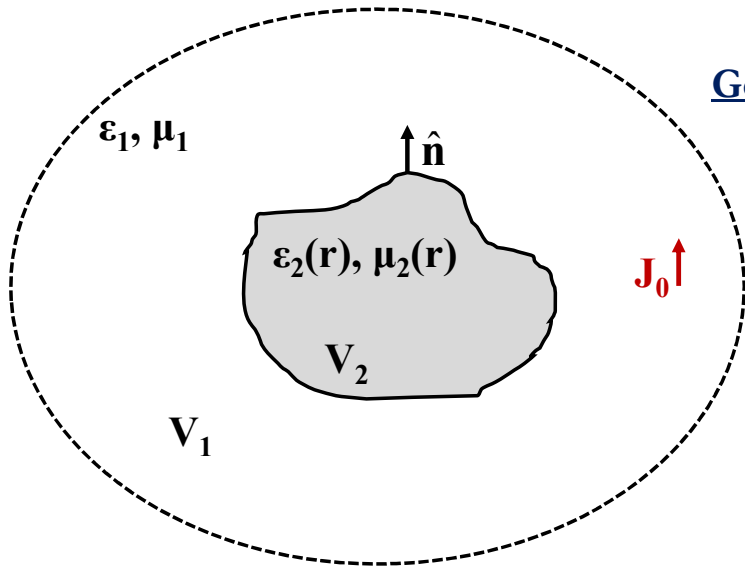
$$\mathbf{J}_i(\mathbf{r}) = \sigma(\mathbf{r}) \mathbf{E}(\mathbf{r}) \quad \text{Ohm's law}$$

constitutive relations

the advantages to work in the frequency domain instead of in the time domain are:

- ❑ the time dependence is neglected
- ❑ the partial time derivative is replaced by the product

Scattering equations

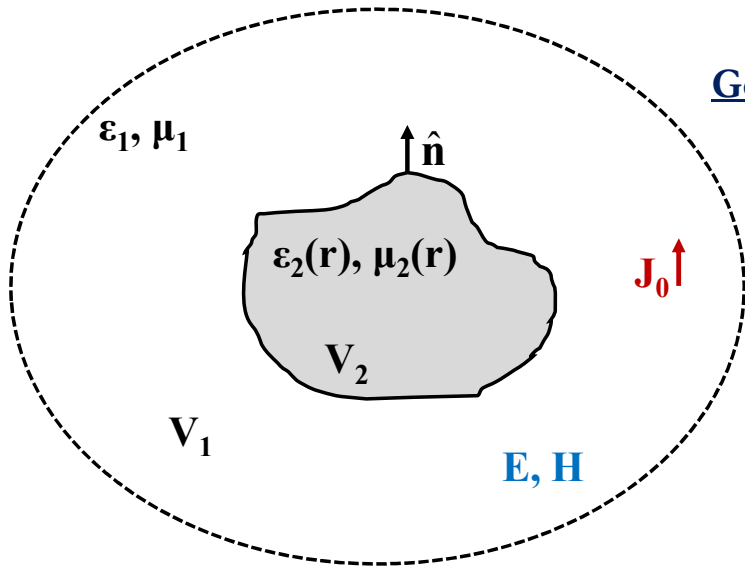


Goal: determine the electric-magnetic fields produced by a current density \mathbf{J}_0 (primary source) which radiates in presence of an object having volume V_2

$$V = V_1 \cup V_2$$

$$\varepsilon(\mathbf{r}) = \begin{cases} \varepsilon_1 & \mathbf{r} \in V_1 \\ \varepsilon_2(\mathbf{r}) & \mathbf{r} \in V_2 \end{cases} \quad \mu(\mathbf{r}) = \begin{cases} \mu_1 & \mathbf{r} \in V_1 \\ \mu_2(\mathbf{r}) & \mathbf{r} \in V_2 \end{cases}$$

Scattering equations



Goal: determine the electric-magnetic fields produced by a current density \mathbf{J}_0 (primary source) which radiates in presence of an object having volume V_2

\mathbf{E}, \mathbf{H} are the electric and magnetic fields in presence of the obstacle

$$\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu(\mathbf{r})\mathbf{H}(\mathbf{r})$$

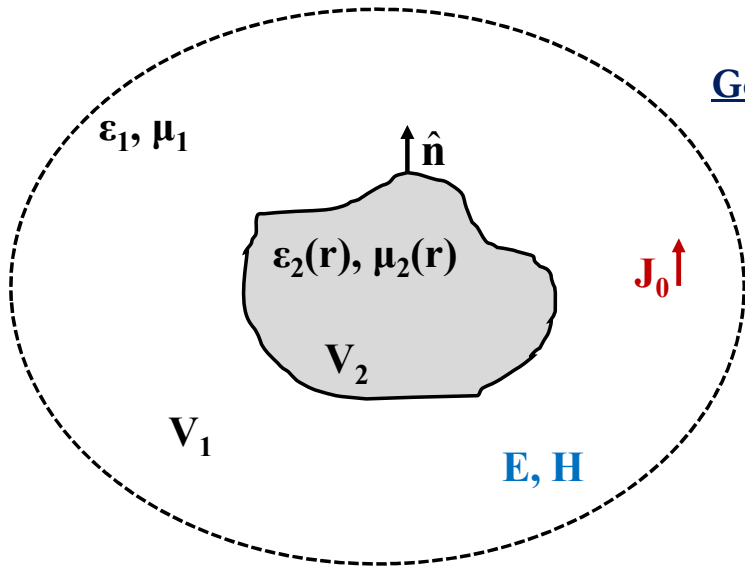
$$\nabla \times \mathbf{H}(\mathbf{r}) = j\omega\epsilon(\mathbf{r})\mathbf{E}(\mathbf{r}) + \mathbf{J}_0(\mathbf{r})$$

Maxwell's equations in a non-homogeneous medium

$$V = V_1 \cup V_2$$

$$\epsilon(\mathbf{r}) = \begin{cases} \epsilon_1 & \mathbf{r} \in V_1 \\ \epsilon_2(\mathbf{r}) & \mathbf{r} \in V_2 \end{cases} \quad \mu(\mathbf{r}) = \begin{cases} \mu_1 & \mathbf{r} \in V_1 \\ \mu_2(\mathbf{r}) & \mathbf{r} \in V_2 \end{cases}$$

Scattering equations



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Maxwell's equations in a non-homogeneous medium

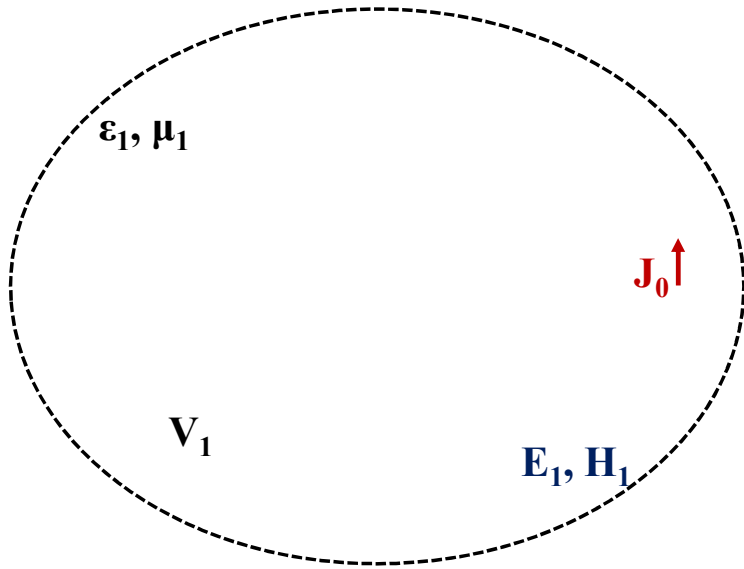
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}^s$$

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}^s$$

$$V = V_1 \cup V_2$$

$$\epsilon(\mathbf{r}) = \begin{cases} \epsilon_1 & \mathbf{r} \in V_1 \\ \epsilon_2(\mathbf{r}) & \mathbf{r} \in V_2 \end{cases} \quad \mu(\mathbf{r}) = \begin{cases} \mu_1 & \mathbf{r} \in V_1 \\ \mu_2(\mathbf{r}) & \mathbf{r} \in V_2 \end{cases}$$

Scattering equations



$$V = V_1$$

$$\epsilon(\mathbf{r}) = \epsilon_1 \quad \mathbf{r} \in V \quad \mu(\mathbf{r}) = \mu_1 \quad \mathbf{r} \in V$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}^s$$
$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}^s$$

$\mathbf{E}_1, \mathbf{H}_1$ are the electric and magnetic fields in absence of the obstacle

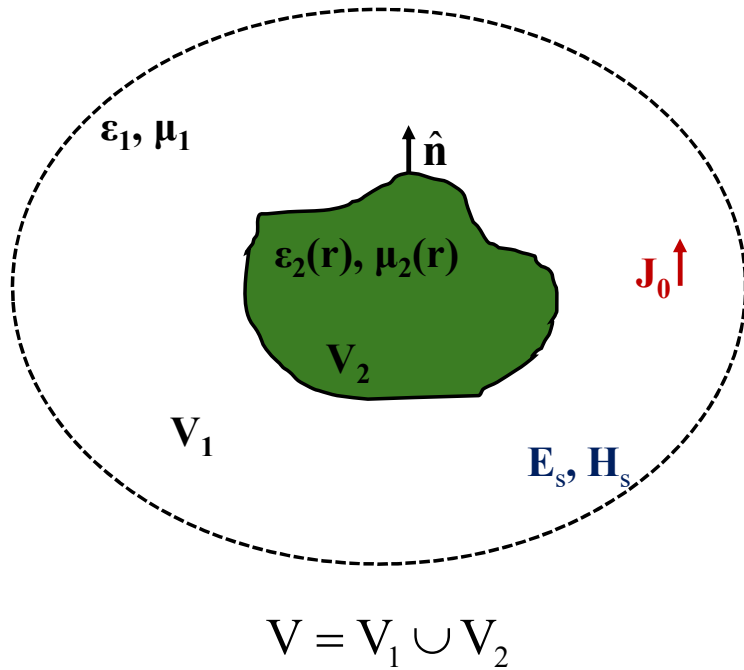
$$\nabla \times \mathbf{E}_1(\mathbf{r}) = -j\omega\mu_1\mathbf{H}_1(\mathbf{r})$$

$$\nabla \times \mathbf{H}_1(\mathbf{r}) = j\omega\epsilon_1\mathbf{E}_1(\mathbf{r}) + \mathbf{J}_0(\mathbf{r})$$

Maxwell's equations in a homogeneous medium

the incident fields are known quantities if the source is completely characterized

Scattering equations



$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}^s$$

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}^s$$

$\mathbf{E}^s, \mathbf{H}^s$ are the electric and magnetic scattered (perturbed) fields due to the obstacle

$$\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu(\mathbf{r})\mathbf{H}(\mathbf{r}) \quad \forall \mathbf{r} \in V$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = j\omega\epsilon(\mathbf{r})\mathbf{E}(\mathbf{r}) + \mathbf{J}_0(\mathbf{r}) \quad \forall \mathbf{r} \in V$$



$$\nabla \times \mathbf{E}_1(\mathbf{r}) = -j\omega\mu_1\mathbf{H}_1(\mathbf{r}) \quad \forall \mathbf{r} \in V$$

$$\nabla \times \mathbf{H}_1(\mathbf{r}) = j\omega\epsilon_1\mathbf{E}_1(\mathbf{r}) + \mathbf{J}_0(\mathbf{r}) \quad \forall \mathbf{r} \in V$$

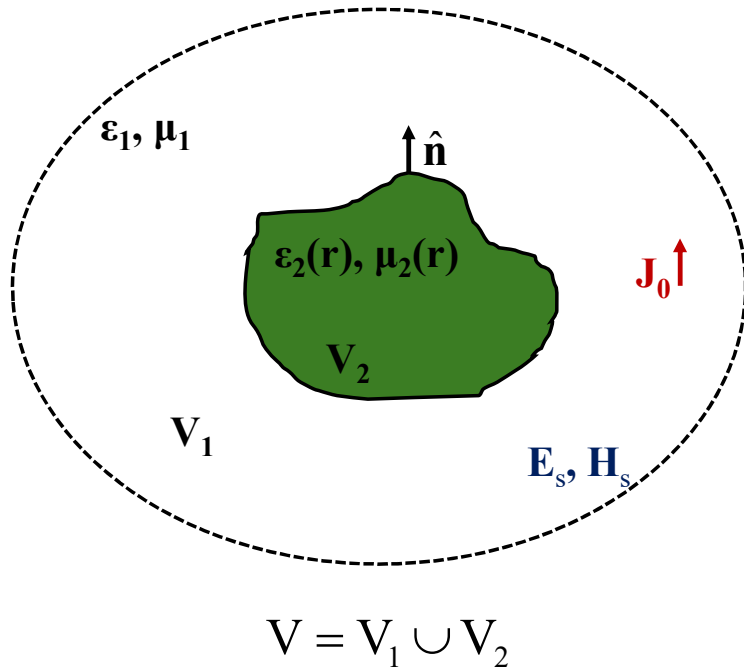
$$\epsilon(\mathbf{r}) = \begin{cases} \epsilon_1 & \mathbf{r} \in V_1 \\ \epsilon_2(\mathbf{r}) & \mathbf{r} \in V_2 \end{cases}$$

$$\mu(\mathbf{r}) = \begin{cases} \mu_1 & \mathbf{r} \in V_1 \\ \mu_2(\mathbf{r}) & \mathbf{r} \in V_2 \end{cases}$$

$$\epsilon(\mathbf{r}) = \epsilon_1 \quad \mathbf{r} \in V$$

$$\mu(\mathbf{r}) = \mu_1 \quad \mathbf{r} \in V$$

Scattering equations



$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}^s$$

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}^s$$

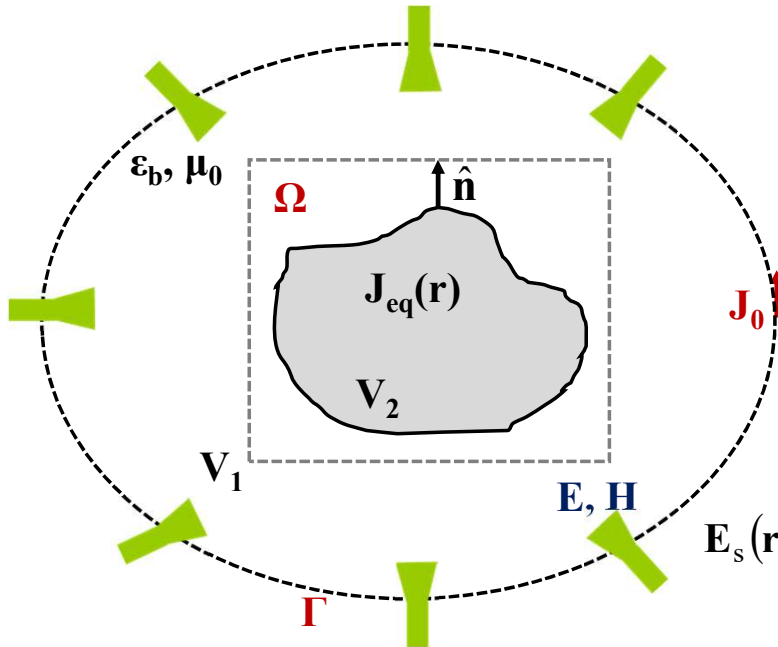
$\mathbf{E}^s, \mathbf{H}^s$ are the electric and magnetic scattered (perturbed) fields due to the obstacle

$$\nabla \times \mathbf{E}_s(\mathbf{r}) = -j\omega \underbrace{[\mu(\mathbf{r}) - \mu_1]}_{\mathbf{M}_{eq}(\mathbf{r}) \in V_2} \mathbf{H}(\mathbf{r}) - j\omega\mu_1 \mathbf{H}_s(\mathbf{r}) \quad \forall \mathbf{r} \in V$$

$$\nabla \times \mathbf{H}_s(\mathbf{r}) = \underbrace{j\omega[\varepsilon(\mathbf{r}) - \varepsilon_1]}_{\mathbf{J}_{eq}(\mathbf{r}) \in V_2} \mathbf{E}(\mathbf{r}) + j\omega\varepsilon_1 \mathbf{E}_s(\mathbf{r}) \quad \forall \mathbf{r} \in V$$

the scattered fields, i.e. the fields ascribed to the object presence and, specifically, to the interaction between the incident field and the object itself, **can be regarded as the field radiated in a homogeneous medium by the equivalent electric-magnetic sources $\mathbf{J}_{eq}(\mathbf{r})$ and $\mathbf{M}_{eq}(\mathbf{r})$**

Scattering equations



Ω : investigated domain

Γ : measurement domain

$$V = V_1 \cup V_2$$

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) - \omega^2 \varepsilon_b \mu_0 \int_{\Omega} \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r} \in V = V_1 \cup \Omega$$

$$\chi(\mathbf{r}) = \begin{cases} 0 & \mathbf{r} \in V_1 \\ \frac{\varepsilon_s(\mathbf{r}) - \varepsilon_b}{\varepsilon_b} & \mathbf{r} \in V_2 \end{cases}$$



$$\mathbf{E}_s(\mathbf{r}_m) = \mathbf{E}(\mathbf{r}_m) - \mathbf{E}_1(\mathbf{r}_m) = j\omega\mu_0 \int_{\Omega} \overline{\mathbf{G}}(\mathbf{r}_m, \mathbf{r}) \mathbf{J}_{eq}(\mathbf{r}) d\mathbf{r} \quad \mathbf{r}_m \in \Gamma$$

external or data equation

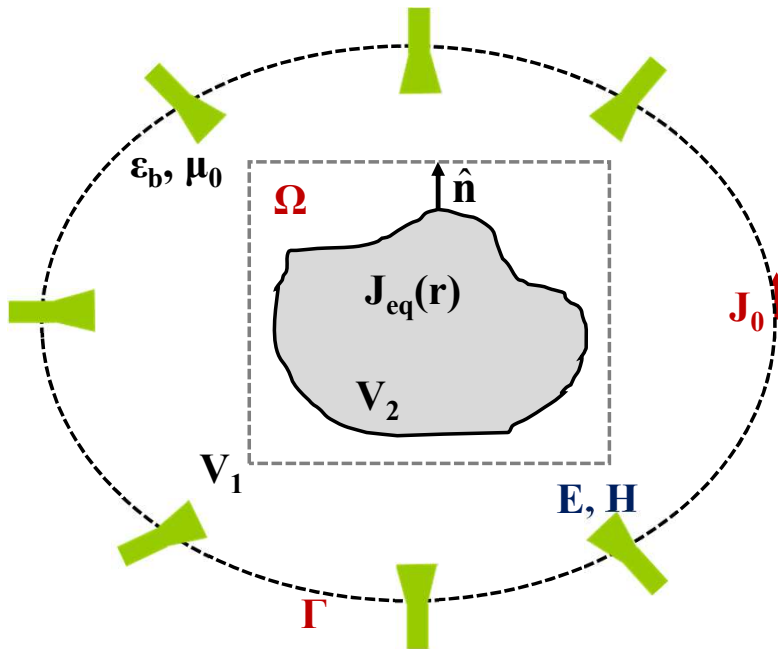
$$\mathbf{J}(\mathbf{r}) = j\omega\varepsilon_b \chi(\mathbf{r}) \mathbf{E}_1(\mathbf{r}) - k_b^2 \chi(\mathbf{r}) \int_{\Omega} \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r} \in \Omega$$

internal or object equation

Contrast Source Integral Equation (CS)

$$k_b^2 = \omega^2 \varepsilon_b \mu_b$$

Scattering equations



Ω : investigated domain

Γ : measurement domain

$$V = V_1 \cup V_2$$

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) - \omega^2 \varepsilon_b \mu_0 \int_{\Omega} \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r} \in V = V_1 \cup \Omega$$

$$\chi(\mathbf{r}) = \begin{cases} 0 & \mathbf{r} \in V_1 \\ \frac{\varepsilon_s(\mathbf{r}) - \varepsilon_b}{\varepsilon_b} & \mathbf{r} \in V_2 \end{cases}$$



$$\mathbf{E}_s(\mathbf{r}_m) = -k_b^2 \int_{V_2} \overline{\mathbf{G}}(\mathbf{r}_m, \mathbf{r}) \chi(\mathbf{r}) \mathbf{E}(\mathbf{r}) d\mathbf{r} \quad \mathbf{r}_m \in V_1$$

external or data equation

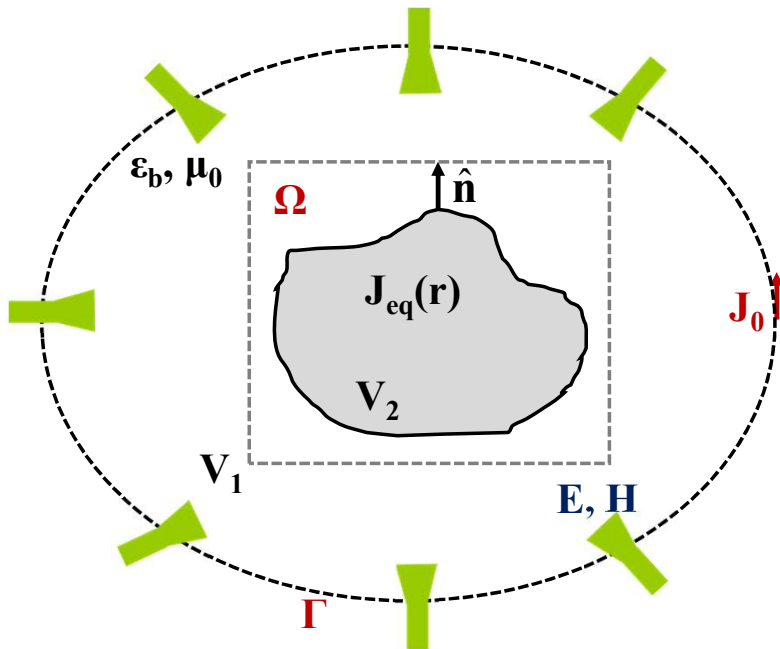
$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) - k_b^2 \int_{V_2} \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r} \in V_2$$

internal or object equation

Electric Field Integral Equation (EFIE)

$$k_b^2 = \omega^2 \varepsilon_b \mu_b$$

Inverse scattering problem



Ω : investigated domain

Γ : measurement domain

$\mathbf{E}(\mathbf{r})$ is assumed to be measurable only for \mathbf{r} in V_1
 the problem can be stated as the determination of
 the unknown e.m. parameters in V_2

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_i(\mathbf{r}) + \mathbf{E}_s(\mathbf{r}) \quad \mathbf{r} \in V_1$$

measured known

$$\mathbf{E}_s(\mathbf{r}_m) = -k_b^2 \int_{V_2} \overline{\mathbf{G}}(\mathbf{r}_m, \mathbf{r}) \chi(\mathbf{r}) \mathbf{E}(\mathbf{r}) d\mathbf{r} \quad \mathbf{r}_m \in V_1$$

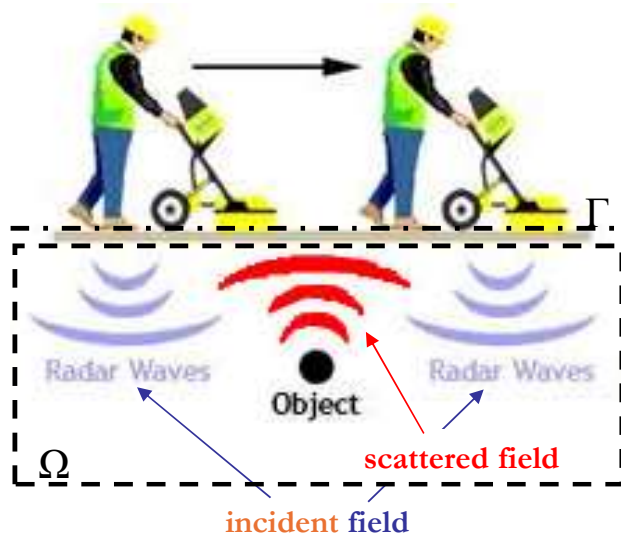
external or data equation

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) - k_b^2 \int_{V_2} \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r} \in V_2$$

internal or object equation

Electric Field Integral Equation (EFIE)

Inverse scattering problem



ideal case

- the transmitting antenna is a line source
- the background is a homogeneous medium, whose e.m. features are known



scattering model

$$E_s(\mathbf{r}_m) = k_b^2 \int_{\Omega} \mathbf{G}(\mathbf{r}_m - \mathbf{r}') \chi(\mathbf{r}') E(\mathbf{r}') d\mathbf{r}' = \boxed{A_e[\chi E]} \quad \mathbf{r}_m \in \Gamma$$

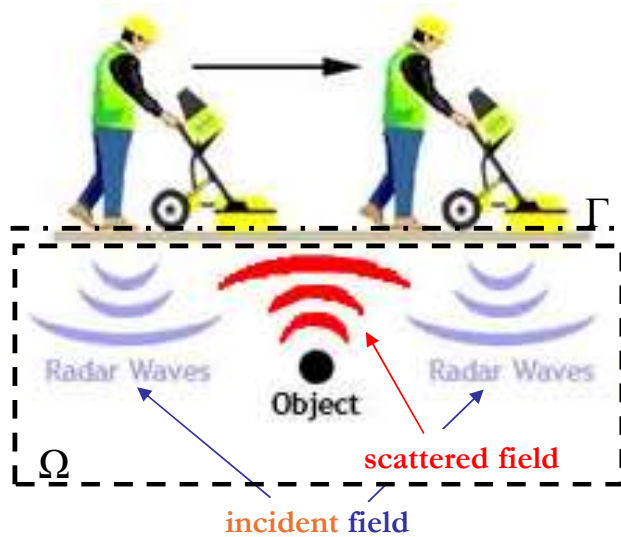
external radiation operator

$$E(\mathbf{r}) - E_{inc}(\mathbf{r}) = k_b^2 \int_{\Omega} \mathbf{G}(\mathbf{r} - \mathbf{r}') \chi(\mathbf{r}') E(\mathbf{r}') d\mathbf{r}' = \boxed{A_i[\chi E]} \quad \mathbf{r} \in \Omega$$

internal radiation operator

Imaging is faced as an inverse scattering problem where the electromagnetic anomalies (targets) are reconstructed by processing the backscattered field

Inverse scattering problem



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- the transmitting antenna is a line source
- the background is a homogeneous medium, whose e.m. features are known



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$$E_s(\mathbf{r}_m) = k_b^2 \int_{\Omega} \mathbf{G}(\mathbf{r}_m - \mathbf{r}') \chi(\mathbf{r}') E(\mathbf{r}') d\mathbf{r}' = \boxed{A_e[\chi E]} \quad \mathbf{r}_m \in \Gamma$$

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internal radiation operator

ill-posed and non-linear problem

$$\boxed{\tilde{\mathbf{E}}_s} = \mathbf{A}_e \left[(\mathbf{I} - \mathbf{A}_i \chi)^{-1} \mathbf{E}_{inc} \right]$$

gathered data

unknown

Inverse scattering problem: ill-posedness

Hadamard's definition (1902, 1923): **a problem is well posed if its solution exists, is unique, and depends continuously on the data (i.e. a small perturbation of the data results in a small perturbation of the solution)**
if one of these conditions is not satisfied, the problem is called ill-posed

general problem: finding f in X , given g in Y , such that

$$Af = g$$

A is an operator mapping elements of the normed space X into elements of the normed space Y

the well-posedness depends essentially on the properties of the operator A

the problem turns out to be well posed if the operator A is bijective (i.e., injective and surjective) and the inverse operator A^{-1} is continuous.

$$f = A^{-1} g$$

- surjectivity guarantees that there is a solution f for any g (existence).
- injectivity ensures that for any g for which the solution exists, such a solution is unique (uniqueness)
- continuity of A^{-1} ensures that the solution depends continuously on the data (stability).

Inverse scattering problem: ill-posedness

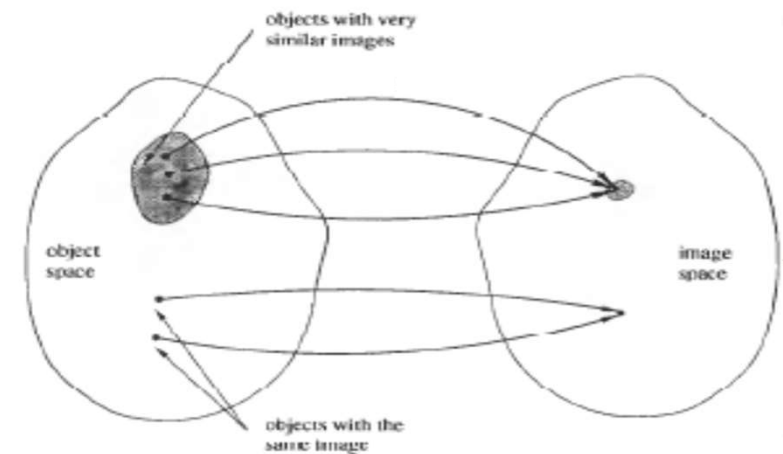
the most critical aspect of an inverse problem is usually its ill-posedness

$$\tilde{\mathbf{E}}_s = \mathbf{A}_e \left[(\mathbf{I} - \mathbf{A}_i \chi)^{-1} \mathbf{E}_{inc} \right]$$

the scattering phenomena involves a loss of information
(the scattering operator \mathbf{A}_e acts as a filter)

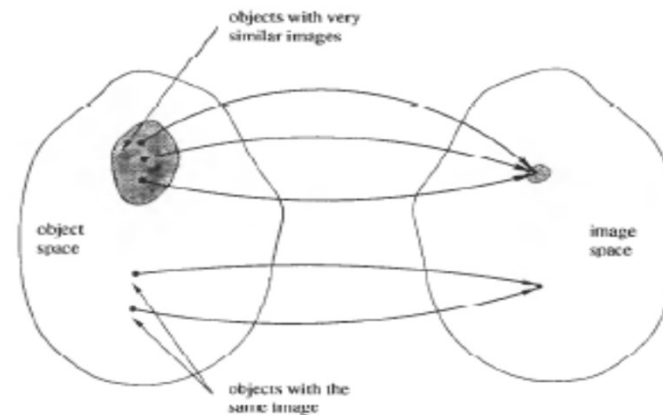
in imaging applications, one measures the scattered field and tries to obtain information on the object subjected to the incident radiation

- ❑ for a given set of measurement values (real data can be affected by noise or measurement errors) → there is no object that produce the prescribed field distribution = the problem lacks in existence
- ❑ two or more different objects produce the same measured data → the problem solution is not unique
- ❑ two very similar sets of measurements are generated by two significantly different objects → the problem solution does not depend continuously on the data = small errors in the measurements result in large errors in the solution



Inverse scattering problem: ill-posedness

regularization procedures are useful tools in controlling ill-posedness.



Applying a regularization procedure means replacing the original ill - posed problem with another well - posed problem, in which some additional information can be added. From this new problem one expects to obtain an approximate solution of the original problem. However, adding further information requires some knowledge of the behavior of the solution to the original problem. This information is usually called a priori information and can be related, in imaging applications, to the physical nature of the body to be inspected, such as its spatial extension, and/or to the noise level of the measured data.

Inverse scattering problem: non-linearity

$$\tilde{\mathbf{E}}_s = \mathbf{A}_e \left[(\mathbf{I} - \mathbf{A}_i \chi)^{-1} \mathbf{E}_{inc} \right]$$

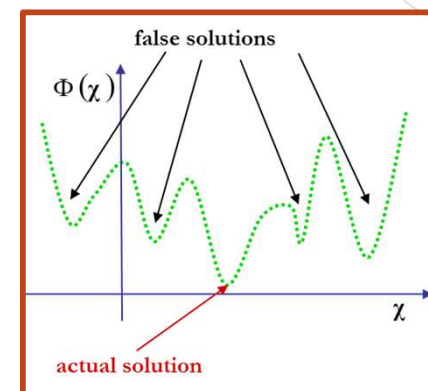
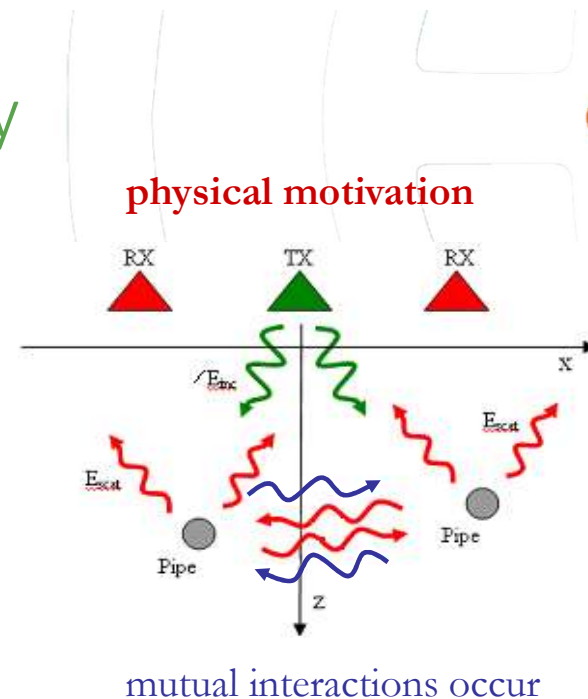
the data-to-unknowns relationship is nonlinear



$$\arg \min \Phi(\chi) = \left\| \mathbf{A}_e \left[(\mathbf{I} - \mathbf{A}_i \chi)^{-1} \mathbf{E}_{inc} \right] - \tilde{\mathbf{E}}_s \right\|^2$$

the imaging is cast as an **optimization problem**

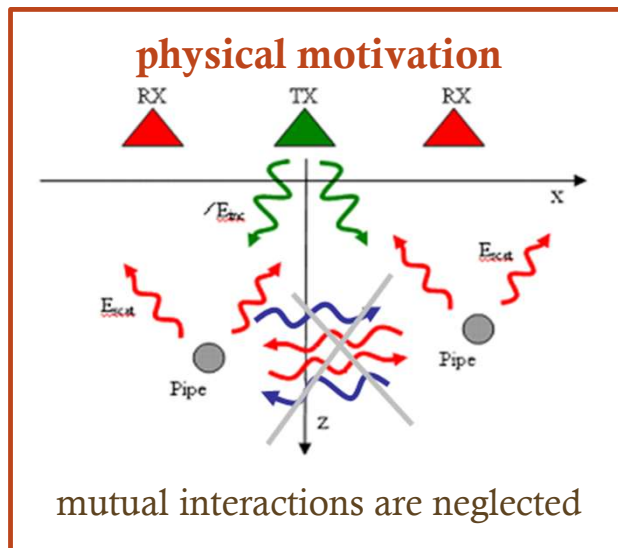
- global optimization approaches
- local optimization strategies



Inverse scattering problem: non-linearity

APPROXIMATE MODEL

- ❑ **Born Approximation** based inversion (penetrable objects)
- ❑ Kirchhoff Approximation based inversion (metallic objects)



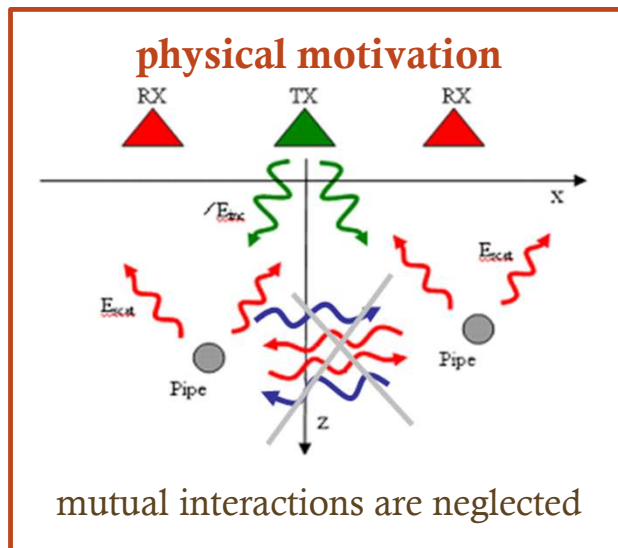
~~$$E(\mathbf{r}) = E_{inc}(\mathbf{r}) + k_b^2 \int_{\Omega} \mathbf{G}(\mathbf{r} - \mathbf{r}') \chi(\mathbf{r}') E(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r} \in \Omega$$~~

$$E_s(\mathbf{r}_m) = k_b^2 \int_{\Omega} \mathbf{G}(\mathbf{r}_m - \mathbf{r}') \chi(\mathbf{r}') E_{inc}(\mathbf{r}') d\mathbf{r}' = L[\chi] \quad \mathbf{r}_m \in \Gamma$$

Inverse scattering problem: non-linearity

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$$E_s(\mathbf{r}_m) = k_b^2 \int_{\Omega} \mathbf{G}(\mathbf{r}_m - \mathbf{r}') \chi(\mathbf{r}') E_{inc}(\mathbf{r}') d\mathbf{r}' = L[\chi] \quad \mathbf{r}_m \in \Gamma$$

quantitative reconstructions are possible only for weak scatterers

reliable qualitative (location, size) information are achieved by using data diversity

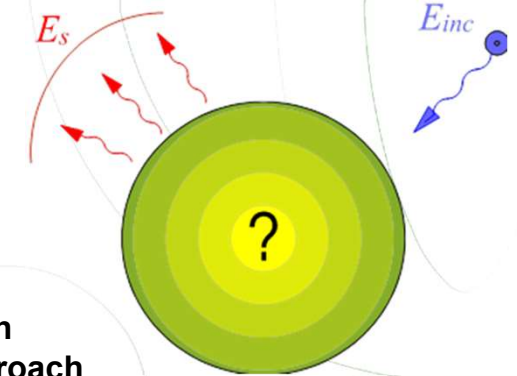
Microwave tomography

IMAGING PROBLEM SOLUTION

Imaging is faced as an inverse scattering problem where the electromagnetic anomalies (targets) are reconstructed by processing the backscattered field

$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

**Born Approximation
based imaging approach**



Microwave tomography

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$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

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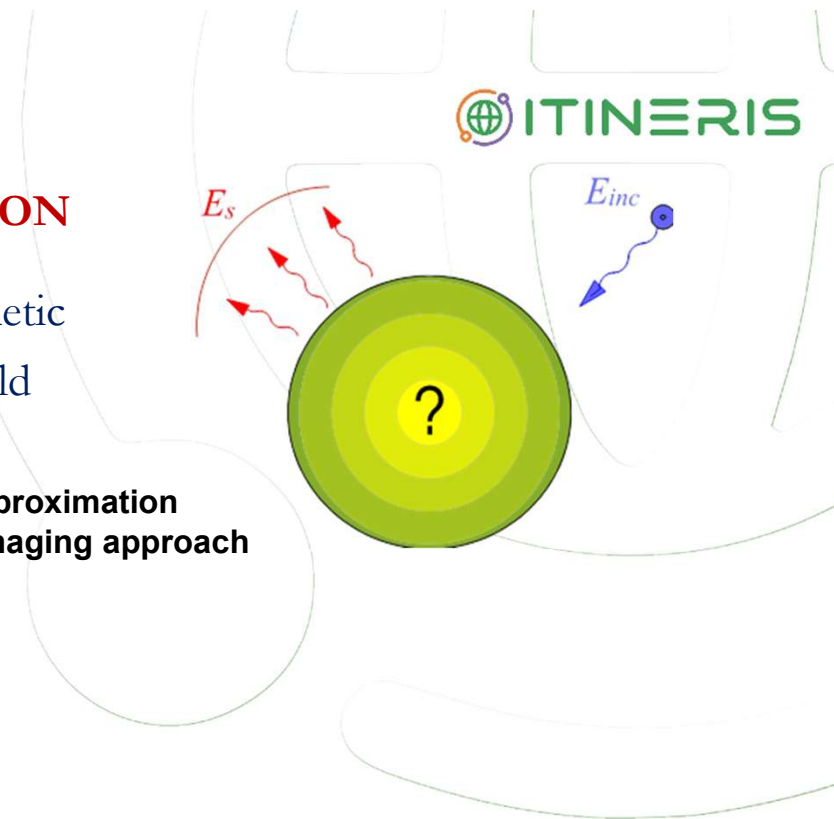


Discretization of the integral equation (MoM)

let $\hat{\mathbf{E}}_s$ be the $(M \times F)$ -dimensional data vector, M and F being the number of work measurement points and frequencies, respectively, the imaging is faced as the solution of the linear system:

$$\hat{\mathbf{E}}_s = \mathbf{L}_{BG}[\chi]$$

where \mathbf{L}_{BG} is the $(M \times F) \times N$ dimensional matrix, N being the number of pixels in Ω , which relates the N -dimensional unknown vector χ to the data vector $\hat{\mathbf{E}}_s$, i.e. it is the discretized version of integral operator



Microwave tomography

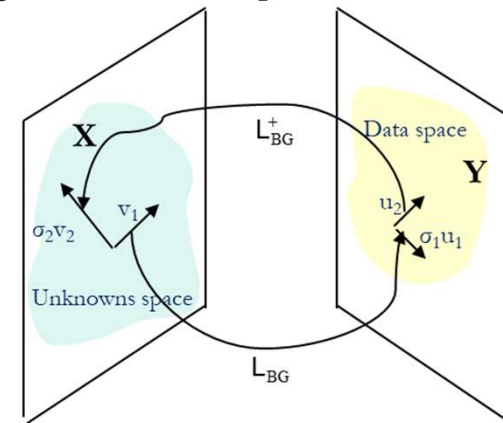
IMAGING PROBLEM SOLUTION

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$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

Born Approximation based imaging approach

Singular Value Decomposition as a solution tool



- $\{v_n\}$ basis vectors to expand the unknowns
- $\{\sigma_n\}$ singular values ordered in a non increasing way
- $\{u_n\}$ basis vectors to expand the data

Discretization of the integral equation (MoM)

Singular Value Decomposition

$$L_{BG} : U\Sigma V^+$$

Microwave tomography

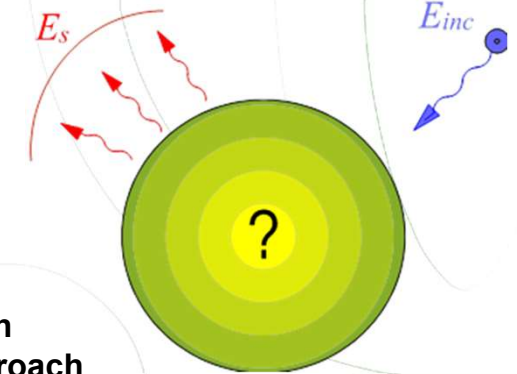
IMAGING PROBLEM SOLUTION

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Discretization of the integral equation (MoM)

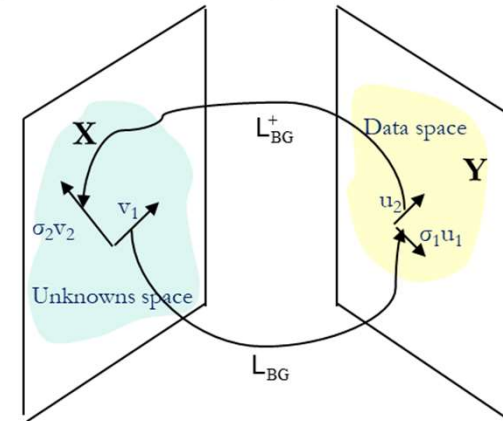
Singular Value Decomposition

$$\mathbf{L}_{BG} : \mathbf{U}\Sigma\mathbf{V}^+$$

$$\mathbf{L}_{BG} \mathbf{v}_n = \sigma_n \mathbf{u}_n$$

$$\mathbf{L}_{BG}^+ \mathbf{u}_n = \sigma_n \mathbf{v}_n$$

σ_n dictates the strength of the image of the basis vectors



- $\{\mathbf{v}_n\}$ basis vectors to expand the unknowns
- $\{\sigma_n\}$ singular values ordered in a non increasing way
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Microwave tomography

IMAGING PROBLEM SOLUTION

Imaging is faced as an inverse scattering problem where the electromagnetic anomalies (targets) are reconstructed by processing the backscattered field

$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$



Discretization of the integral equation (MoM)



Singular Value Decomposition

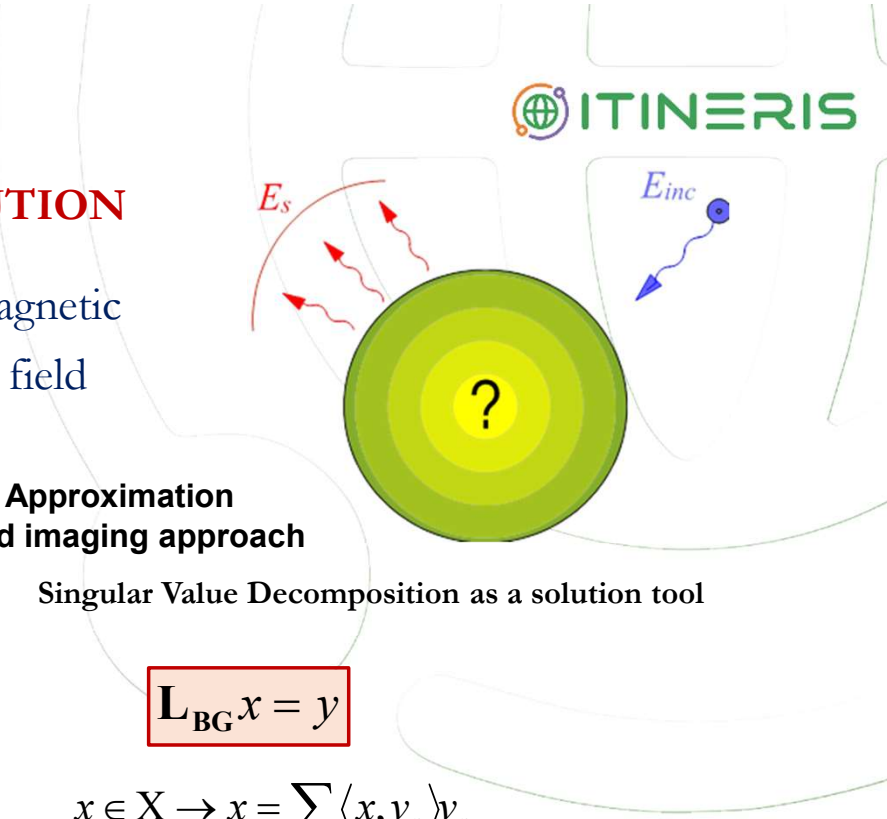
$$\mathbf{L}_{BG} : \mathbf{U}\Sigma\mathbf{V}^+$$

$$\mathbf{L}_{BG} \mathbf{x} = \mathbf{y}$$

$$\mathbf{x} \in \mathbf{X} \rightarrow \mathbf{x} = \sum_n \langle \mathbf{x}, \mathbf{v}_n \rangle \mathbf{v}_n$$

$$\mathbf{y} \in \mathbf{Y} \rightarrow \mathbf{y} = \sum_n \langle \mathbf{y}, \mathbf{u}_n \rangle \mathbf{u}_n$$

$$\mathbf{L}_{BG} \mathbf{x} = \sum_n \sigma_n \langle \mathbf{x}, \mathbf{v}_n \rangle \mathbf{u}_n = \mathbf{y} = \sum_n \langle \mathbf{y}, \mathbf{u}_n \rangle \mathbf{u}_n$$



Born Approximation based imaging approach

Singular Value Decomposition as a solution tool

Microwave tomography

IMAGING PROBLEM SOLUTION

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$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

Born Approximation based imaging approach

Singular Value Decomposition as a solution tool

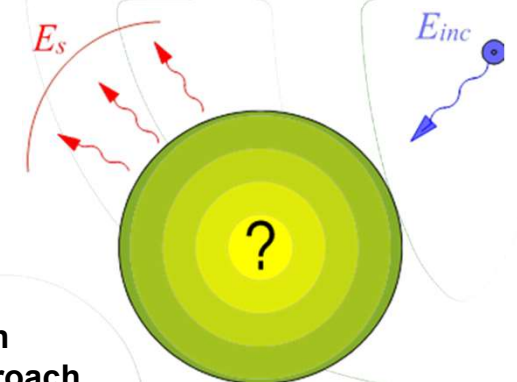
$$\mathbf{L}_{BG} x = y$$

$$x \in X \rightarrow x = \sum_n \langle x, v_n \rangle v_n$$

$$y \in Y \rightarrow y = \sum_n \langle y, u_n \rangle u_n$$

$$\mathbf{L}_{BG} x = \sum_n \underbrace{\sigma_n \langle x, v_n \rangle}_{\text{}} u_n = y = \sum_n \underbrace{\langle y, u_n \rangle}_{\text{}} u_n$$

$$\langle x, v_n \rangle = \sum_n \frac{1}{\sigma_n} \langle y, u_n \rangle$$



Discretization of the integral equation (MoM)

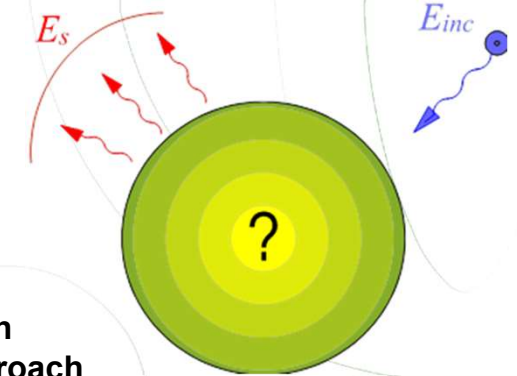
Singular Value Decomposition

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Microwave tomography

IMAGING PROBLEM SOLUTION

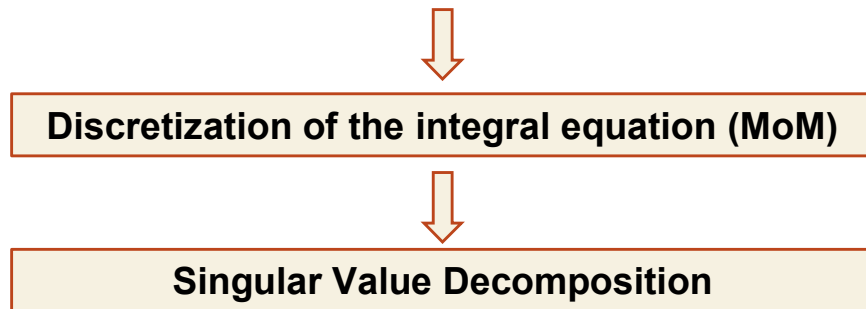
Imaging is faced as an inverse scattering problem where the electromagnetic anomalies (targets) are reconstructed by processing the backscattered field



$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

Born Approximation based imaging approach

Singular Value Decomposition as a solution tool



$$\mathbf{L}_{BG} \mathbf{x} = \mathbf{y}$$

$$\mathbf{x} \in X \rightarrow \mathbf{x} = \sum_n \langle \mathbf{x}, \mathbf{v}_n \rangle \mathbf{v}_n$$

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$$\mathbf{L}_{BG} : U \Sigma V^+$$

$$\mathbf{L}_{BG} \mathbf{x} = \sum_n \sigma_n \langle \mathbf{x}, \mathbf{v}_n \rangle \mathbf{u}_n = \mathbf{y} = \sum_n \langle \mathbf{y}, \mathbf{u}_n \rangle \mathbf{u}_n$$

$$\mathbf{x} = \sum_n \frac{1}{\sigma_n} \langle \mathbf{y}, \mathbf{u}_n \rangle \mathbf{v}_n$$

$$\langle \mathbf{x}, \mathbf{v}_n \rangle = \sum_n \frac{1}{\sigma_n} \langle \mathbf{y}, \mathbf{u}_n \rangle$$

Microwave tomography

IMAGING PROBLEM SOLUTION

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$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

Born Approximation
based imaging approach

Singular Value Decomposition as a solution tool

Discretization of the integral equation (MoM)

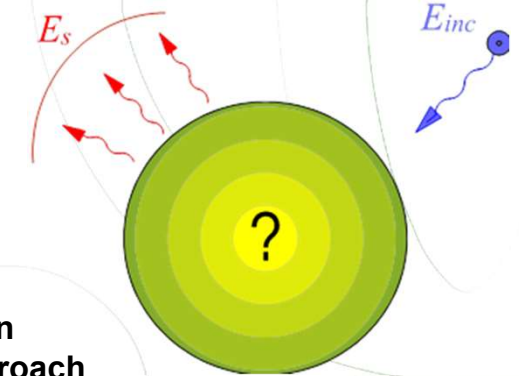
Singular Value Decomposition

Regularised Inversion of the matrix (TSVD)

$$\tilde{\chi} = \sum_{n=1}^P \frac{1}{\sigma_n} \langle \hat{\mathbf{E}}_s, \mathbf{v}_n \rangle \mathbf{u}_n$$

$$P = \min \{ (M \times F), N \}$$

the solution of the imaging is
expressed in a closed form



Microwave tomography

IMAGING PROBLEM SOLUTION

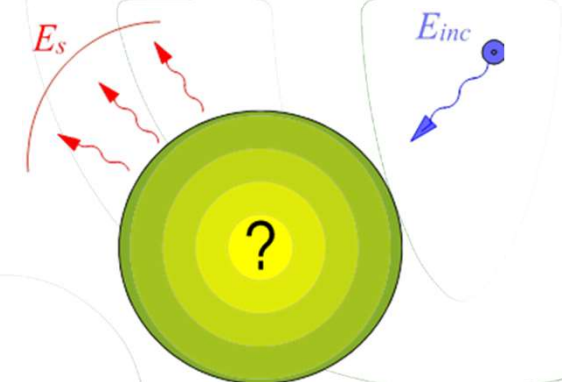
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$$\tilde{\chi} = \sum_{n=1}^P \frac{1}{\sigma_n} \langle \hat{\mathbf{E}}_s, \mathbf{v}_n \rangle u_n$$

$$P = \min \{ (M \times F), N \}$$

the solution of the imaging is expressed in a closed form

pseudo-inverse solution
approximated expression of the
contrast function



the pseudo-inverse solution solves unique-ness and nonexistence problems, but numerical instability may still affect the solution

$$\hat{\mathbf{E}}_s = \mathbf{E}_s + \text{noise} \implies \tilde{\chi} = \sum_{n=1}^Q \frac{1}{\sigma_n} \langle \mathbf{E}_s, \mathbf{v}_n \rangle u_n + \sum_{n=1}^Q \frac{1}{\sigma_n} \langle \text{noise}, \mathbf{v}_n \rangle u_n$$

$\sigma_n \rightarrow 0$ for increasing n \implies the solution can be unstable
(ill posedness)

Microwave tomography

IMAGING PROBLEM SOLUTION

Imaging is faced as an inverse scattering problem where the electromagnetic anomalies (targets) are reconstructed by processing the backscattered field

Noise on data entails the necessity to use a regularization strategy

$$\tilde{\chi} = \sum_{n=1}^T \frac{1}{\sigma_n} \langle \hat{\mathbf{E}}_s, \mathbf{v}_n \rangle u_n$$

T (< Q) is a trade off between the accuracy and stability

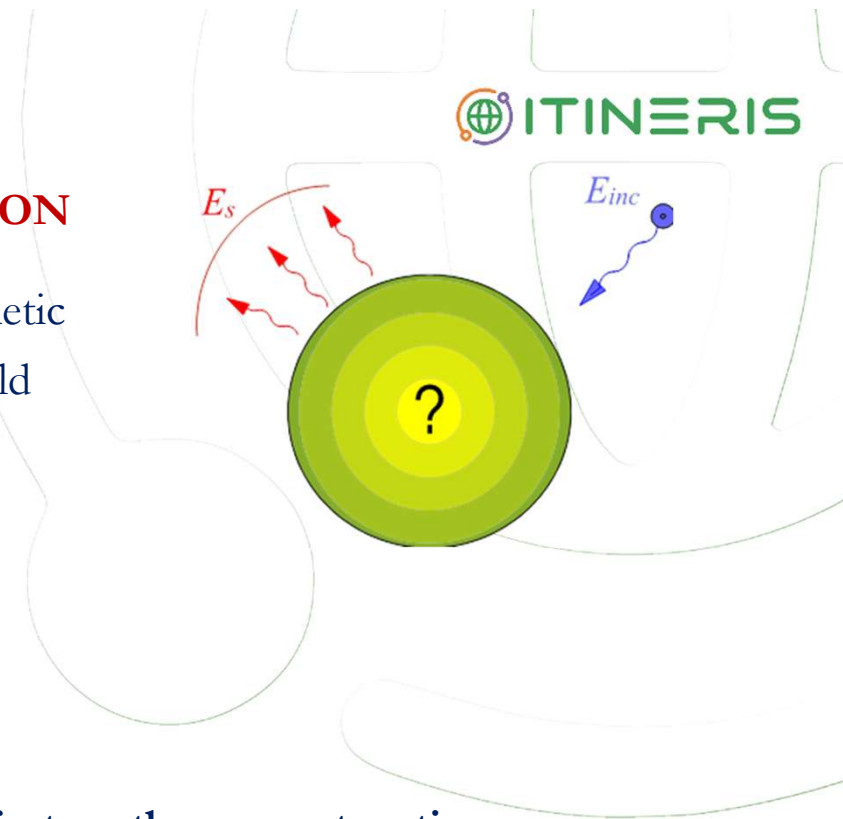
The necessity of a stable solution leads to a filtering effect on the reconstruction

- Low-pass filtering along the measurement direction

Role of the synthetic aperture

- Band-pass filtering along the depth direction

Role of the work frequency band



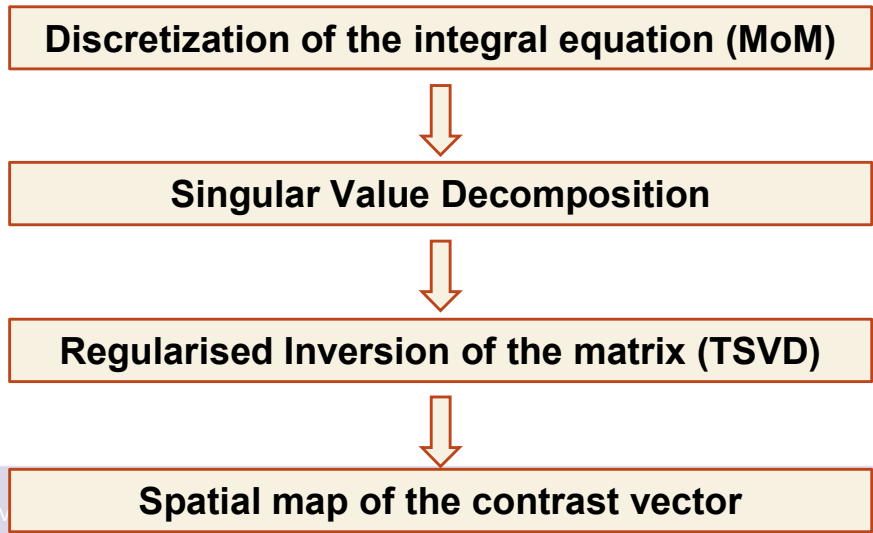
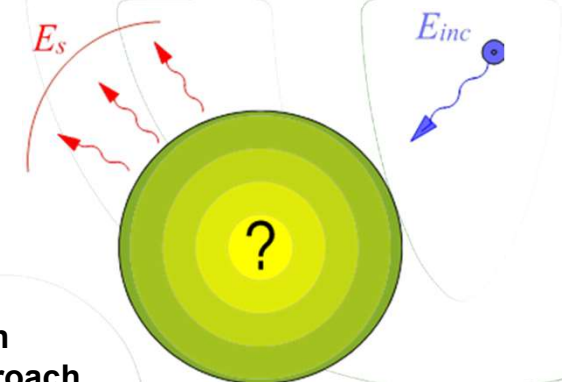
Microwave tomography

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$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

Born Approximation based imaging approach



Singular Value Decomposition as a solution tool

$$\tilde{\chi} = \sum_{n=1}^T \frac{1}{\sigma_n} \langle \hat{\mathbf{E}}_s, \mathbf{v}_n \rangle u_n$$

Overview

Part I

- 🌐 Ground Penetrating Radar (GPR) vs Radar
 - 🌐 similarities and differences
 - 🌐 constitutive properties of materials

- 🌐 GPR measurement configurations

- 🌐 GPR data

Part II

- 🌐 Scattering equations

- 🌐 Inverse scattering problem

- 🌐 definition
- 🌐 non-linearity
- 🌐 ill-posedness

- 🌐 Microwave Tomography

Part III

- 🌐 Microwave tomography: a flexible tool for GPR imaging

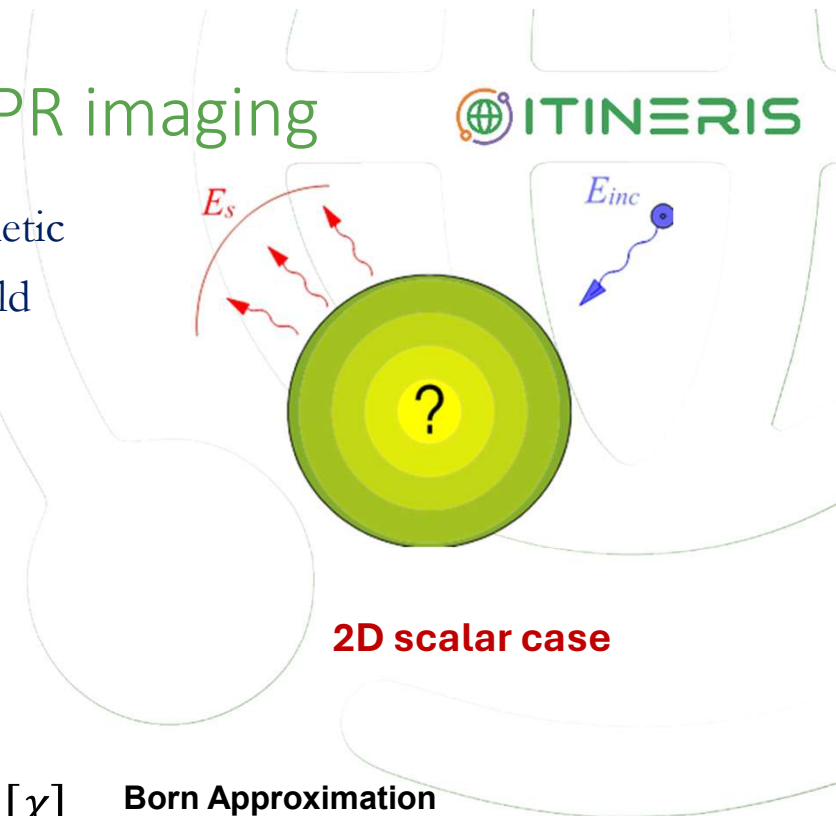
- 🌐 Applicative examples



Microwave tomography: a flexible tool for GPR imaging

Imaging is faced as an inverse scattering problem where the electromagnetic anomalies (targets) are reconstructed by processing the backscattered field

- non magnetic materials
- cylindrical objects of arbitrary cross section
- TM polarization
- multi-bistatic configuration
- **Born approximation** $\Rightarrow J(x,z) = \chi(x,z)E_{inc}(x,z)$
- filamentary line source



$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

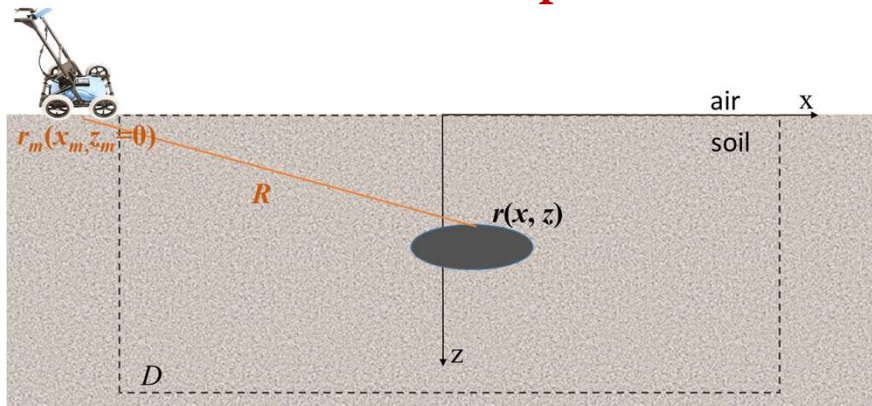
**Born Approximation
based imaging approach**

flexible formulation easily adaptable whatever is the measurement configuration and the reference scenario

Green's function and incident field depend on the adopted scattering model

Microwave tomography: a flexible tool for GPR imaging

in situ setup



the signal propagation occurs into a homogeneous, isotropic, non-magnetic medium

homogeneous scattering model

Born Approximation

$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

electric field radiated in \mathbf{r}_m by a filamentary line source located in \mathbf{r}

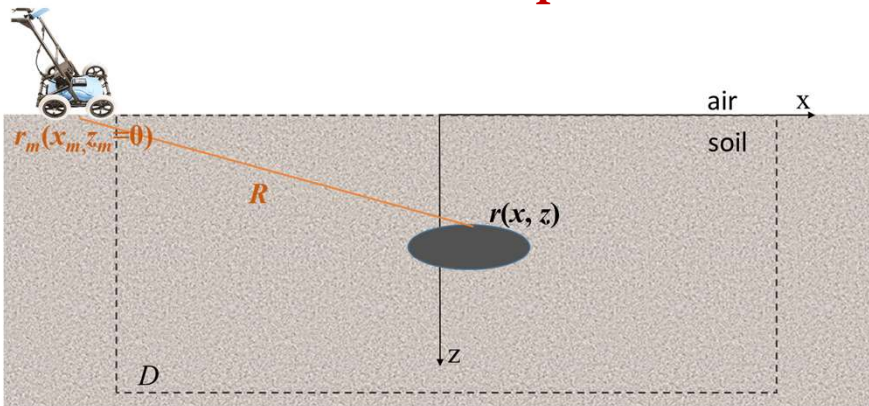
$$g(\mathbf{r}_m, \mathbf{r}, \omega) = \frac{j}{4} H_0^2(k_s R)$$

the transmitting antenna is a line source fed by an electric current having intensity I

$$E_i(\mathbf{r}, \mathbf{r}_m, \omega) = -j\omega\mu_0 I g(\mathbf{r}_m, \mathbf{r}, \omega)$$

Microwave tomography: a flexible tool for GPR imaging

in situ setup



the signal propagation occurs into a homogeneous, isotropic, non-magnetic medium

homogeneous scattering model

Born Approximation

$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

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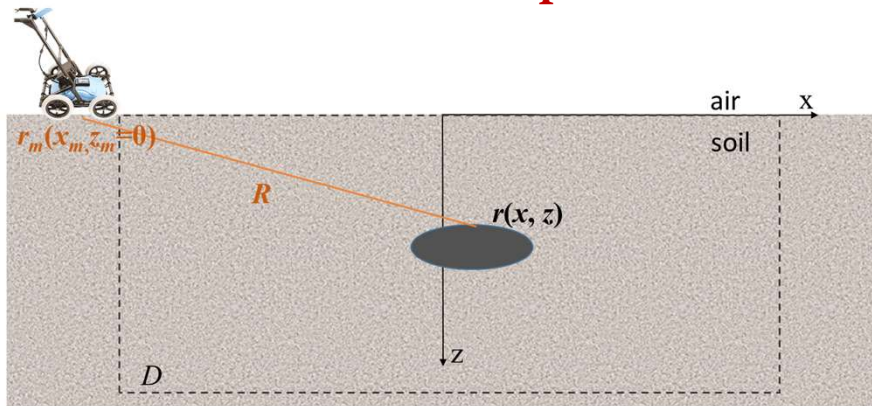
the transmitting antenna is a line source fed by an electric current having intensity I

$$E_i(\mathbf{r}, \mathbf{r}_m, \omega) = -j\omega\mu_0 I g(\mathbf{r}_m, \mathbf{r}, \omega)$$

$$E_s(\mathbf{r}_m, \omega) \propto \int_D [H_0^2(k_s R)]^2 \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

Microwave tomography: a flexible tool for GPR imaging

in situ setup



the signal propagation occurs into a homogeneous, isotropic, non-magnetic medium

homogeneous scattering model

Born Approximation

$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

$$g(\mathbf{r}_m, \mathbf{r}, \omega) = \frac{j}{4} H_0^2(k_s R)$$

$$E_i(\mathbf{r}, \mathbf{r}_m, \omega) = -j\omega\mu_0 I g(\mathbf{r}_m, \mathbf{r}, \omega)$$

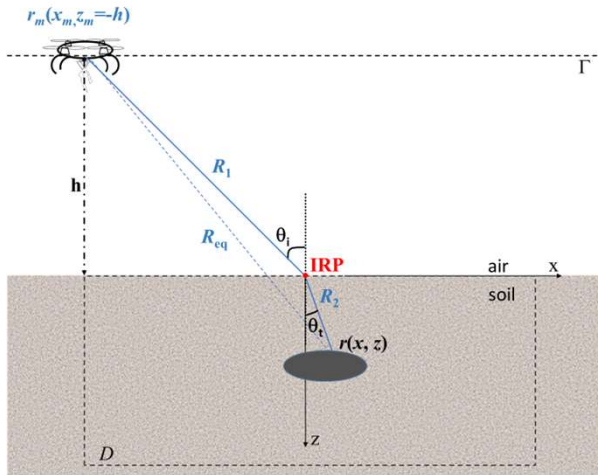
Far-field approximation

$$R = |\mathbf{r}_m - \mathbf{r}| \rightarrow \infty \Rightarrow H_0^2(k_s R) \rightarrow \sqrt{\frac{2j}{\pi R}} e^{-jR}$$

$$E_s(\mathbf{r}_m, \omega) \propto \int_D \chi(\mathbf{r}) e^{-j2R} d\mathbf{r}$$

Microwave tomography: a flexible tool for GPR imaging

contactless setup



the signal propagation occurs into different homogeneous, isotropic, non-magnetic media

half-space scattering model

Born Approximation

$$E_s(\mathbf{r}_m, \omega) \propto \int_D \underbrace{g(\mathbf{r}_m, \mathbf{r}, \omega)}_{\text{Green's function}} \underbrace{E_i(\mathbf{r}, \mathbf{r}_m, \omega)}_{\text{incident field}} \chi(\mathbf{r}) d\mathbf{r} = L[\chi]$$

Spectral domain (SP) model - exact representation

$$E_s(x_m, h, \omega) = \frac{j\omega\mu_0 I k_s^2}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j(u+v)x_m} e^{-j(u+v)x} \times \\ \times \frac{e^{-j(k_{zs}(u) + k_{zs}(v))z} e^{-j(k_{z0}(u) + k_{z0}(v))h}}{(k_{zs}(u) + k_{z0}(u))(k_{zs}(v) + k_{z0}(v))} \chi(x, z) du dv dx dz$$

(plane-wave spectrum expansion of the field radiated by an electric line source)

Ray-based (RB) model - far-field approximation

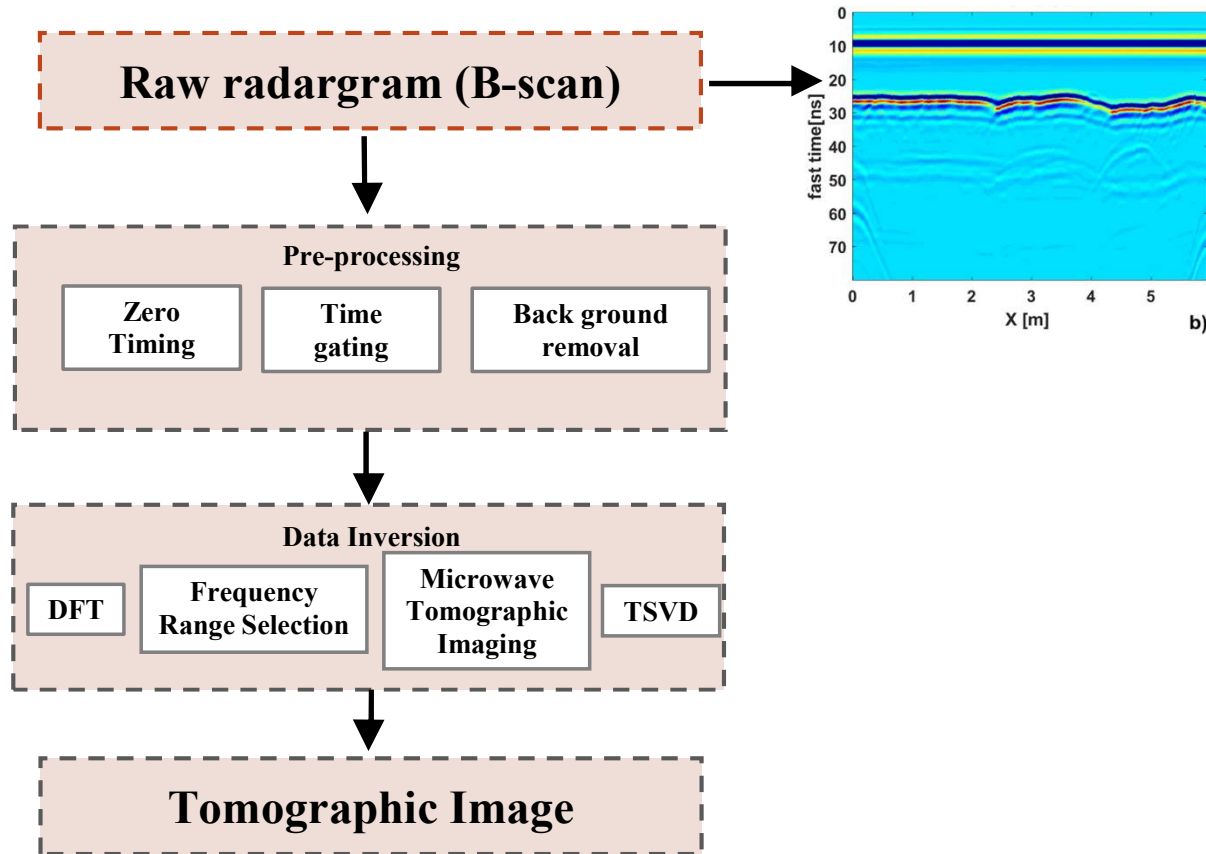
$$E_s(x_m, h, \omega) = \frac{-jk_s^2 \eta_s I}{8\pi} \int \int_D T_{12} T_{21} \frac{e^{-j2k_0(R_1 + n_s R_2)}}{R_1 + R_2} \chi(x, z) dx dz$$

Equivalent permittivity (EP) model - far-field approximation and $x_m \sim x$

$$E_s(x_m, h, \omega) = \frac{-jk_s^2 \eta_s I}{8\pi} \int \int_D T_{12} T_{21} \frac{e^{-j2k_{seq} R}}{R} \chi(x, z) dx dz$$

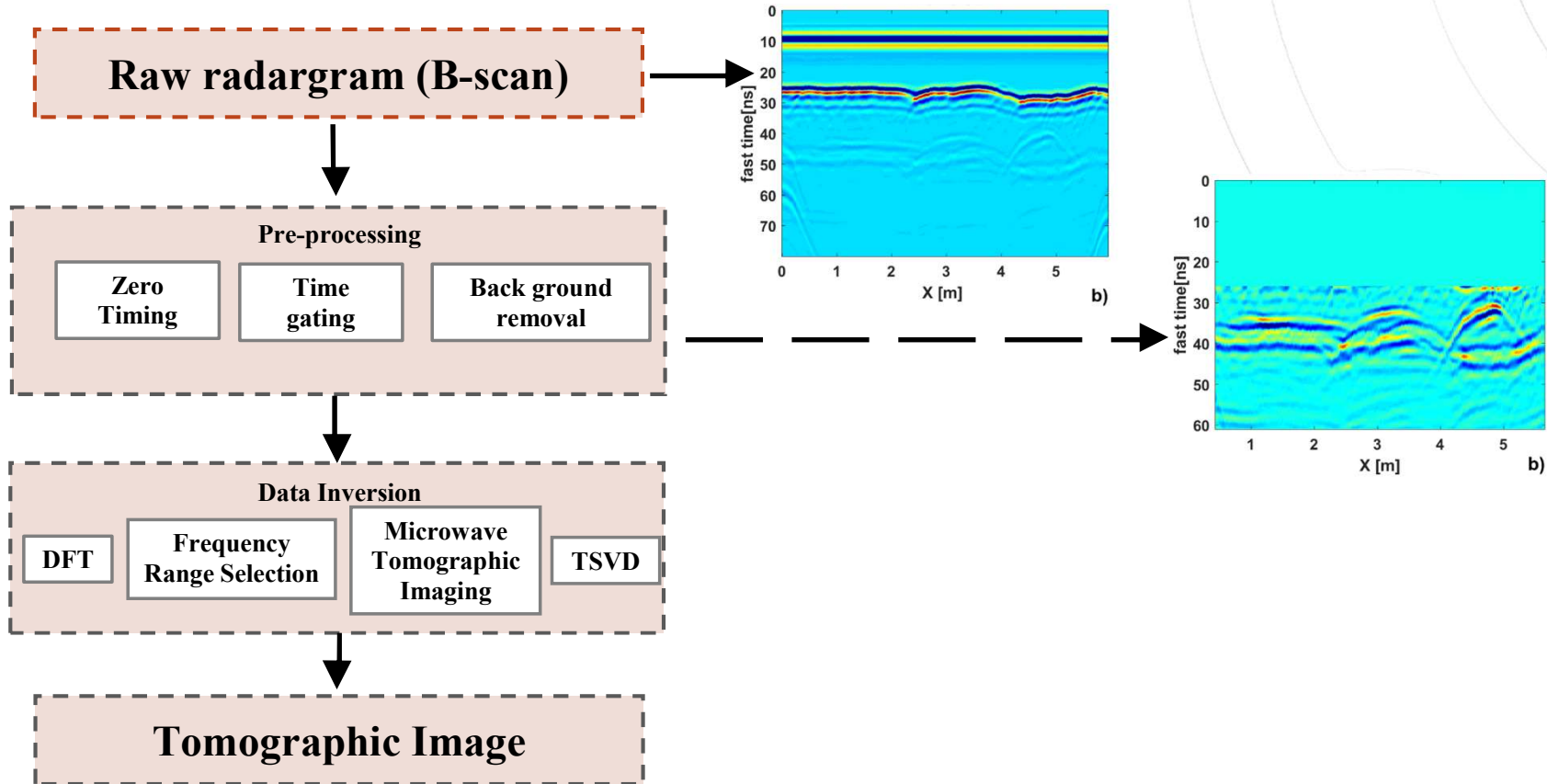
Microwave tomography: a flexible tool for GPR imaging

MWT ENHANCED GPR DATA PROCESSING



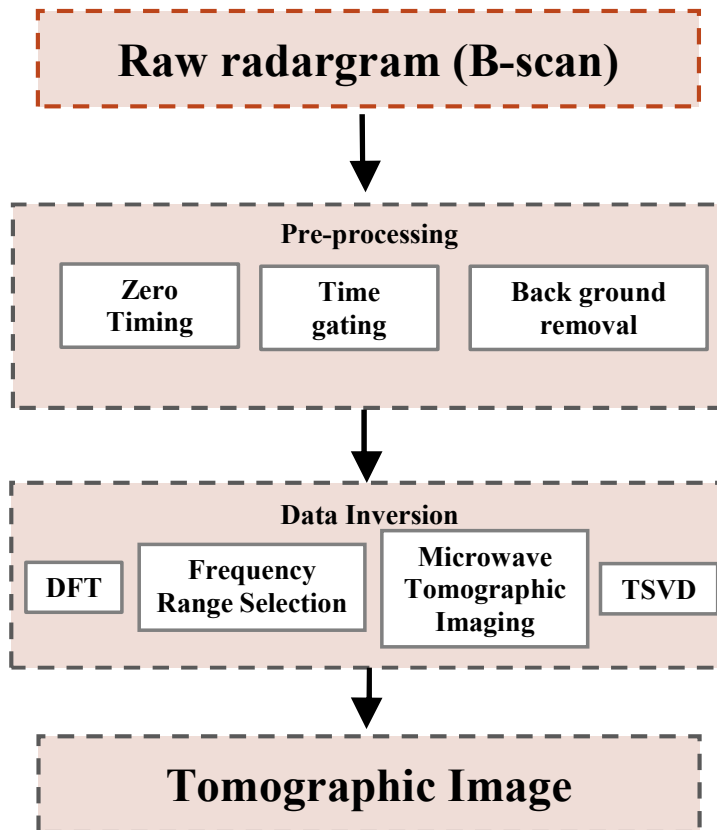
Microwave tomography: a flexible tool for GPR imaging

MWT ENHANCED GPR DATA PROCESSING

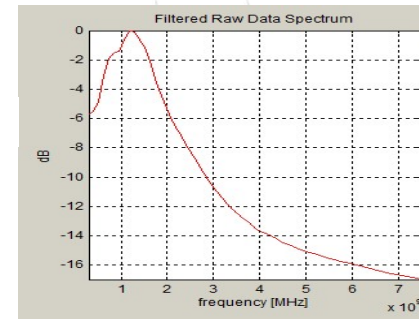


Microwave tomography: a flexible tool for GPR imaging

MWT ENHANCED GPR DATA PROCESSING



Frequency Range Selection

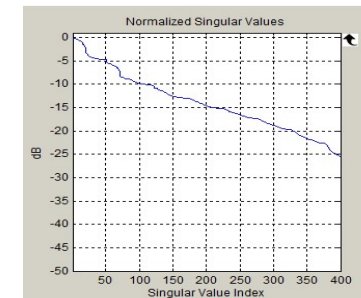


Microwave Tomographic Imaging

$$E_s(x_s, z_s, f) \propto \int_{\Omega} \underbrace{G(x_s, z_s, x', z', f)}_{\text{Green function}} \underbrace{E_{inc}(x', z', f)}_{\text{Incident field}} \chi(x', z') d\Omega = L[\chi]$$

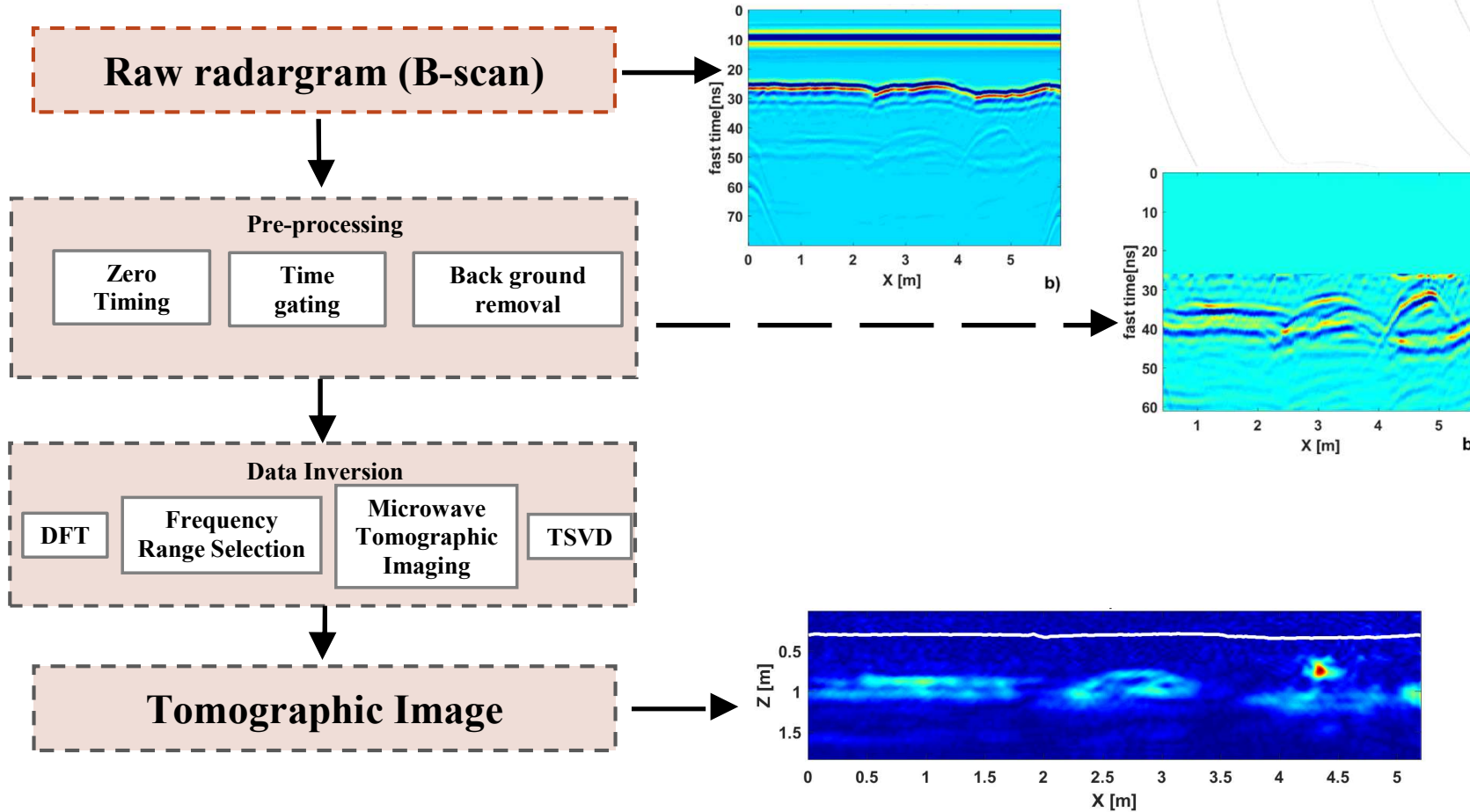
Regularized Inversion of the matrix

$$R\chi = \sum_{n=1}^N \frac{1}{\sigma_n} \langle E_s, v_n \rangle u_n$$



Microwave tomography: a flexible tool for GPR imaging

MWT ENHANCED GPR DATA PROCESSING



Applicative examples

GPR IN SITU SETUP & MWT DATA PROCESSING: APPLICATIONS

Subsoil and vertical structures diagnostic

Urban areas and structural surveys

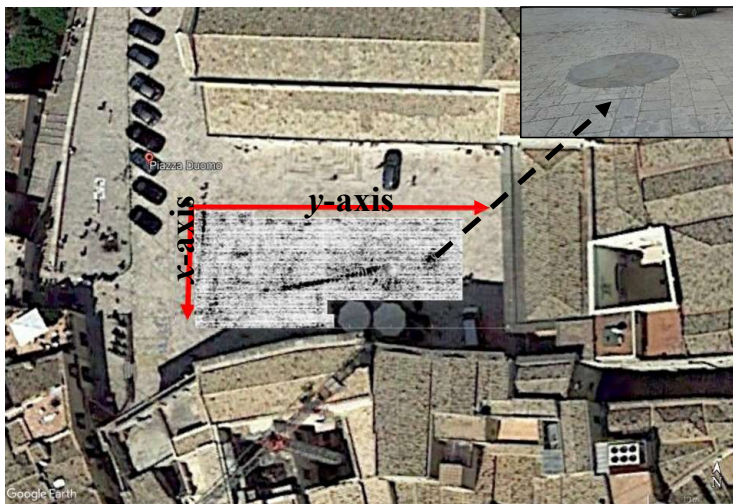
Environment Monitoring (natural and anthropic risks)

Cultural heritage and Archaeological surveys

Applicative examples

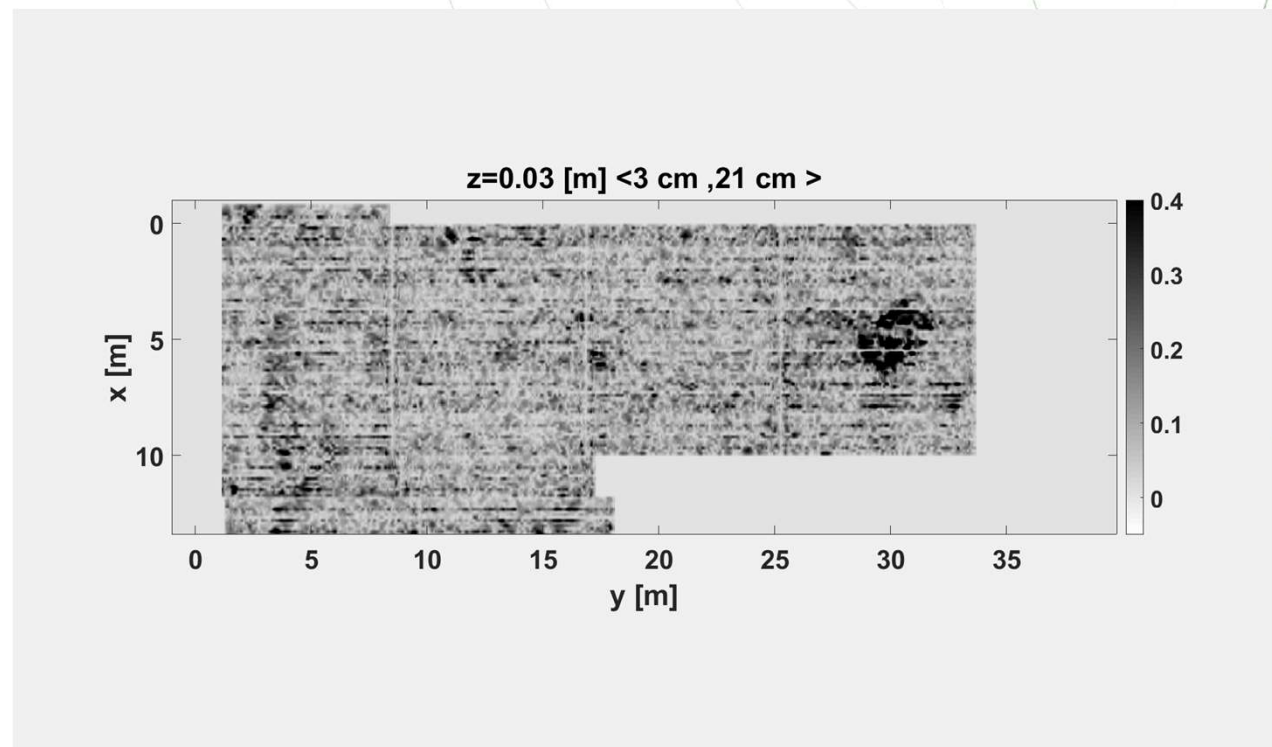
Urban areas and structural surveys

GPR surveys at Matera – Piazza Duomo



Pseudo 3D reconstruction

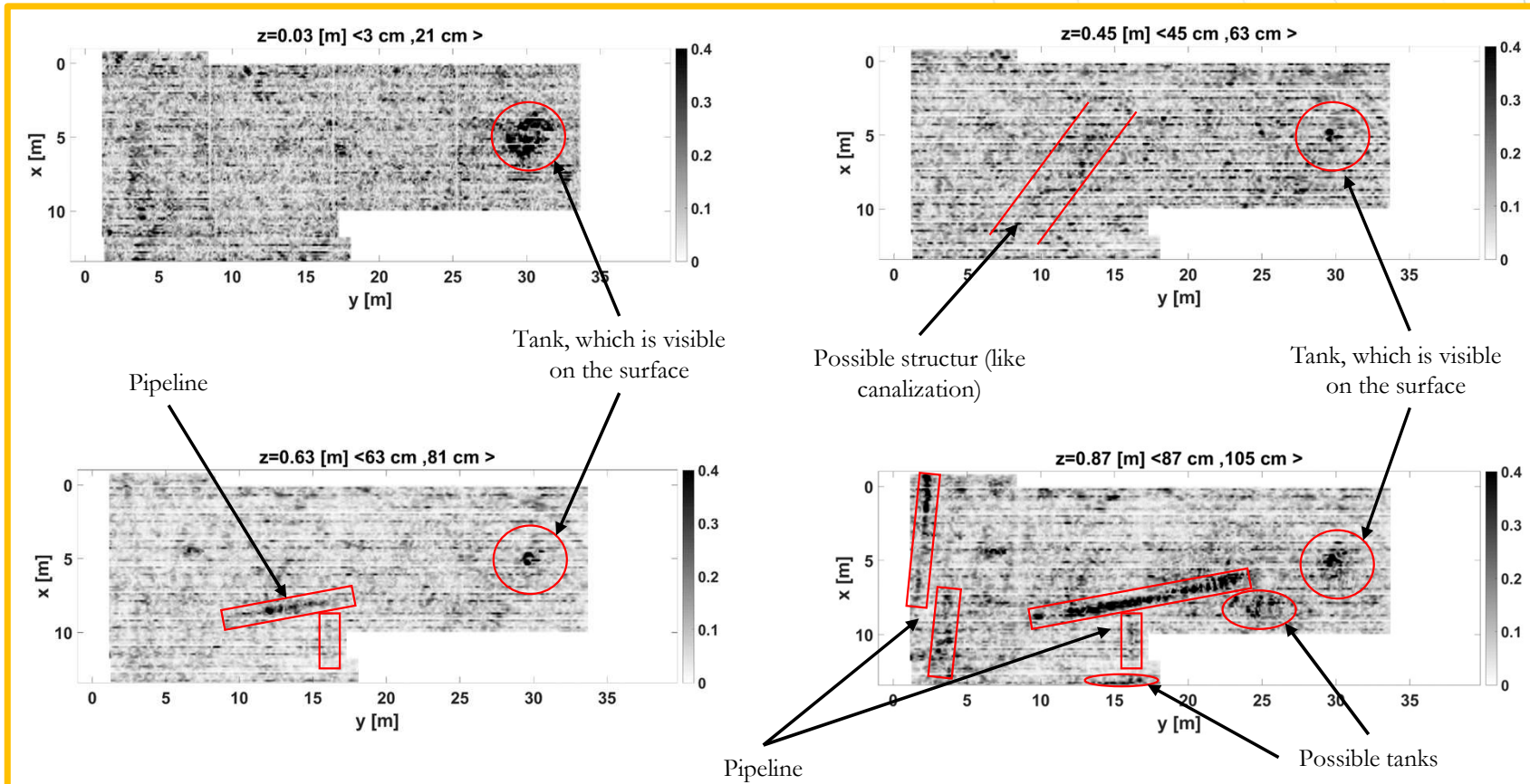
The single 2D profiles have been separately processed and the relative tomographic reconstructions have been joined to generate volumetric representation



Applicative examples

Urban areas and structural surveys

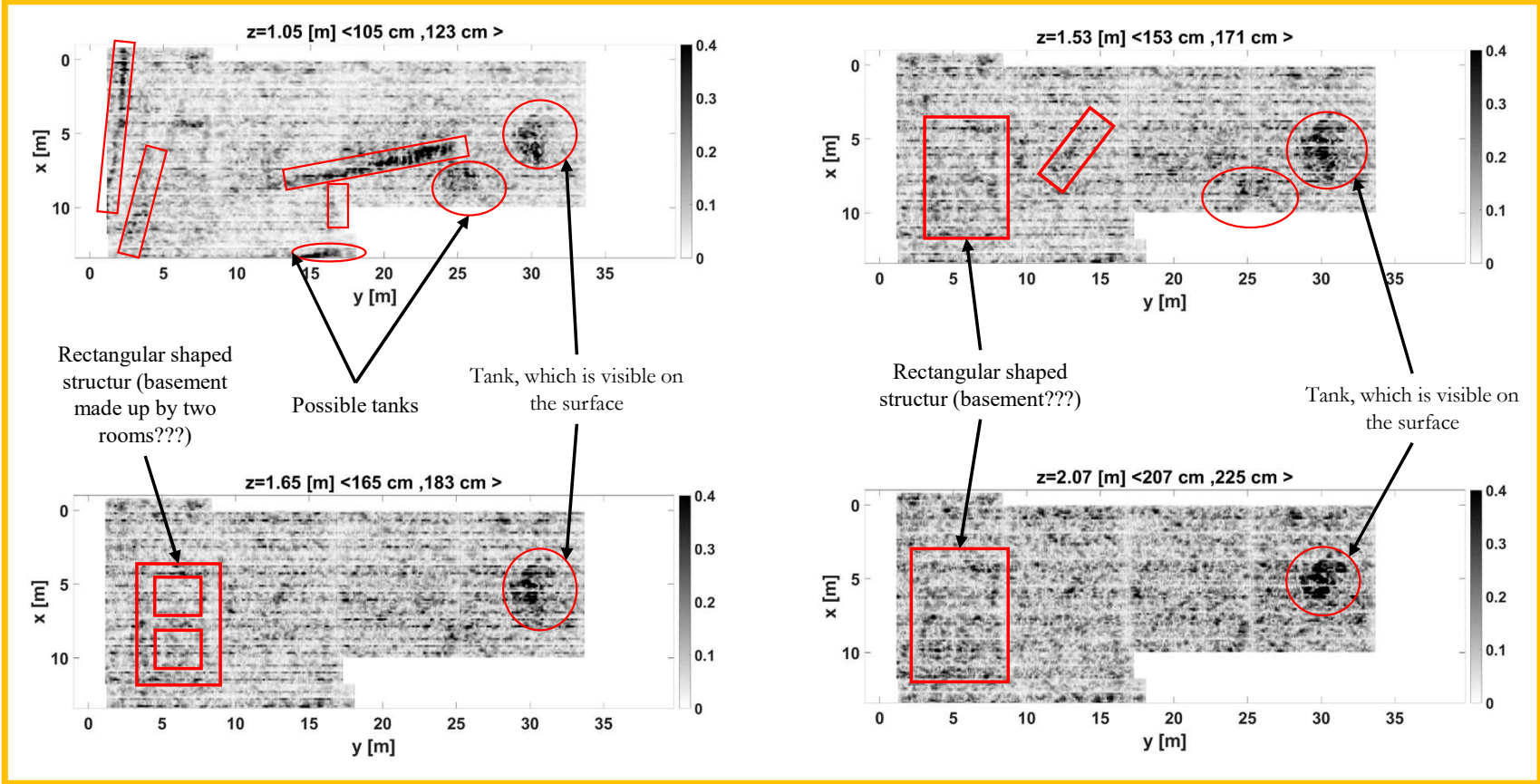
GPR surveys at Matera – Piazza Duomo



Applicative examples

Urban areas and structural surveys

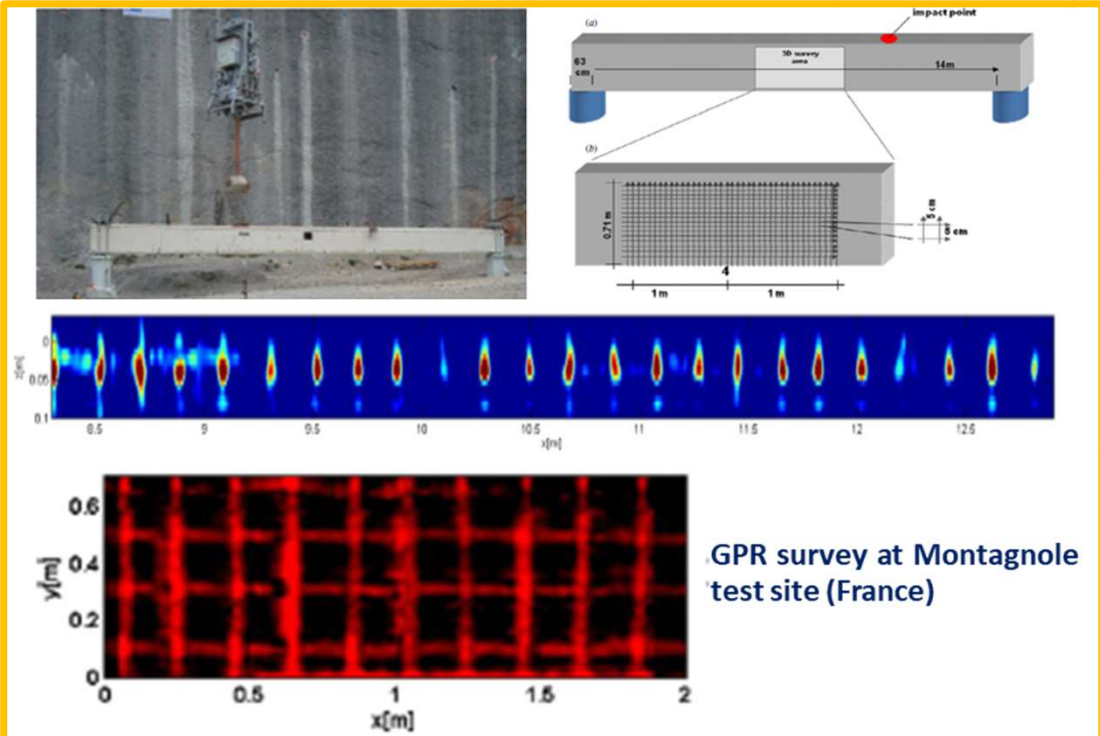
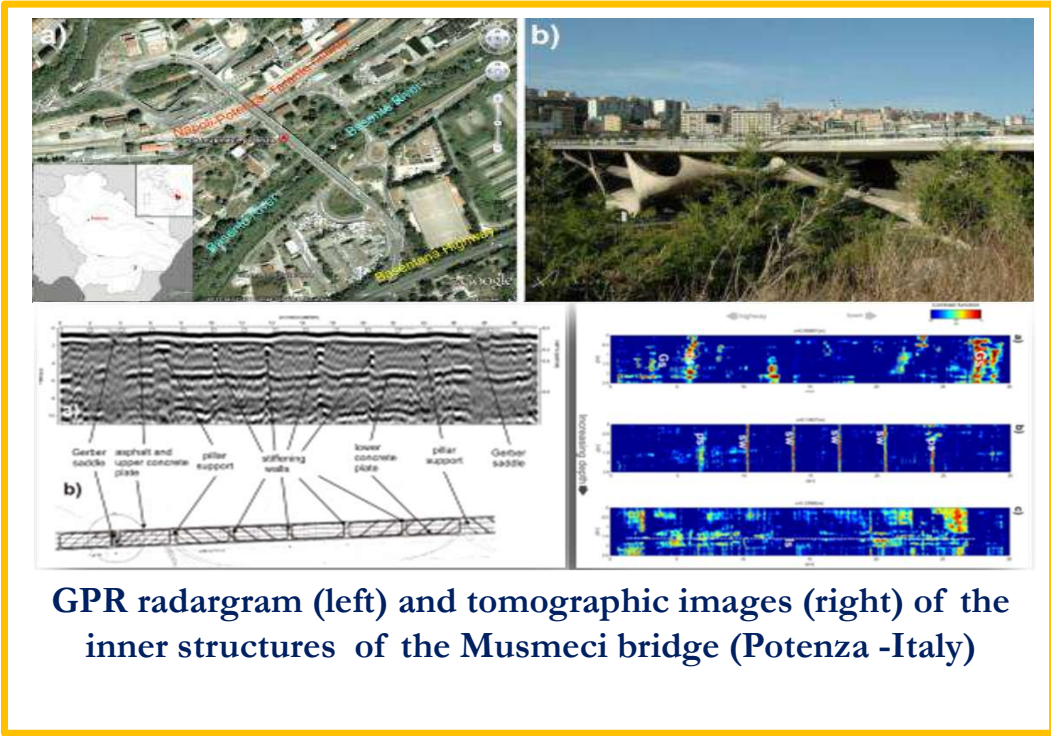
GPR surveys at Matera – Piazza Duomo



Applicative examples

Urban areas and structural surveys

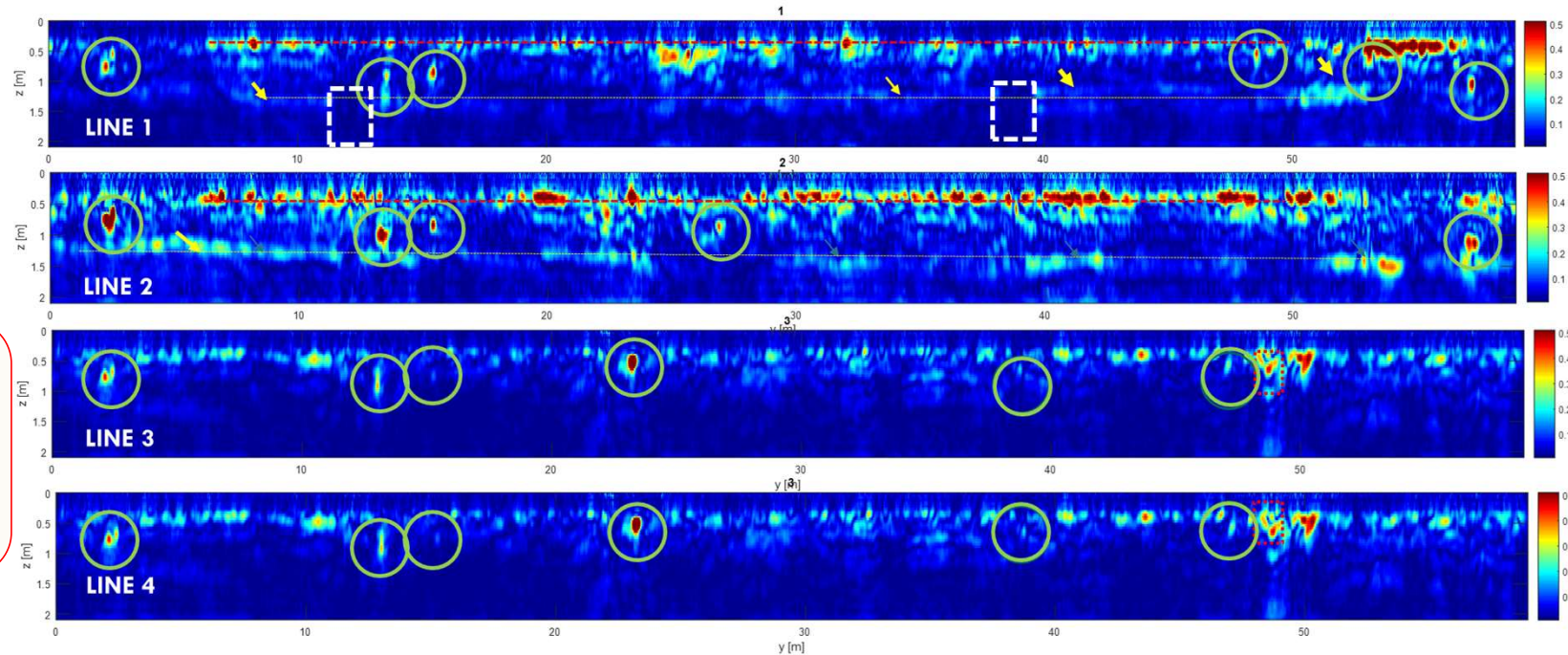
GPR surveys for infrastructure assessment



Applicative examples

Urban areas and structural surveys

GPR surveys for water utility monitoring at Napoli, Soccavo quarter

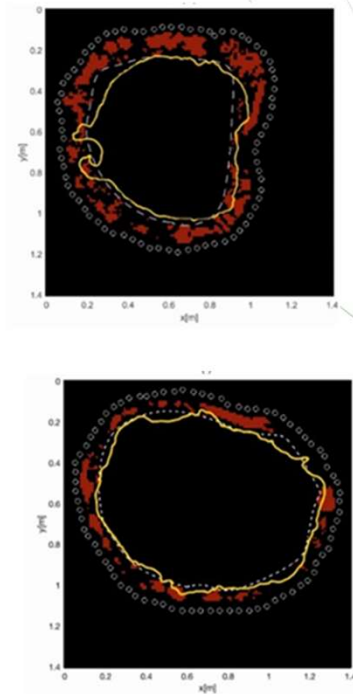


- SIDEWALK
- MEASUREMENT LINE PARALLEL PIPELINE
- MEASUREMENT LINE ORTHOGONAL PIPELINE
- MANHOLE
- ANOMALY

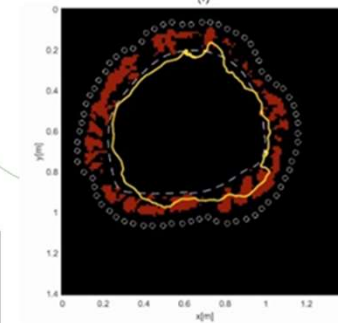
Applicative examples

Environment Monitoring (natural and anthropic risks)

Assessment of the Internal Structure of Hollow Trees



2 GHz GPR data

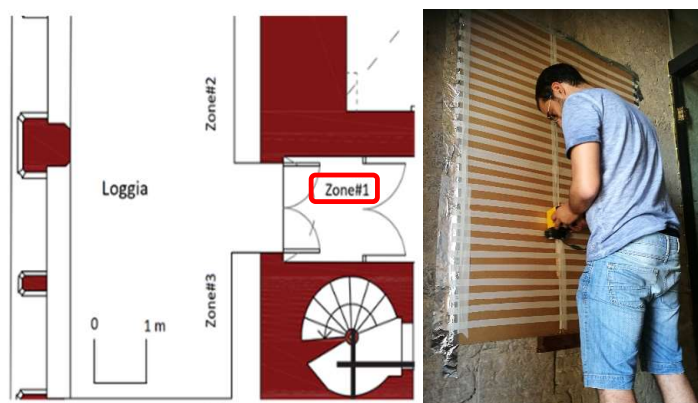


Binary images converted from the tomographic images ($h = 0 \text{ m}, 0.6 \text{ m}, 1.3 \text{ m}$)

Applicative examples

Cultural heritage and Archaeological surveys

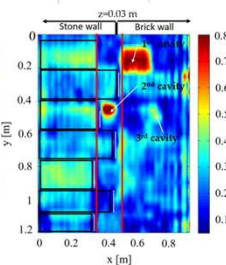
Consoli Palace – Gubbio, Italy



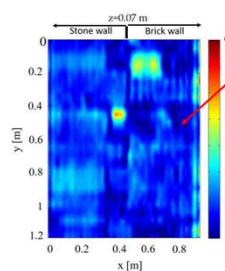
Loggia cross hass - survey 12th July 2017



The crack pattern, the three main cavities and the arrangement of the stone are clearly recognizable both by means of a visual inspection (left panel) and the tomographic image reconstruction (right panel). The red lines delimit the junction area

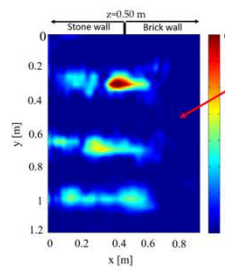
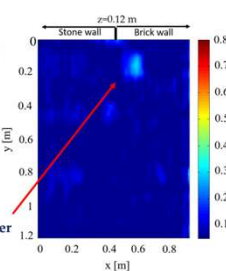


Focused images of the crack pattern on the wall at the cross-hall leading to the Loggia are given by the microwave tomographic reconstructions which allow us to infer the depth of the cavities and the arrangement of the stone blocks.



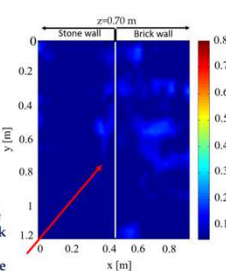
The 3rd cavity is not well visible, it is less deep than the other 2 ones and the crack pattern

The first cavity is still present and the crack reaches a depth no higher than about 0.10 m



three long stone blocks, which maybe were adopted for the junction of the walls

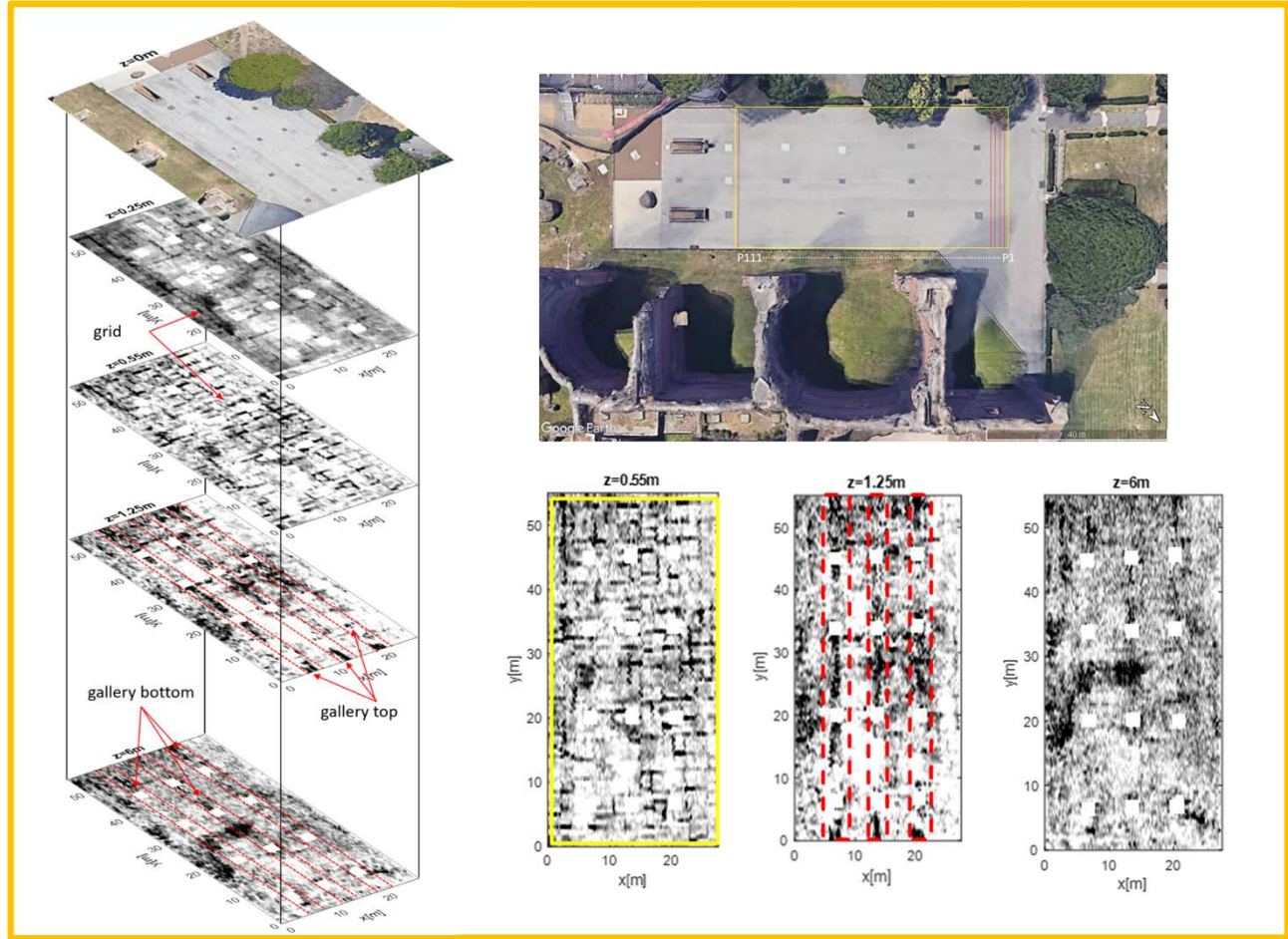
Interface between stone and brick wall (the brick wall texture is recognizable due to the occurrence of the brick eall/air interface)



Applicative examples

Cultural heritage and Archaeological surveys

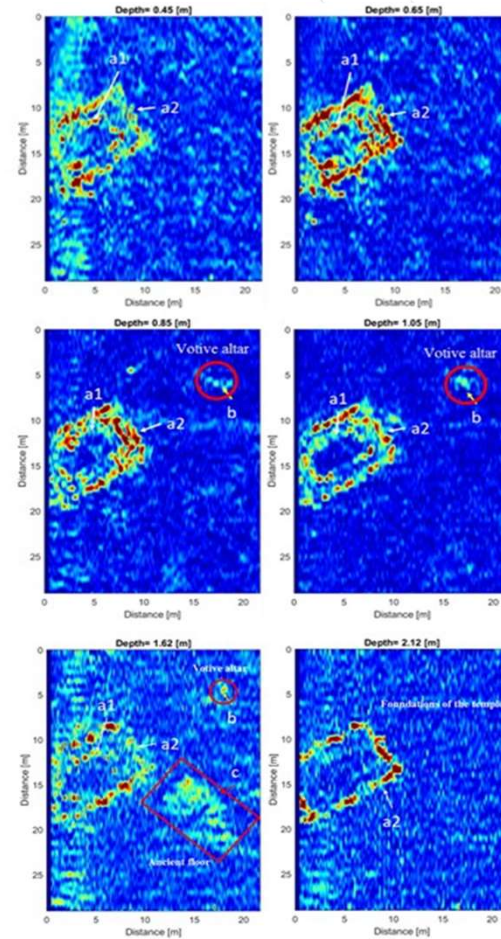
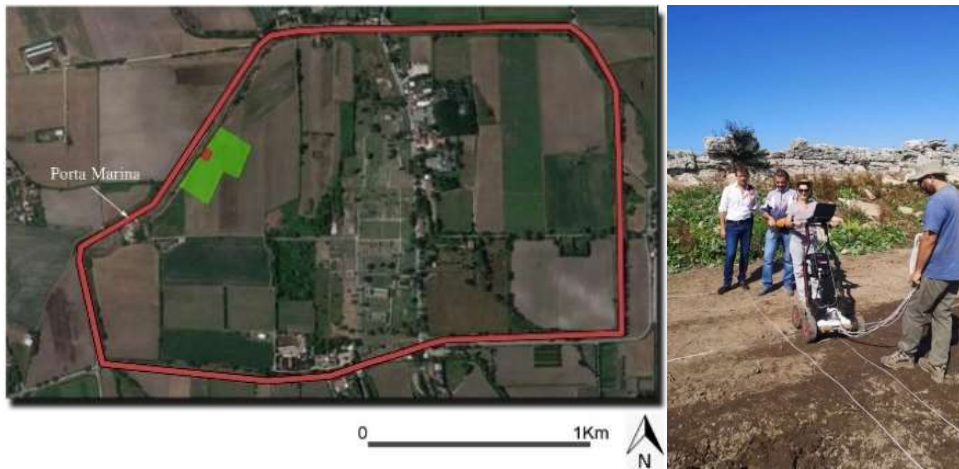
Terme di Caracalla – Roma, Italy



Applicative examples

Cultural heritage and Archaeological surveys

Archeological Park of Paestum – Salerno (Italy)



The GPRT results allowed not only the Temple localization but also an impressive visualization of its basement, which was fully confirmed by the subsequent archeological excavation campaign



Applicative examples

GPR CONTACTLESS SETUP

rapid surveys of large, even inaccessible, areas with high data density

- ice and snow monitoring
- geophysical and hydrological surveys
- demining
- avalanche victim detection
- borders surveillance



Applicative examples

GPR CONTACTLESS SETUP

rapid surveys of large, even inaccessible, areas with high data density

open issues

hardware complications

- payload constrains
- positioning problems
- electronic unit and antenna design

imaging complications

- air-medium interface effect
- achievable performance and resolution limits



Applicative examples

GPR contactless setup

rapid surveys of large, even inaccessible, areas with high data density

- ice and snow monitoring
- geophysical and hydrological surveys
- demining
- avalanche victims detection
- borders surveillance

open issues

hardware complications

- Payload constrains
- Positioning problems
- Electronic unit and antenna design

imaging complications

- Air-medium interface effect
- Achievable performance and resolution limits



Applicative examples

GPR CONTACTLESS SETUP & MWT DATA PROCESSING: APPLICATIONS

Subsoil and vertical structures diagnostic

Urban areas and structural surveys

Environment Monitoring (natural and anthropic risks)

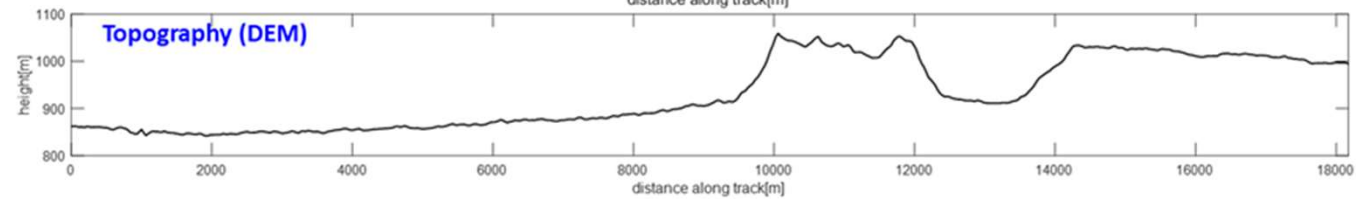
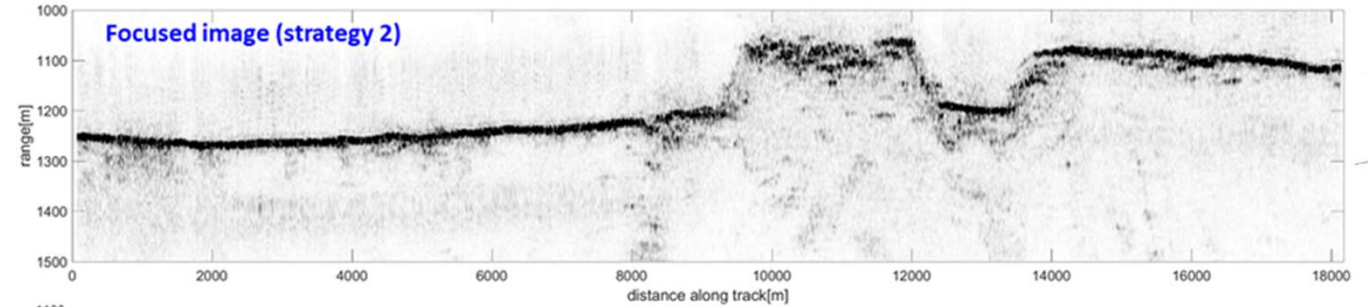
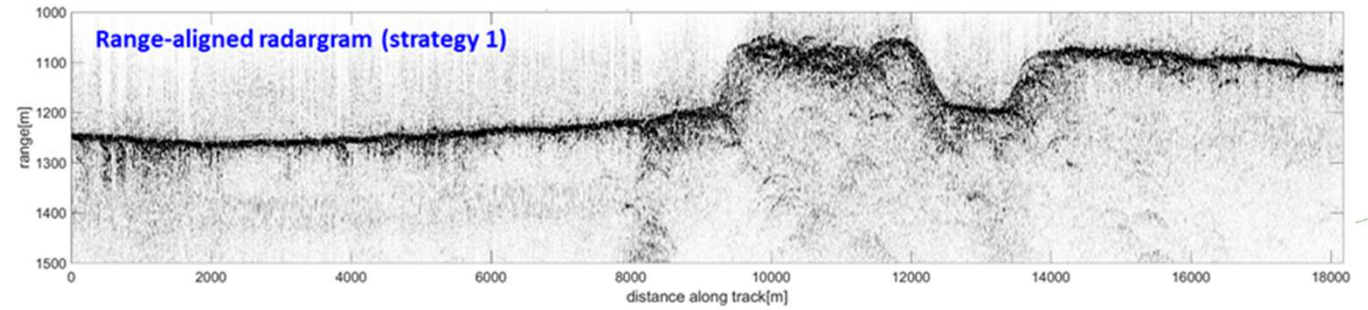
Cultural heritage and Archaeological surveys

Suspicious targets and explosive detection

Planetary explorations

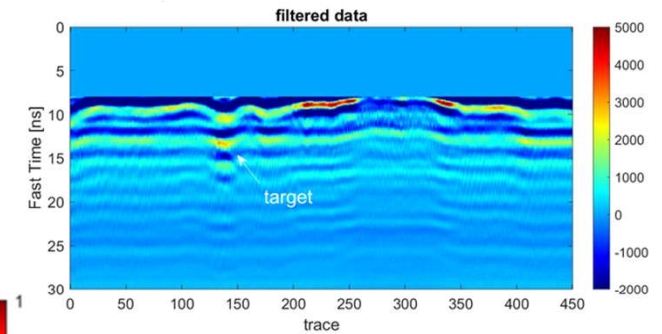
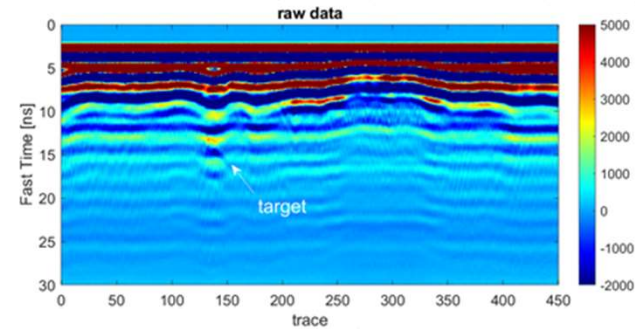
Applicative examples

Survey on Morocco desert

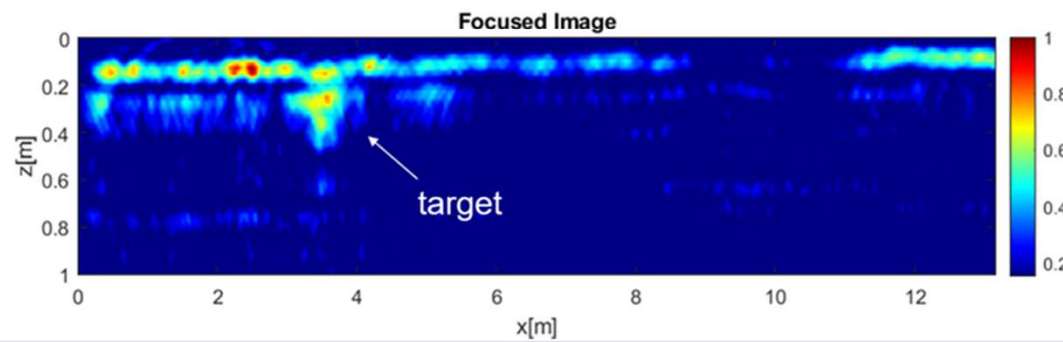


Applicative examples

Controlled experiment at Marsico Nuovo (Pz), South Italy – buried target



Radar type	TD - SOUNDER
Frequency range	200 MHz – 800 MHz
Measurement configuration Tx/RX	Monostatic
Voltage	200 V
Flight height	> 0.5 m
Antenna depression angle	0°
Acquisition rate	High speed mode (100 sample/s)
Payload weight	~ 15 kg



Applicative examples

test site with buried mine simulants



Big mine (PMN-1)
 Diameter top: 11.2 cm
 Diameter bottom: 11.0 cm
 Height: 5.5 cm
 Plastic, rubber cap, low metal

Medium mine (Test target)
 Diameter top: 9.3 cm
 Diameter bottom: 9.8 cm
 Height: 3 cm
 Plastic, no metal, wax filling

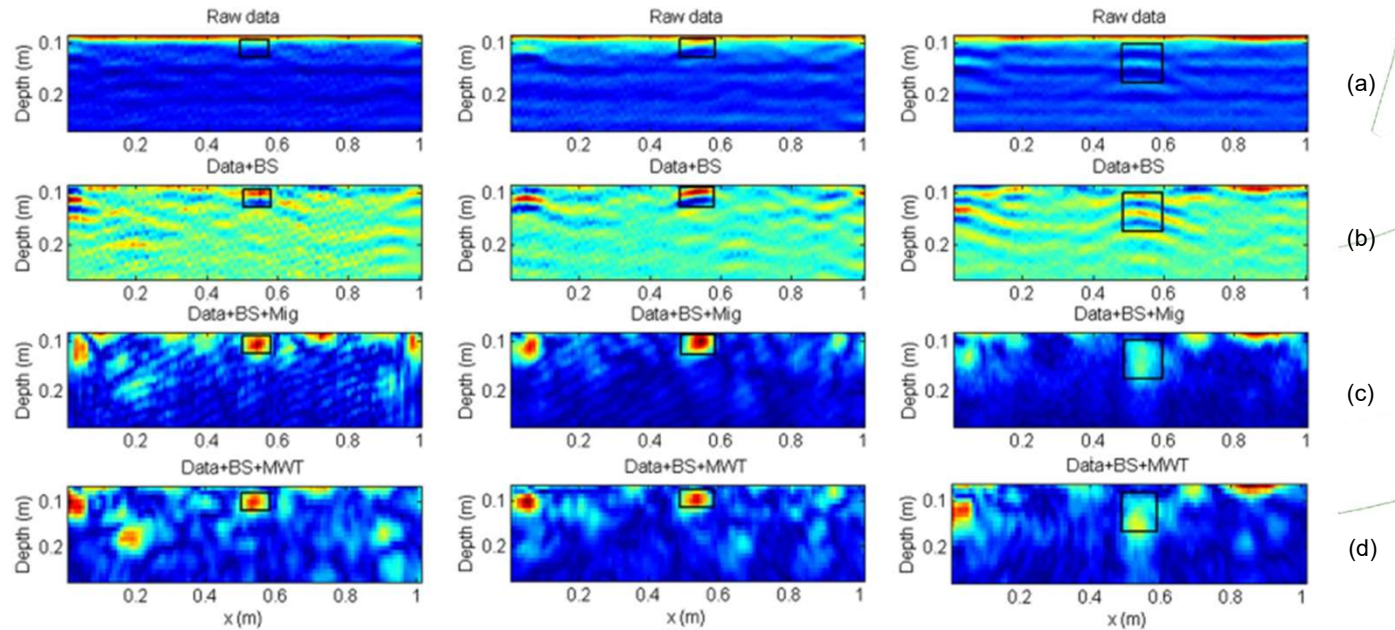
Small mine (Type-72A)
 Diameter top: 7.7 cm
 Diameter bottom: 7.0 cm
 Height: 3.7 cm
 Plastic, rubber cap, low metal

measurement layout: red squares denote the surveyed areas, red points indicate targets' positions

ACQUISITION PARAMETERS OF THE GPR SYSTEM

Central frequency	2 GHz ($\lambda_{air} = 0.15\text{m}$)
PRF	1 MHz
Pulse length	0.5 ns
Sampling time	25 ps
Spatial sampling in Y/X	1 cm/4 cm
Antenna configuration	Perpendicular broadside
Antenna height	4-8 cm
Samples/scan	512/A-scan

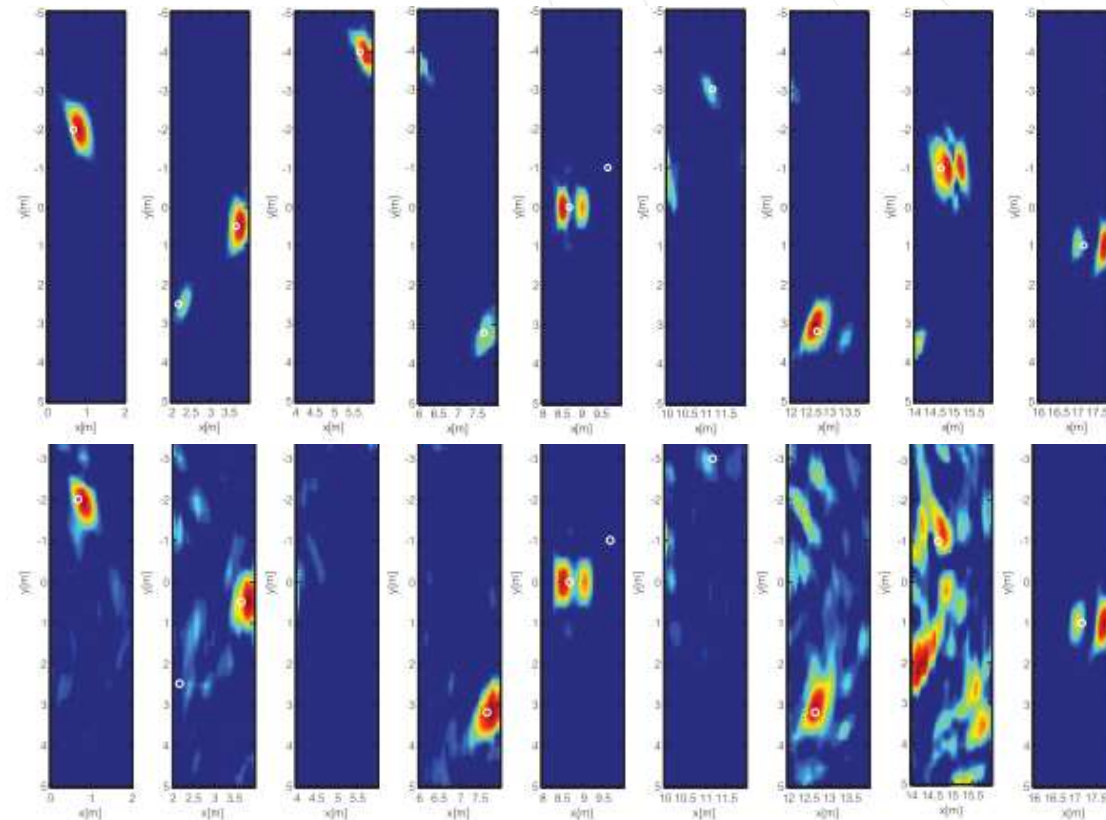
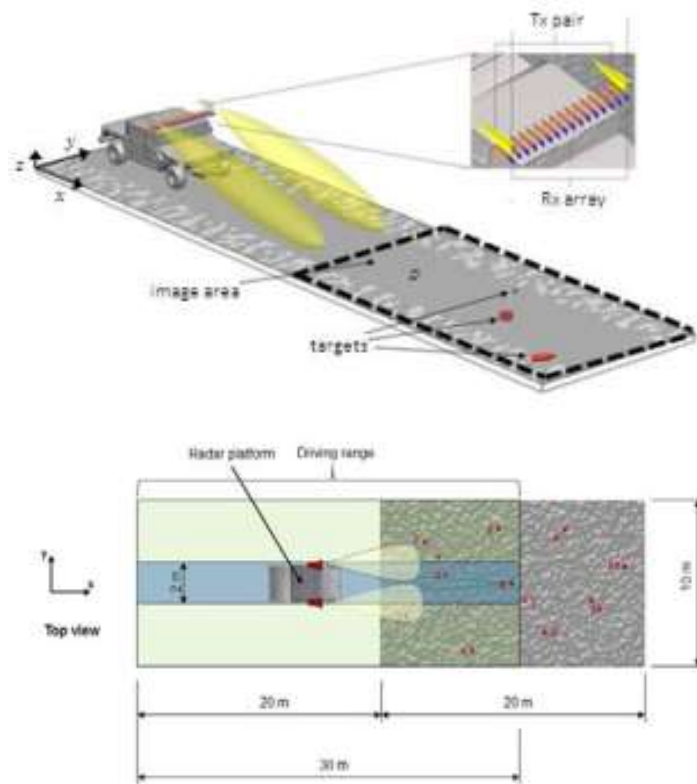
Demining



B-scans with buried targets: (a) raw data ; (b) average background removal (Data +BS); (c) BS and Migration (Data +BS+Mig); (d) BS and Microwave tomography (Data +BS+ MWT) (left) small mines; (middle) medium mines; (right) big mines

Applicative examples

Demining – Forward looking MIMO system

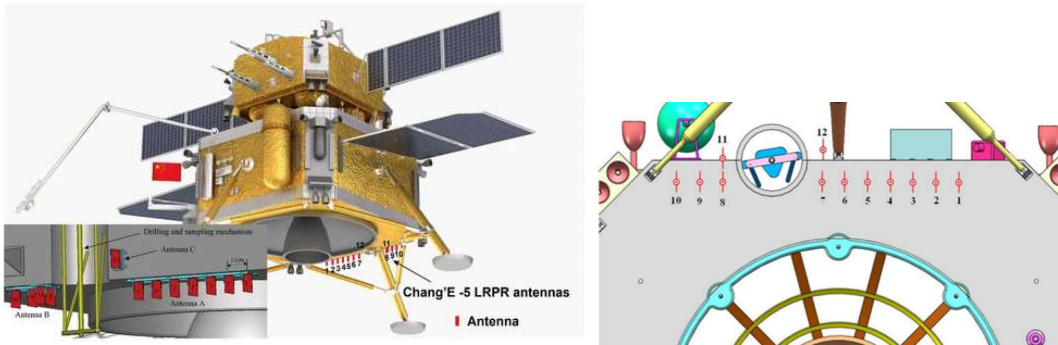


Flat surface

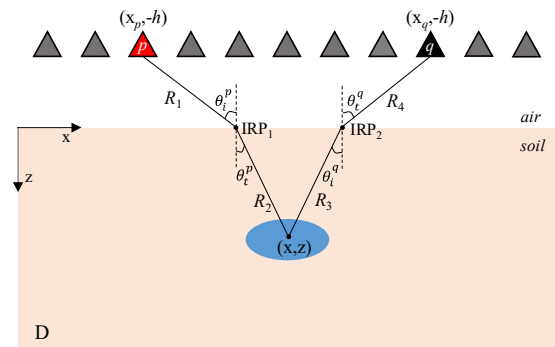
Rough surface

Applicative examples

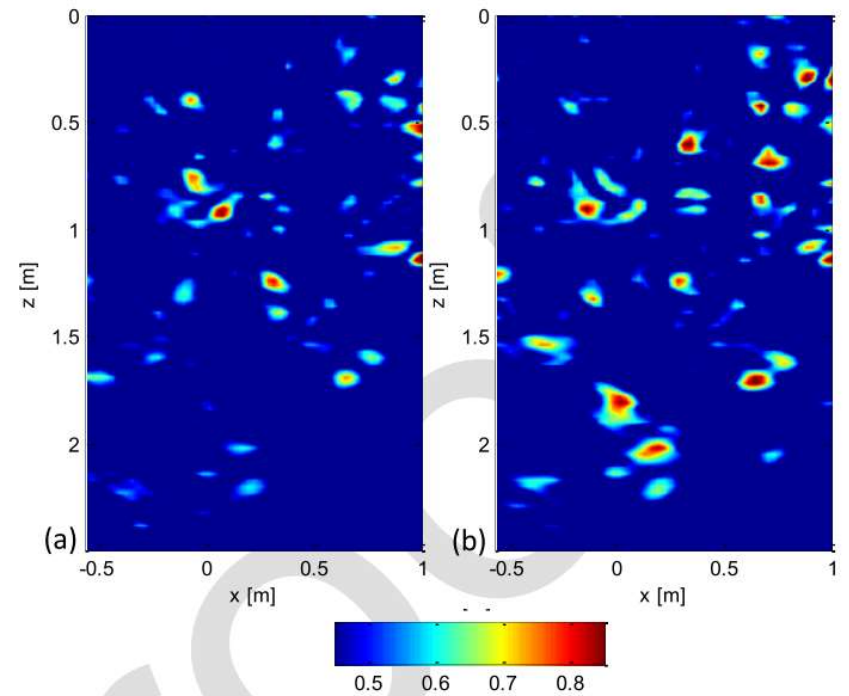
Planetary explorations



CE-5 lander and layout of LRPR antennas around the drilling core (side view and bottom view of the antenna array)



Sketch of the signal propagation when the antenna p transmits and the antenna q receives the signal



Real experiment - tomographic images. (a) Pre-drilling and (b) post-drilling

References

BOOKS

- W. C. Chew, *Waves and Fields in Inhomogeneous Media*, 2nd ed. New York: IEEE, 1995
- M. Bertero and P. Boccacci, *Introduction to Inverse Problems in Imaging*, Bristol Philadelphia, U.K.: AIP, 1998.
- C. A. Balanis, *Advanced Engineering Electromagnetics*. Hoboken, NJ, USA: Wiley, 1989
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- R. Persico, *Introduction to Ground Penetrating Radar: Inverse Scattering and Data Processing*, Hoboken, NJ, USA: Wiley, 2014.

INTERNATIONAL JOURNAL PAPERS

- Catapano, I., Gennarelli, G., Ludeno, G., & Soldovieri, F. (2019). Applying ground-penetrating radar and microwave tomography data processing in cultural heritage: State of the art and future trends. *IEEE Signal Processing Magazine*, 36(4), 53-61
- Webster, J.G., Catapano, I., Gennarelli, G., Ludeno, G., Soldovieri, F. and Persico, R. (2019). Ground-Penetrating Radar: Operation Principle and Data Processing. In *Wiley Encyclopedia of Electrical and Electronics Engineering*, J.G. Webster (Ed.). <https://doi.org/10.1002/047134608X.W8383>.
- I. Catapano, G. Gennarelli, G. Ludeno, C. Noviello, G. Esposito and F. Soldovieri, "Contactless Ground Penetrating Radar Imaging: State of the art, challenges, and microwave tomography-based data processing," in *IEEE Geoscience and Remote Sensing Magazine*, vol. 10, no. 1, pp. 251-273, March 2022, doi: 10.1109/MGRS.2021.3082170.



THANKS!

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(D.D. n. 130/2022 - CUP B53C22002150006) Funded by EU - Next Generation EU PNRR-
Mission 4 "Education and Research" - Component 2: "From research to business" - Investment
3.1: "Fund for the realisation of an integrated system of research and innovation infrastructures"

