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Multichannel Analysis of Surface Waves (MASW)

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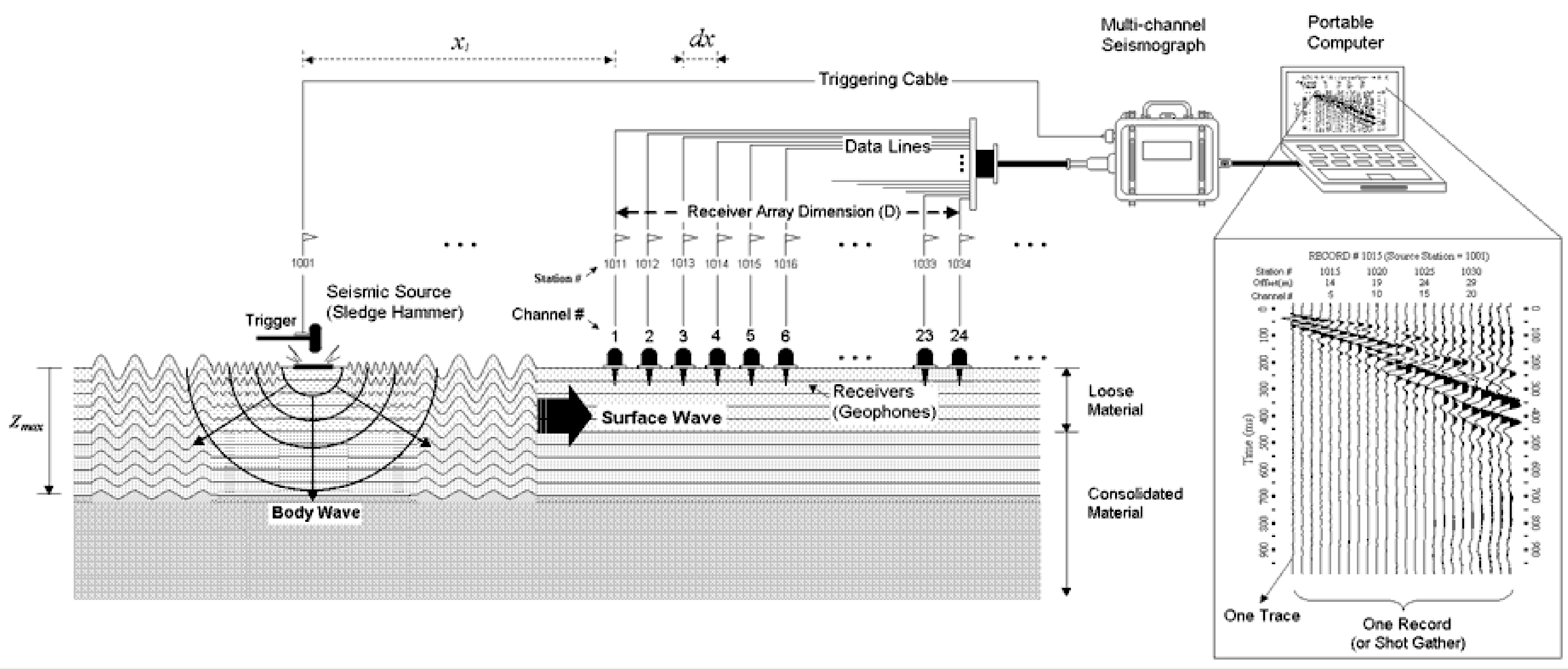
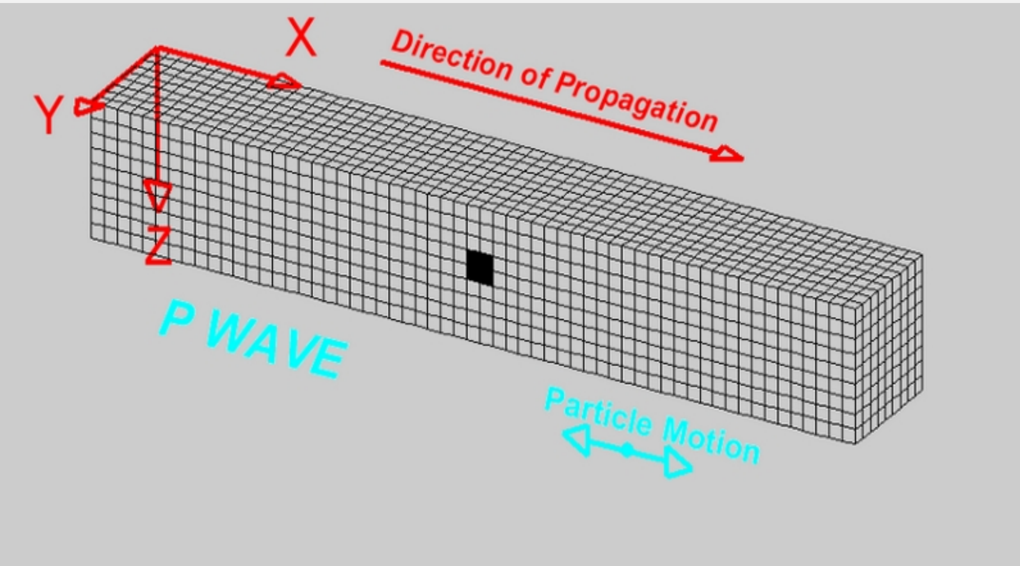
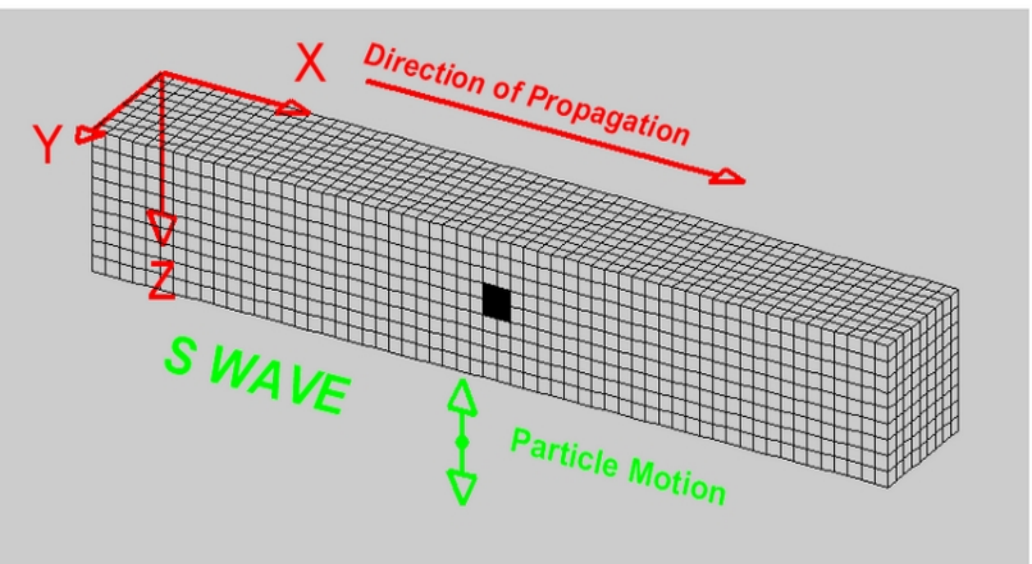
Acquisition Scheme and Equipment



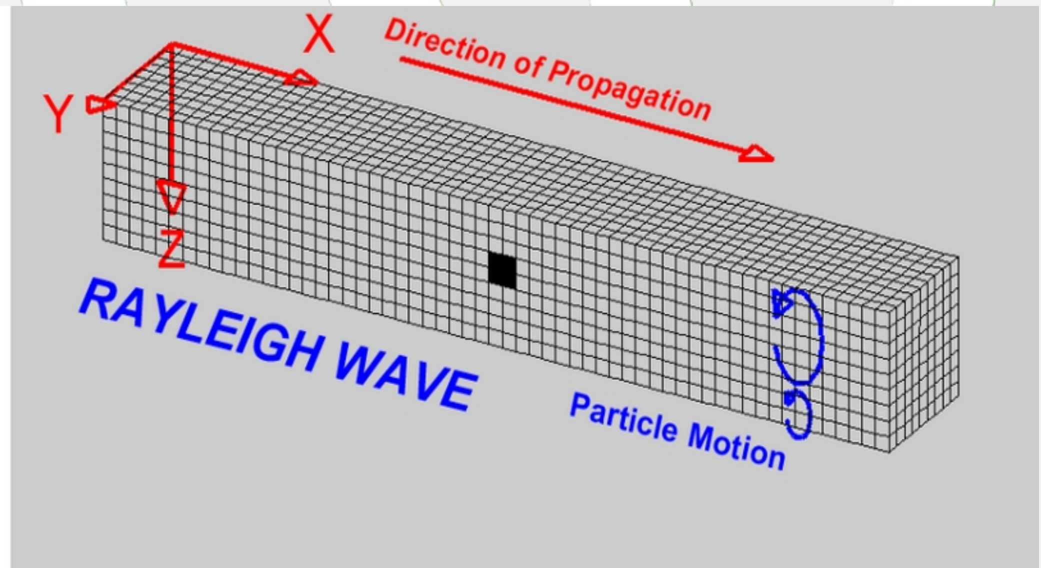
Seismic Waves

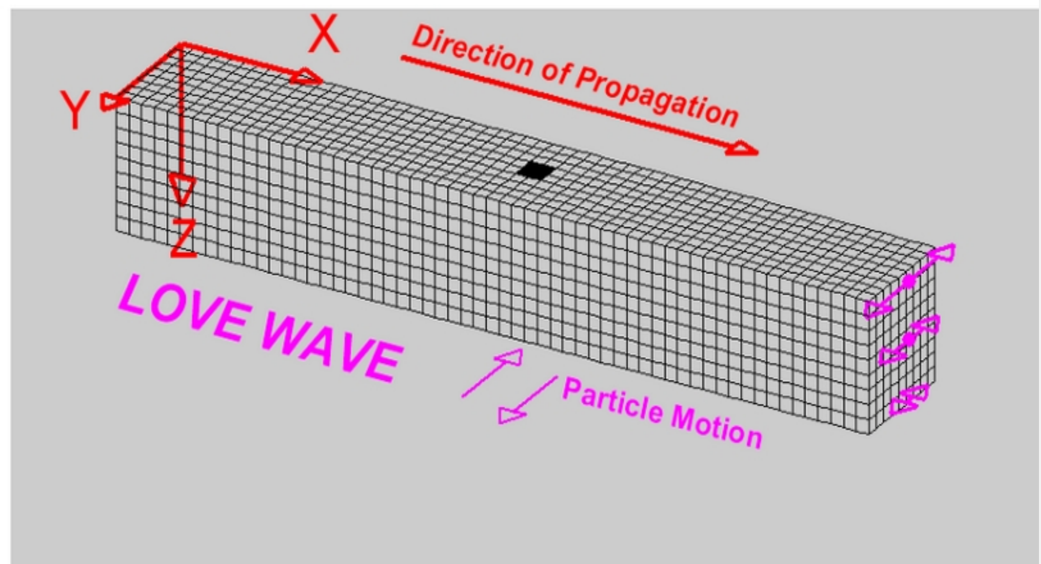
Body Waves





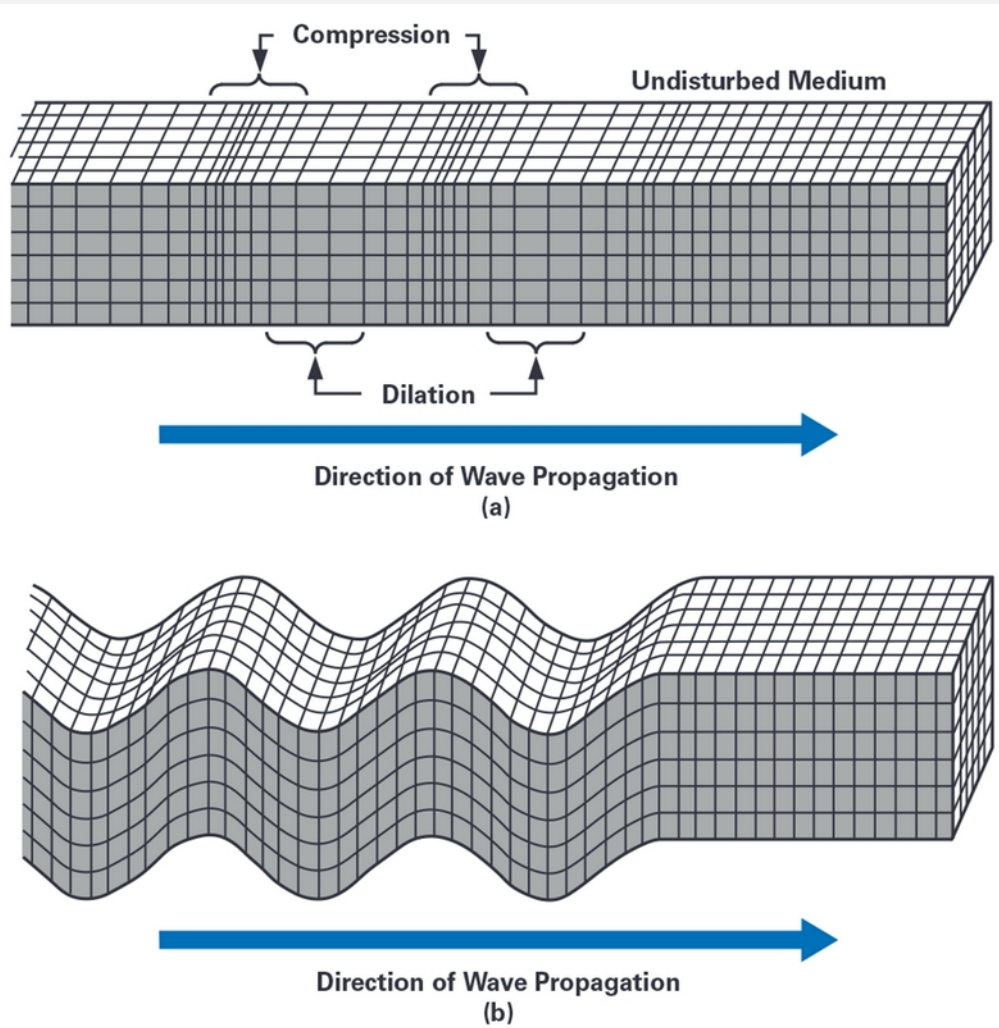
Surface Waves



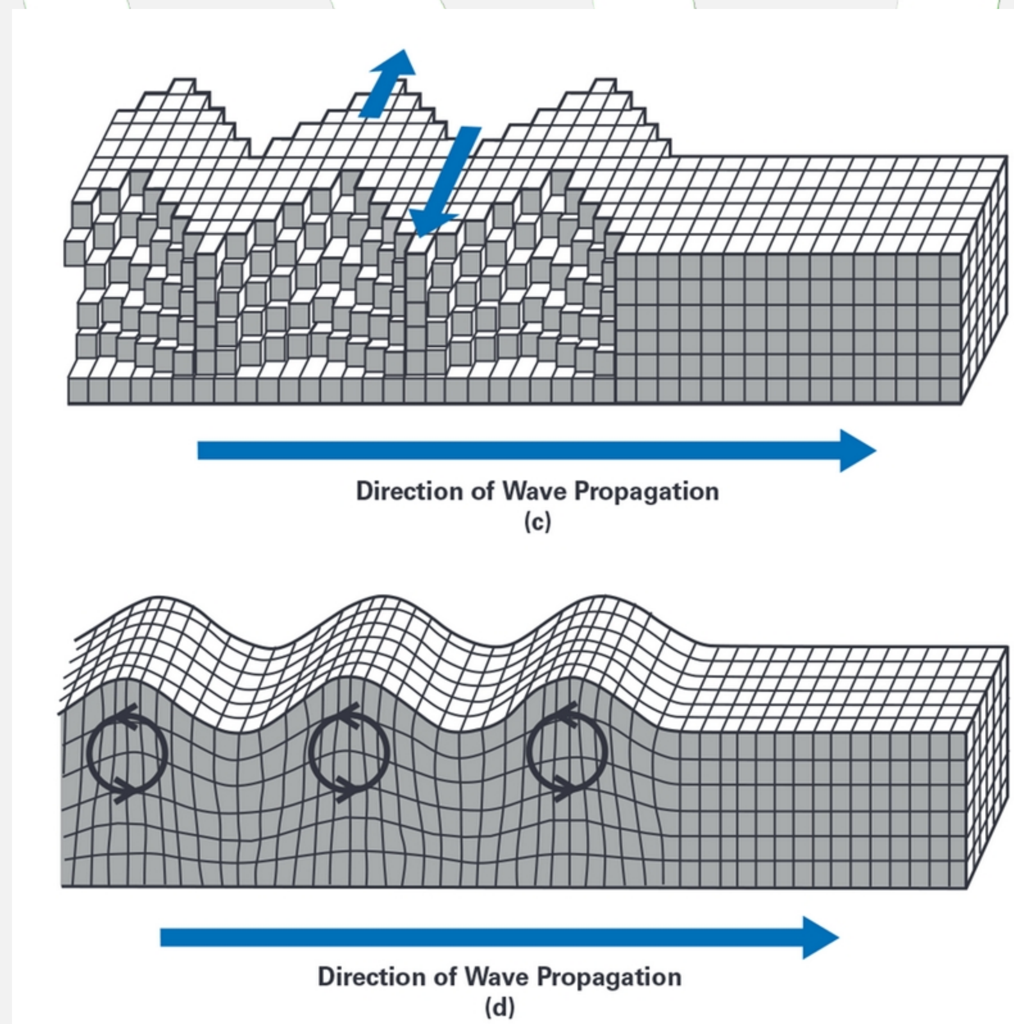


Seismic Waves

Body Waves



Surface Waves



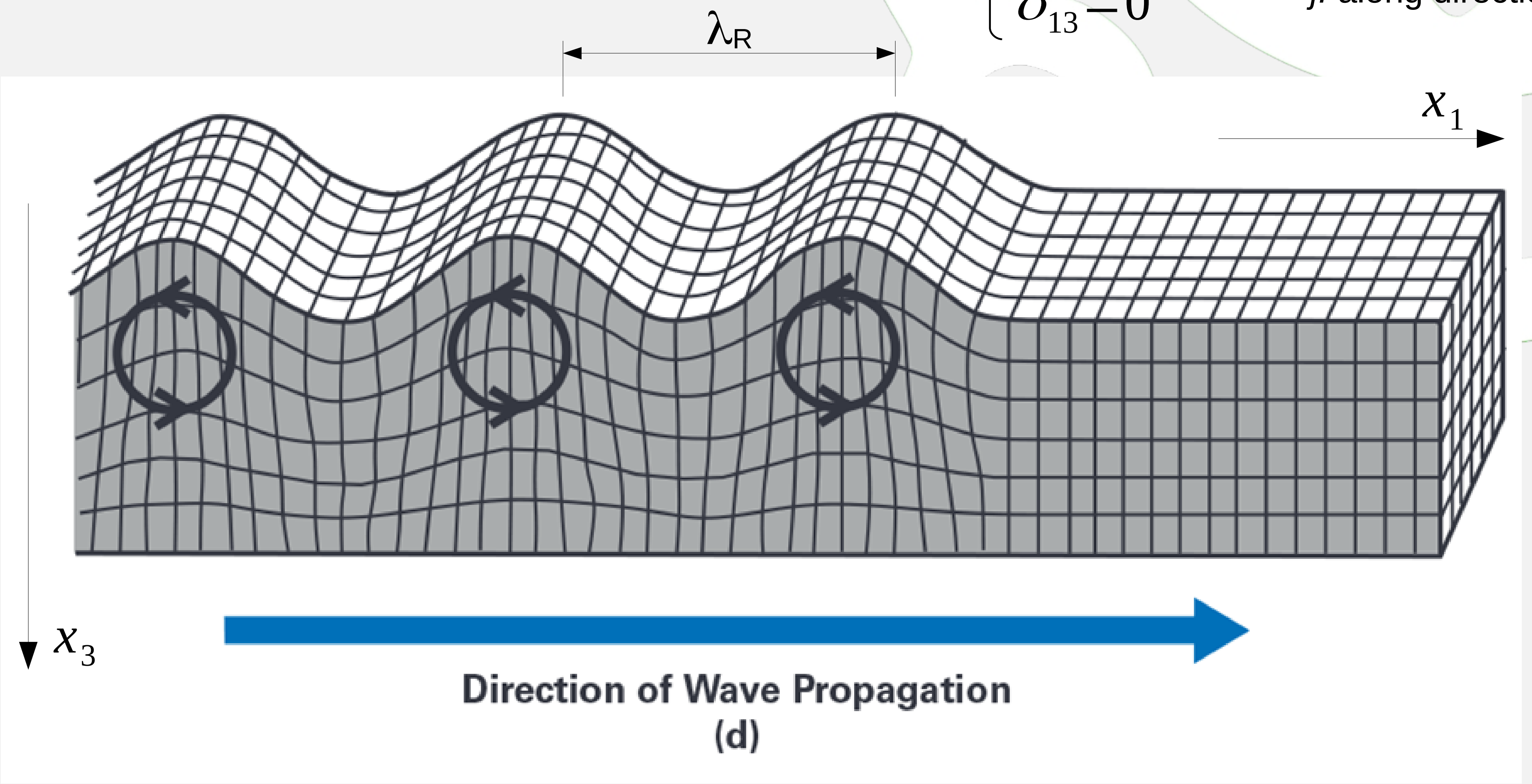
Types of seismic waves: (a) primary waves; (b) secondary waves; (c) Love waves; (d) Rayleigh waves.

Rayleigh Waves

Rayleigh Waves originate from the interaction of incident P and SV waves imposing the free surface boundary conditions on a homogeneous half-space

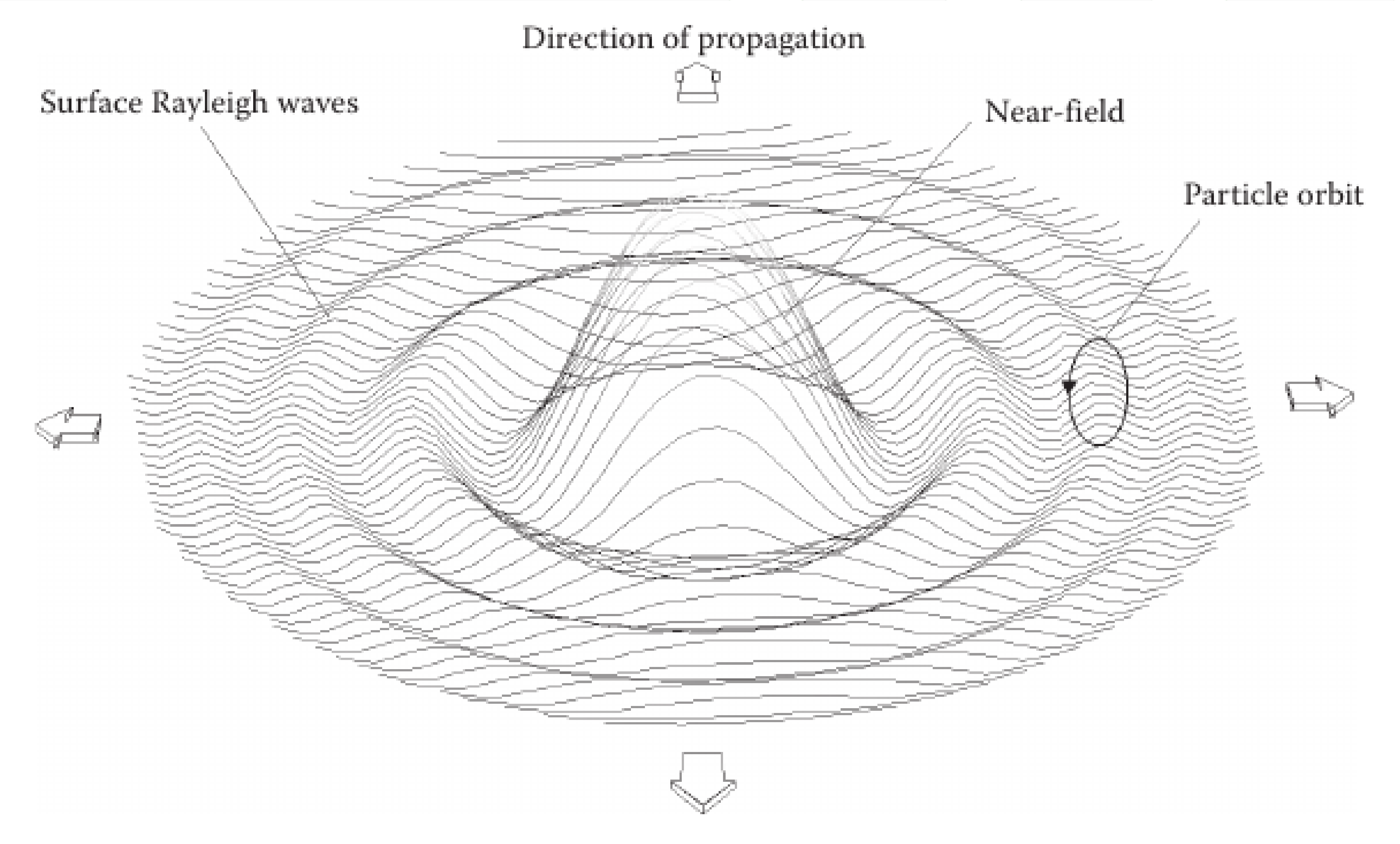
$$\begin{cases} \sigma_{33} = 0 \\ \sigma_{13} = 0 \end{cases}$$

σ_{ij} — *i:* applied to the face orthogonal to *i*; *j:* along direction *j*



See for a detailed description: Lay and Wallace, 1995, Modern Global Seismology, Chapter 4

Rayleigh Waves



2D radiation pattern of Rayleigh surface waves generated by a vertical point source (cylindrical waveform)

See:Foti and al., 2015 - Surface Wave Methods for Near-Surface Site Characterization

Propagation velocities

Fastest P wave $$V_p = \sqrt{\frac{(\lambda + 2\mu)}{\rho}} = \sqrt{\frac{(K + 4/3\mu)}{\rho}} = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$$

Slower S wave $$V_s = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2\rho(1+\nu)}}$$

Slowest Rayleigh wave $$V_R \approx 0.9 V_s$$

Elastic homogeneous medium

From V_p and V_s ⇒ $$\frac{V_p}{V_s} = \sqrt{\frac{2(1-\nu)}{1-2\nu}}$$

For most materials: $0 \le \nu \le 0.5$

⇒ $$\sqrt{2} \le \frac{V_p}{V_s} \le \infty$$ and $$0.87 \le \frac{V_R}{V_s} \le 0.96$$

$V_p = \alpha \quad V_s = \beta$

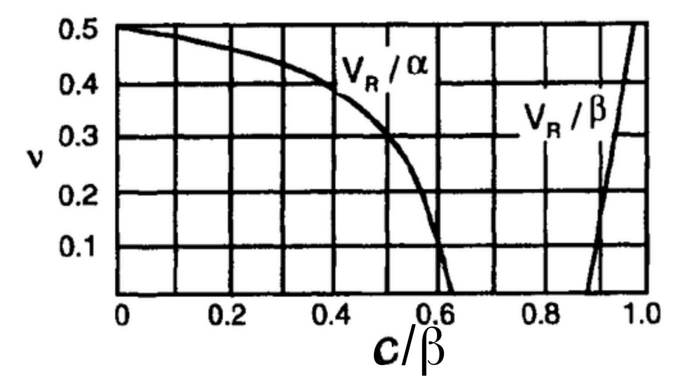
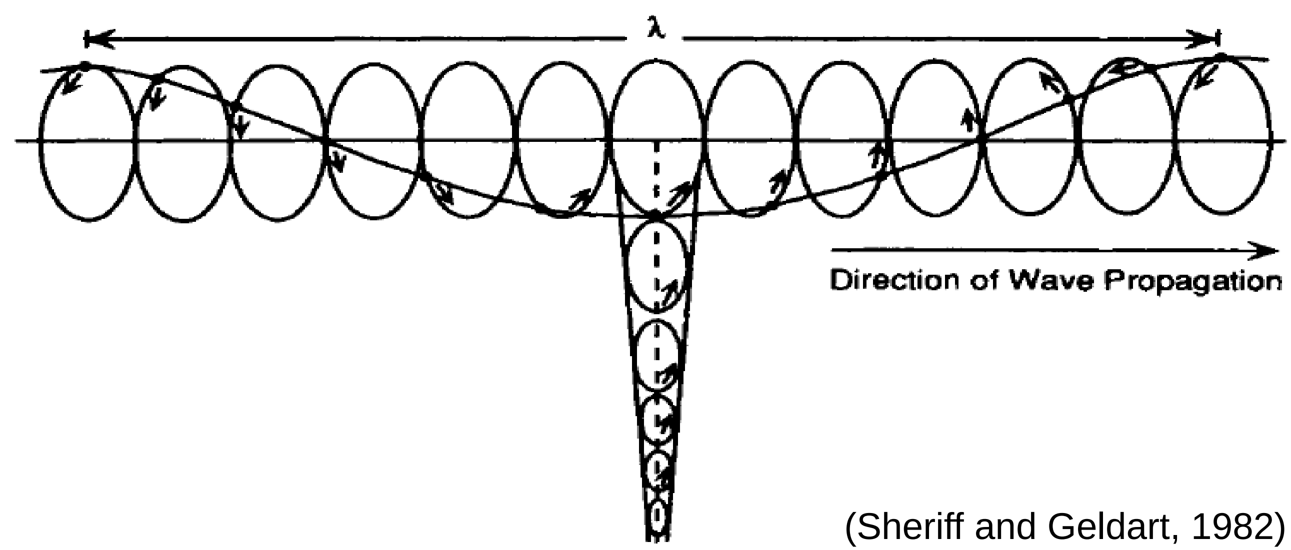


FIGURE 4.4 Half-space Rayleigh-wave velocity c as a function of Poisson's ratio, ν, where $\nu = [(\alpha^2/\beta^2) - 2]/2[(\alpha^2/\beta^2) - 1]$. For a fluid, $\beta = 0$ and $\nu = 0.5$, in which case $c = 0$. For a Poisson solid, $\alpha = \sqrt{3}\beta$, $\nu = 0.25$, and $c = 0.9194\beta$. (From Sheriff and Geldart, "Exploration Seismology," Vol. 1, History, theory, and data acquisition. Copyright©1982. Reprinted with the permission of Cambridge University Press.)

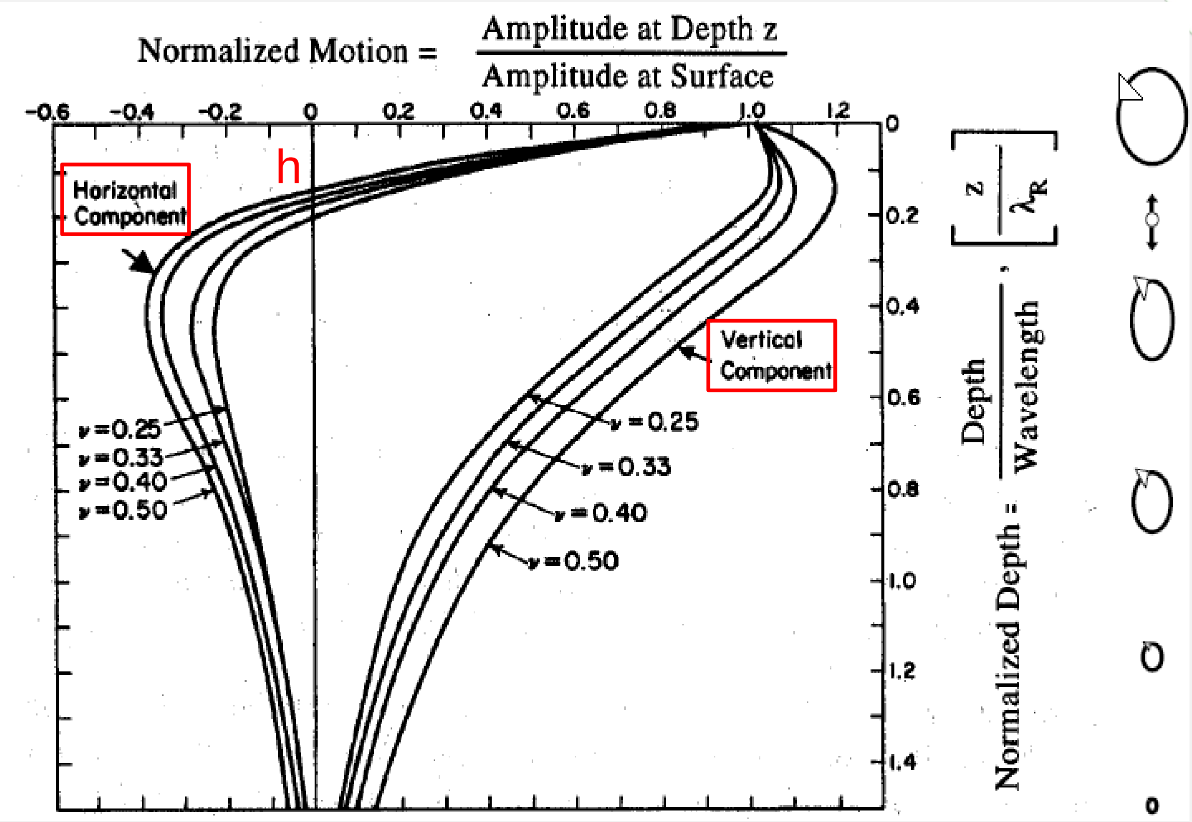
Lay and Wallace, 1995

Rayleigh waves displacements

Rayleigh-wave particle motions over one wavelength along the surface and as a function of depth



(Sheriff and Geldart, 1982)

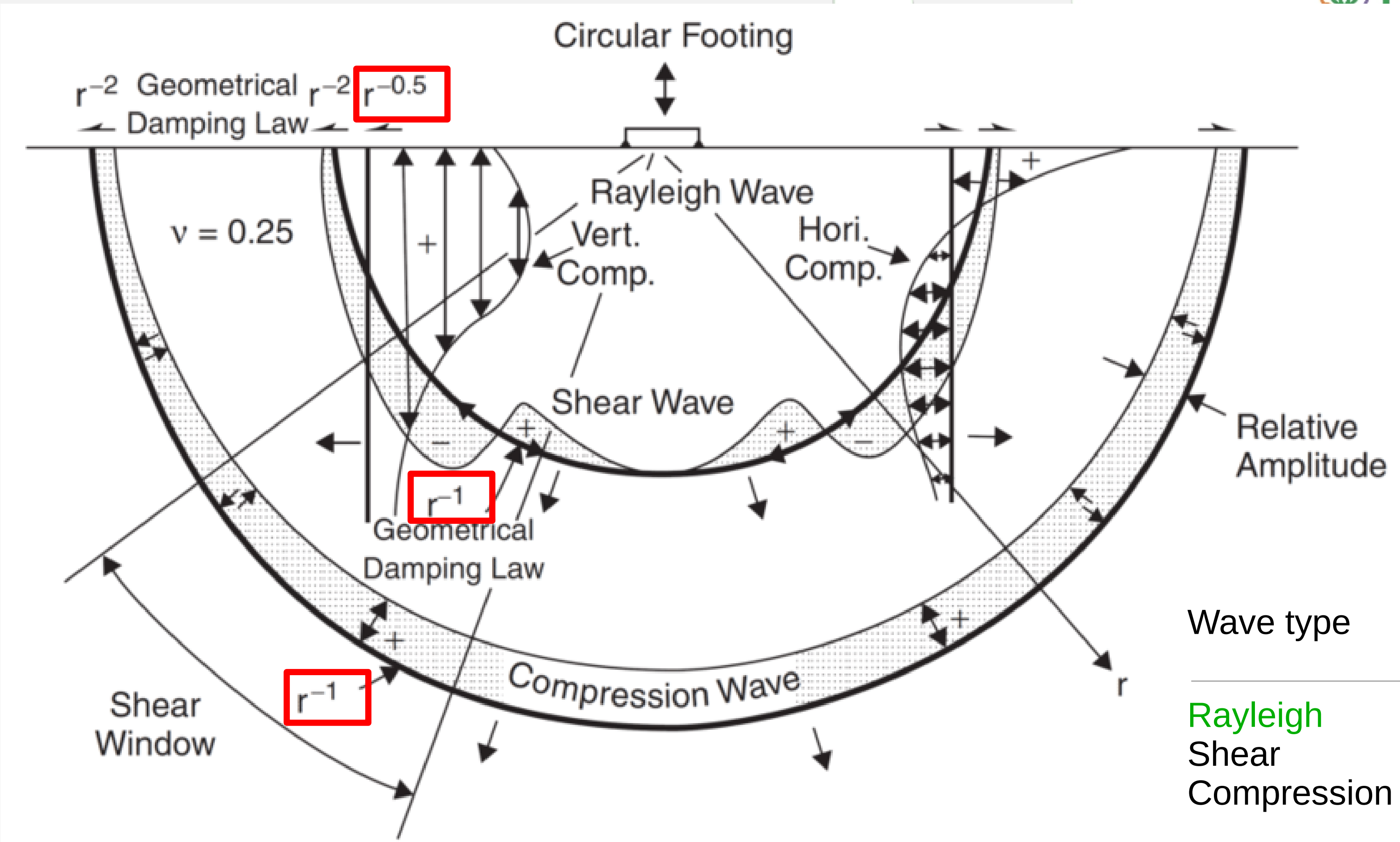


Horizontal and vertical displacements of Rayleigh waves in a homogeneous half-space. The particle motion is retrograde elliptical above depth h and prograde elliptical at greater depth

Amplitude decay:
exponential with depth

(Richart et al.,1970)

Geometrical Spreading



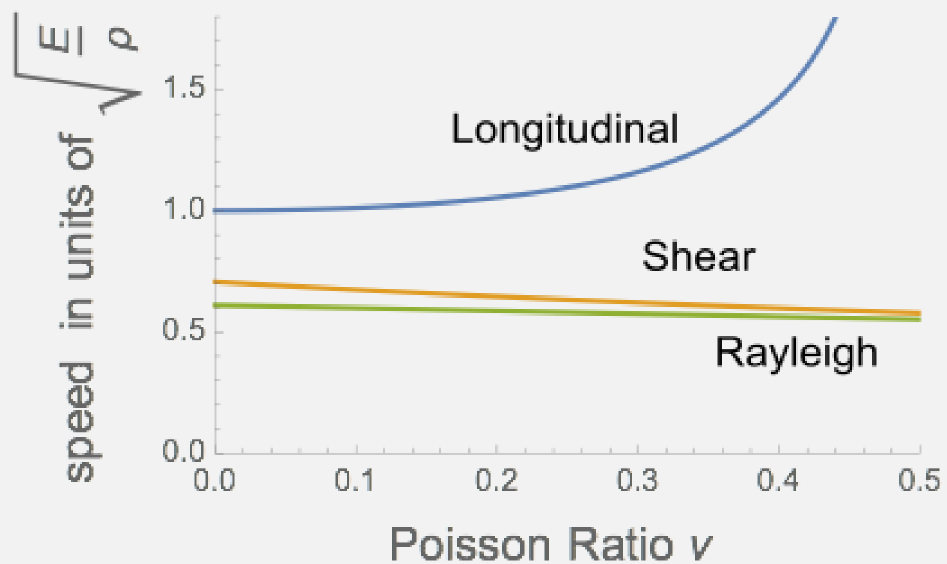
Wave type	% of total energy
Rayleigh	67%
Shear	26%
Compression	7%

Source type	Detection location	Wave type: Body	Wave type: Surface
Point	Within space	$1/r$	N/A
	At surface	$1/r^2$	$1/\sqrt{r}$

Geometric attenuation rates of body and surface wave vibration amplitudes with distance r in a half-space

Some Surface Waves Propagation Characteristics

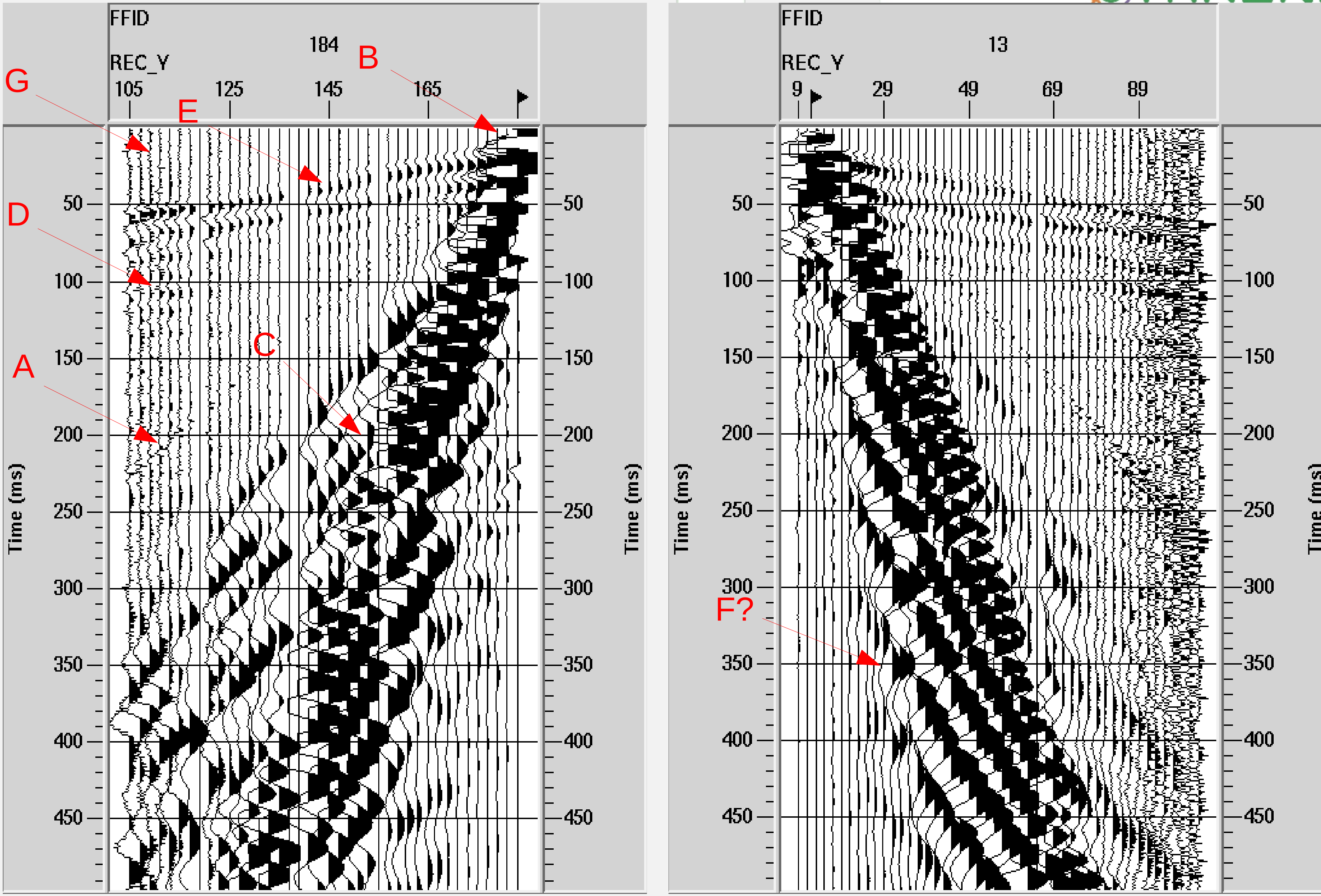
- Surface waves propagate parallel to the earth's surface without spreading energy through the earth's interior
- Rayleigh waves are a mix of P- and SV-waves
- Particle motion is in a retrograde elliptical sense in a vertical plane with respect to the surface (above a depth h)
- There are no SH-modes within Rayleigh waves.
- They have cylindrical wavefronts: amplitude decreases as $r^{-1/2}$
- Their amplitude decreases exponentially with the depth
- Most of the energy propagates in a shallow zone (≤ λ)



Comparison of the Rayleigh wave speed with shear and longitudinal wave speeds for an isotropic elastic material. The speeds are shown in dimensionless units.

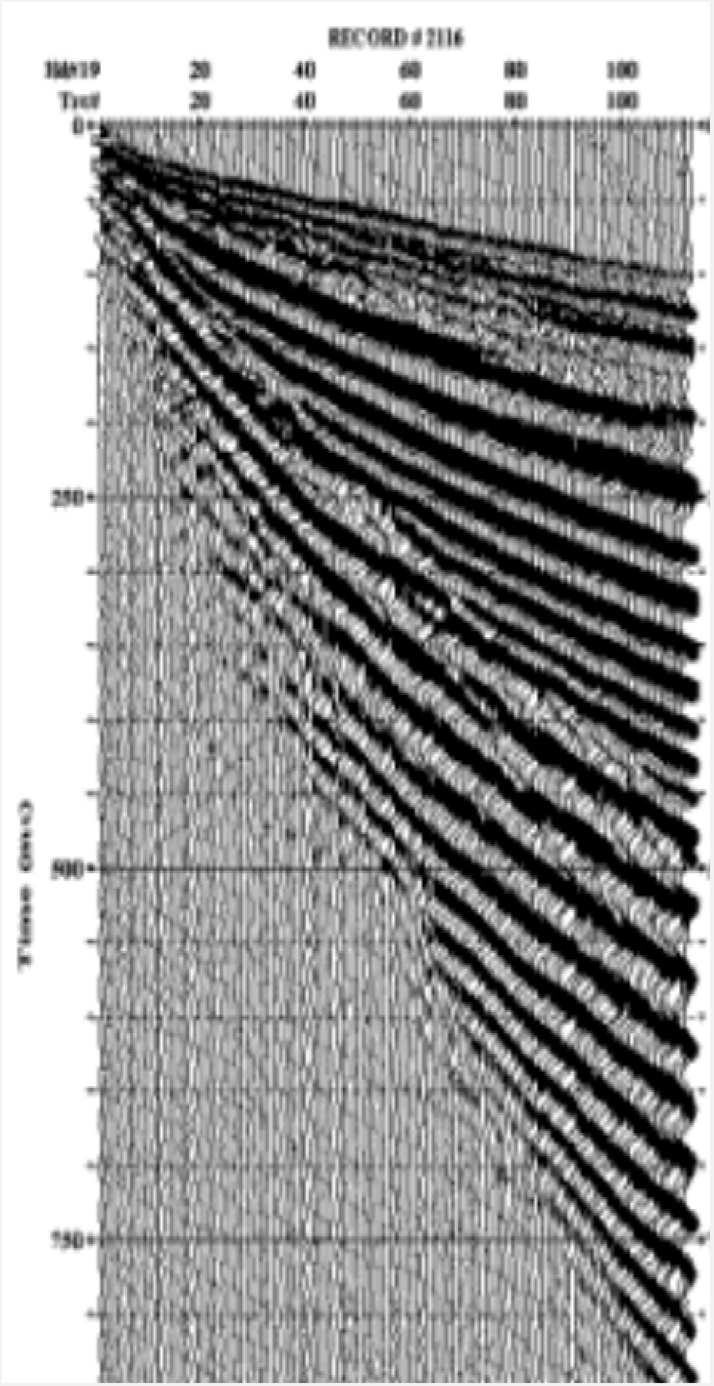
https://en.wikipedia.org/wiki/Rayleigh_wave

Types of Recorded Waves



A: Air wave
B: Direct wave
C: Surface wave
D: Reflection
E: Refraction
F: Back scattering of Surface wave
G: Ambient noise

MASW Method



(Kansas Geological Survey)

Body wave

Surface waves (higher mode), superimposed on body waves

Surface waves (fundamental mode) superimposed on body waves

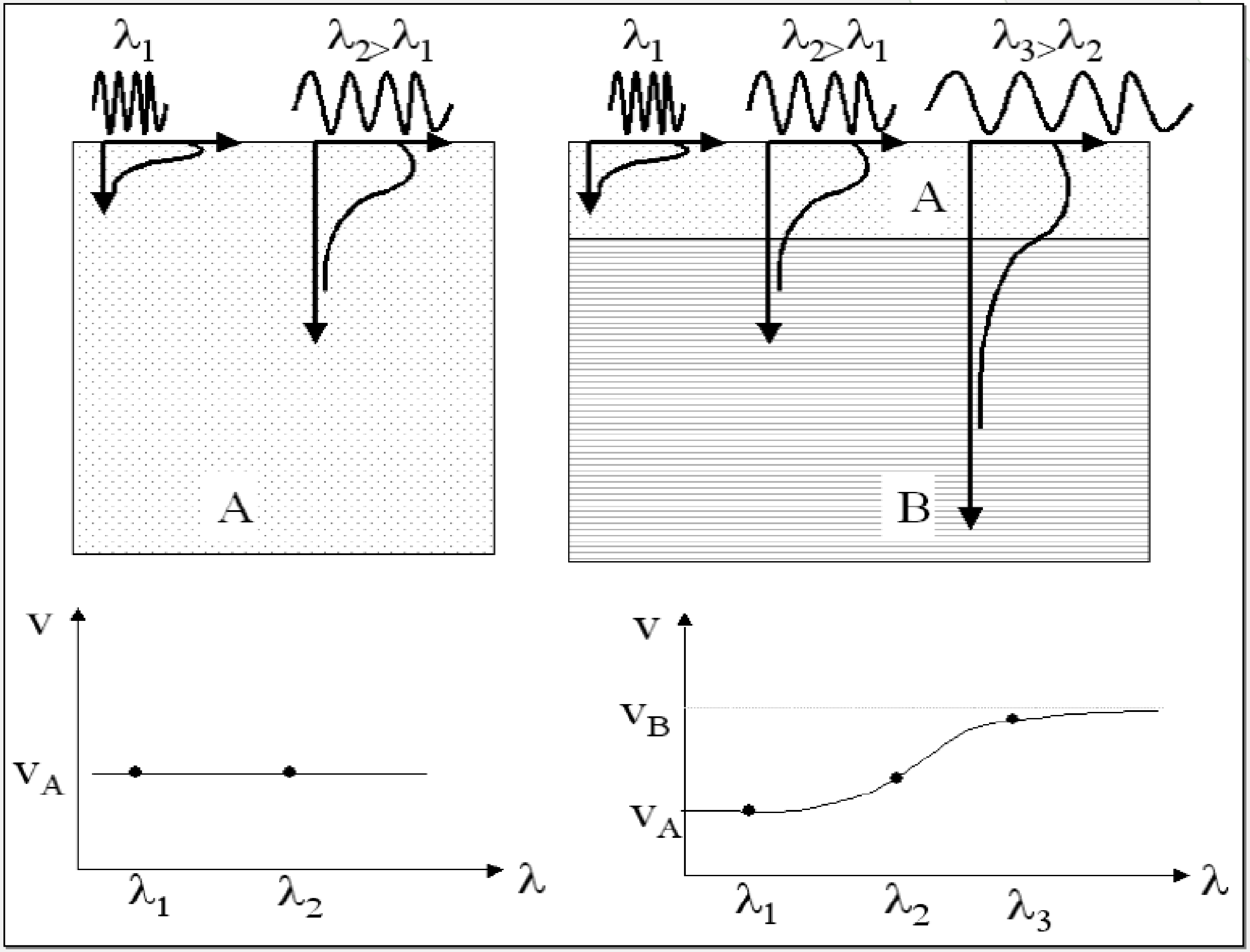
The MASW method considers Surface waves as useful information

For seismic reflection the surface waves are considered as coherent noise to be removed

Dispersion

The velocity of propagation of Rayleigh waves in a homogeneous, isotropic, linear elastic half-space is a function of the mechanical properties of the medium, not a function of frequency.

homogeneous medium heterogeneous medium



(Strobbia, 2003)

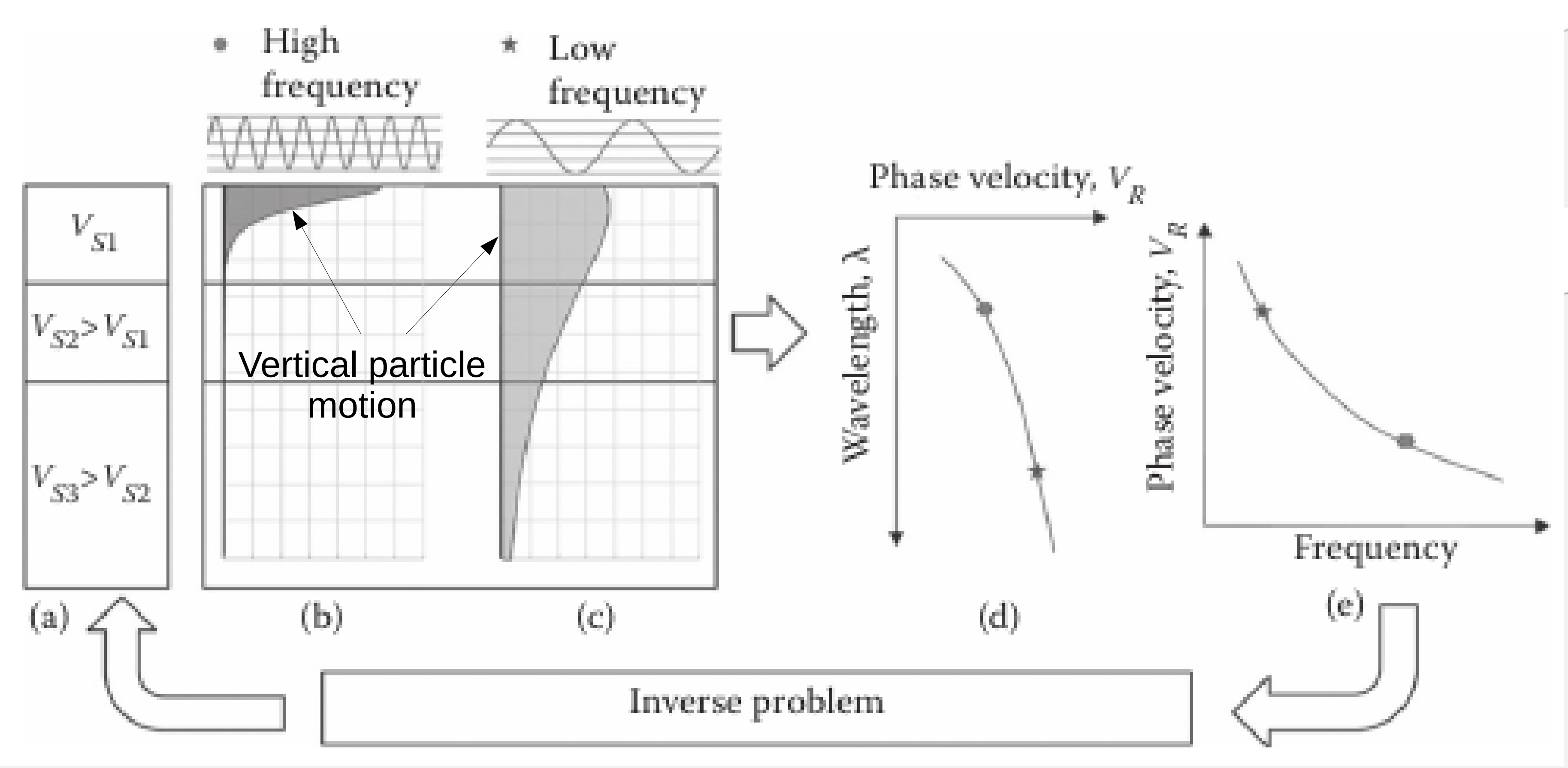
In vertically heterogeneous media, the phenomenon of geometric dispersion arises, which results in the phase velocity of Rayleigh waves being frequency dependent.

The dispersive nature of Rayleigh waves propagating in a vertically heterogeneous medium forms the basis of surface wave testing.

Dispersion

Dispersion = Different frequencies travel at different velocities

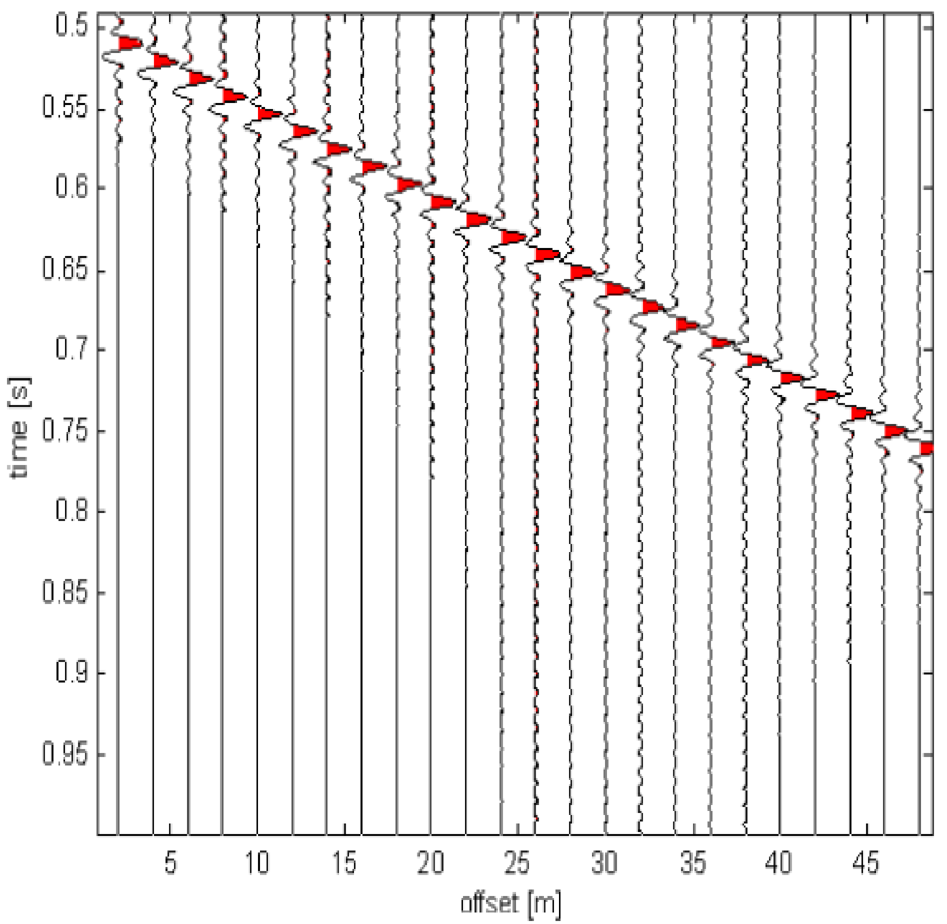
Longer wavelengths “sample” greater depths



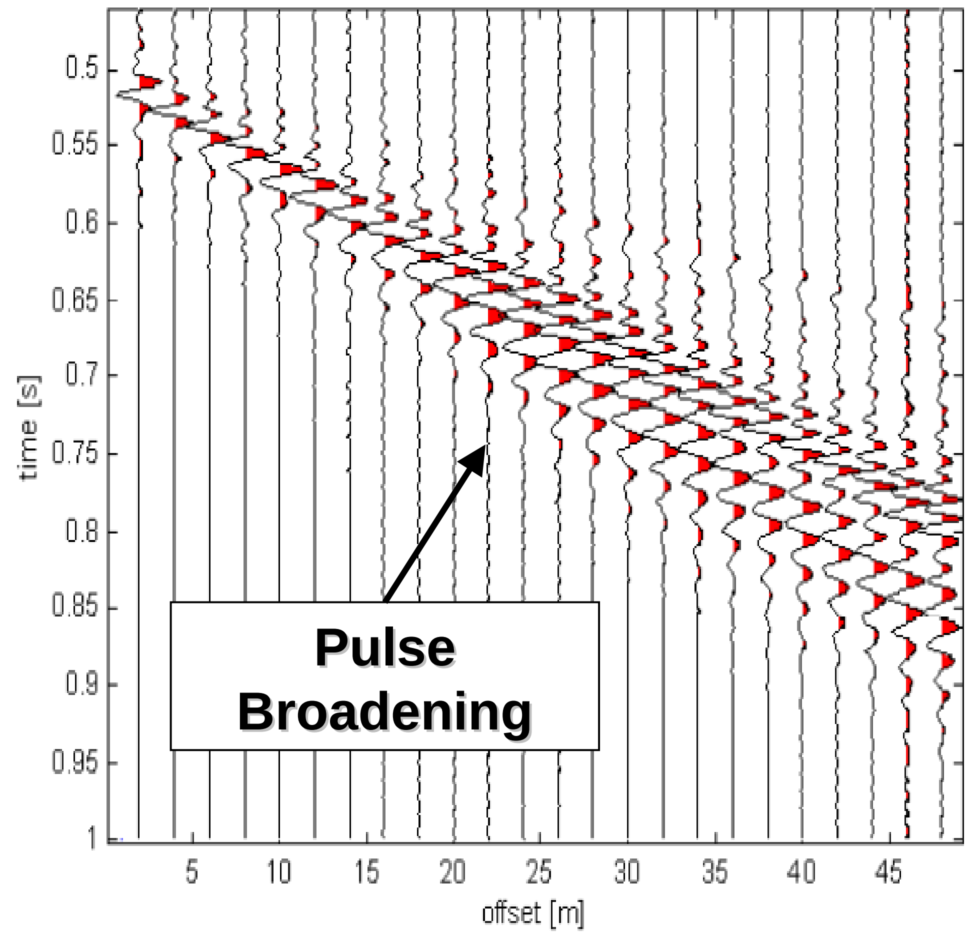
Parameter identification on the basis of geometric dispersion (Foti, 2015)

Dispersion

Homogeneous medium



Layered medium



NO DISPERSION

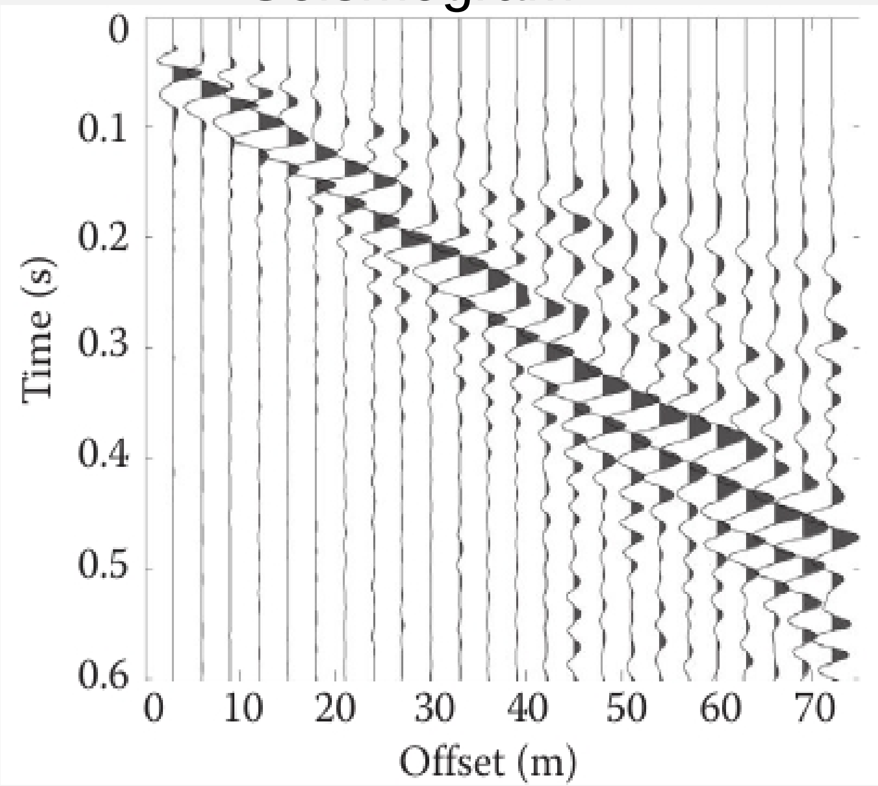
DISPERSION

(Strobbia, 2003)

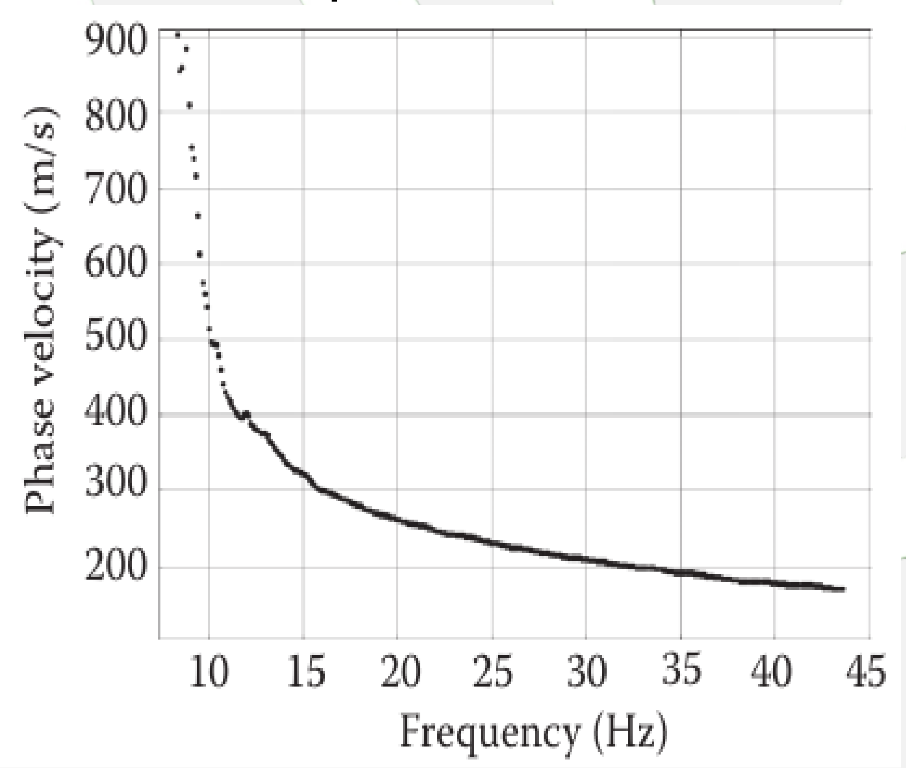
Dispersion Curve

Dispersion curve = relation between frequency and phase velocity

Seismogram



Dispersion curve



At high frequency phase velocity is the Rayleigh velocity of the uppermost layer

At low frequency the effect of deeper layers becomes important, and the phase velocity tends asymptotically to the Rayleigh velocity of the deepest material (the half space)

Dispersion Curves Computation

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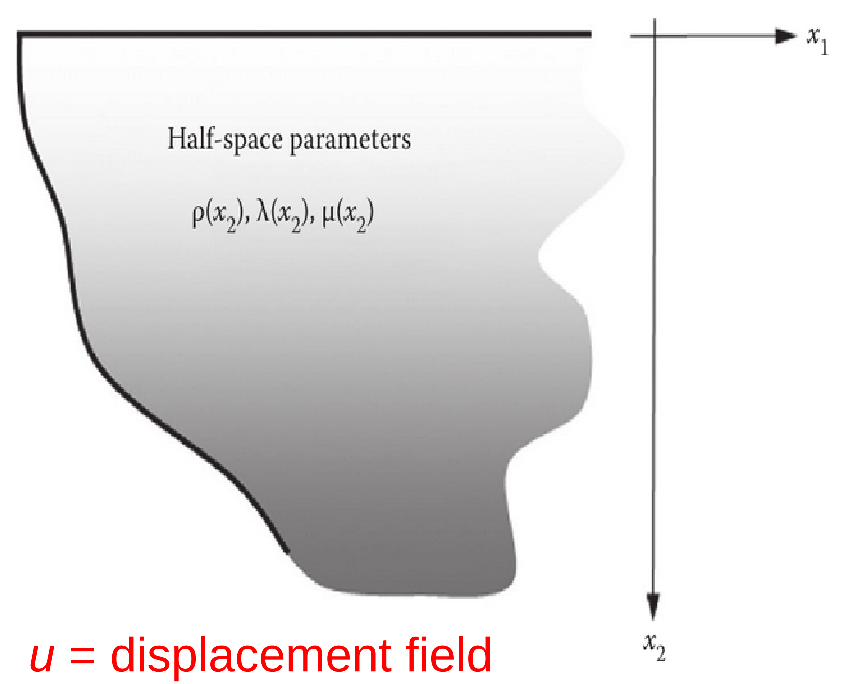
Seeking solutions of the wave equation of the form:

$$\text{For } \textit{Love waves}: \begin{cases} u_1 = 0 \\ u_2 = 0 \\ u_3 = l_1(x_2, k, \omega) \cdot e^{i(kx_1 - \omega t)} \end{cases}$$

$$\text{For } \textit{Rayleigh waves}: \begin{cases} u_1 = r_1(x_2, k, \omega) \cdot e^{i(kx_1 - \omega t)} \\ u_2 = i \cdot r_2(x_2, k, \omega) \cdot e^{i(kx_1 - \omega t)} \\ u_3 = 0 \end{cases}$$



u = displacement field

with the following boundary conditions:

$$\begin{cases} \boldsymbol{\sigma}(\mathbf{x}) \cdot \mathbf{n} = 0 \quad \text{at} \quad x_2 = 0 \\ \mathbf{u}(\mathbf{x}) \to 0 \quad \boldsymbol{\sigma}(\mathbf{x}) \to 0 \quad \text{as} \quad x_2 \to \infty \end{cases}$$

$$\mathbf{u}(x_1, x_2^-, x_3) = \mathbf{u}(x_1, x_2^+, x_3)$$

$$\boldsymbol{\sigma}(x_1, x_2^-, x_3) \cdot \mathbf{n} = \boldsymbol{\sigma}(x_1, x_2^+, x_3) \cdot \mathbf{n}$$

An equation of this form is obtained:

$$\Phi_{L/R}\,[\lambda(x_2), \mu(x_2), \rho(x_2), k, \omega] = 0$$

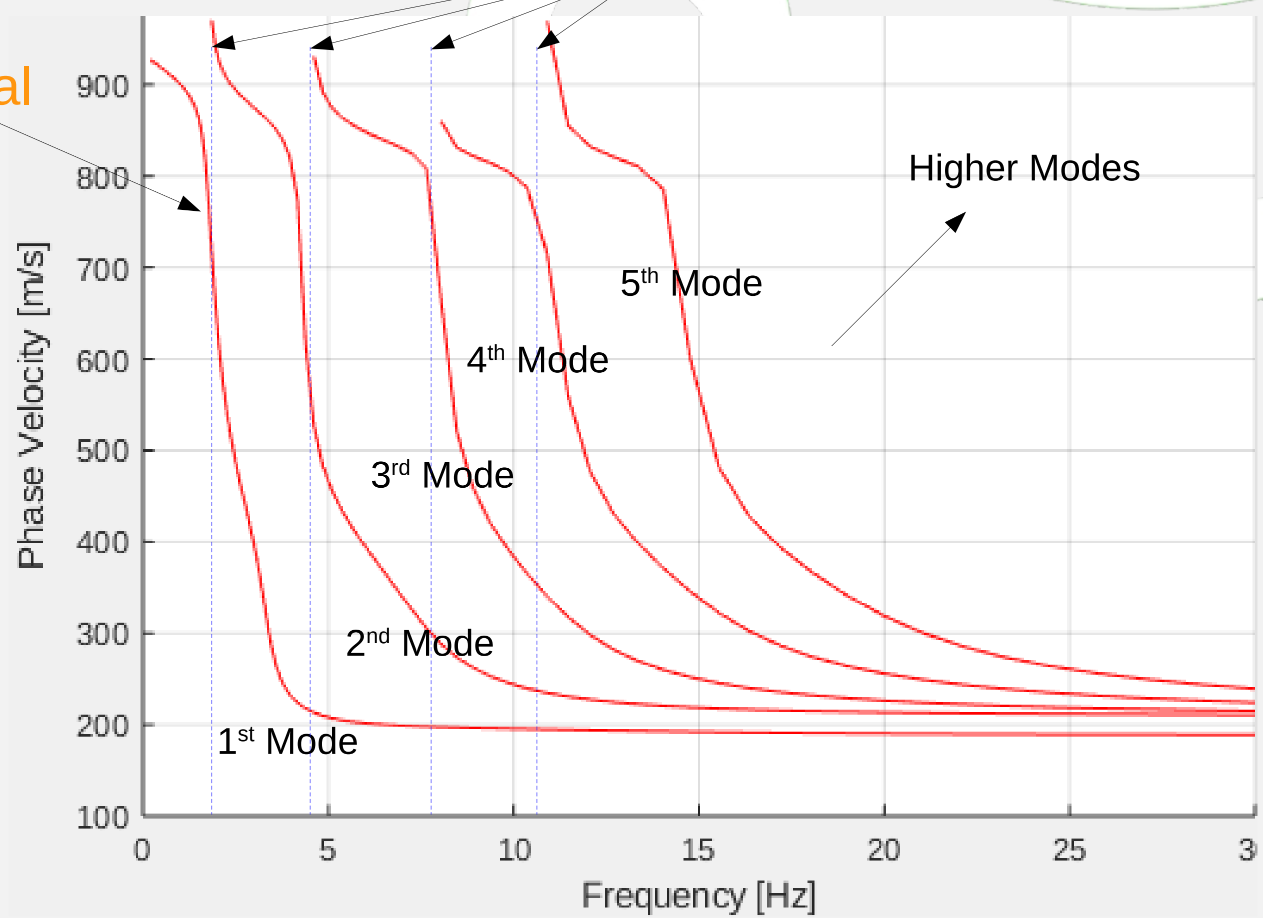
for each ω different *k*'s satisfy the equation

It states that in vertically inhomogeneous media, the velocity of propagation of surface Love and Rayleigh waves is a multivalued function of frequency

Foti et al. 2015

Multi-modal phenomenon

- The Rayleigh wave propagation in vertically heterogeneous media (1D), is a multi-modal phenomenon: at each frequency different wavelengths can propagate
- The excitation of different modes depends on the model layering



Model

One line per layer:

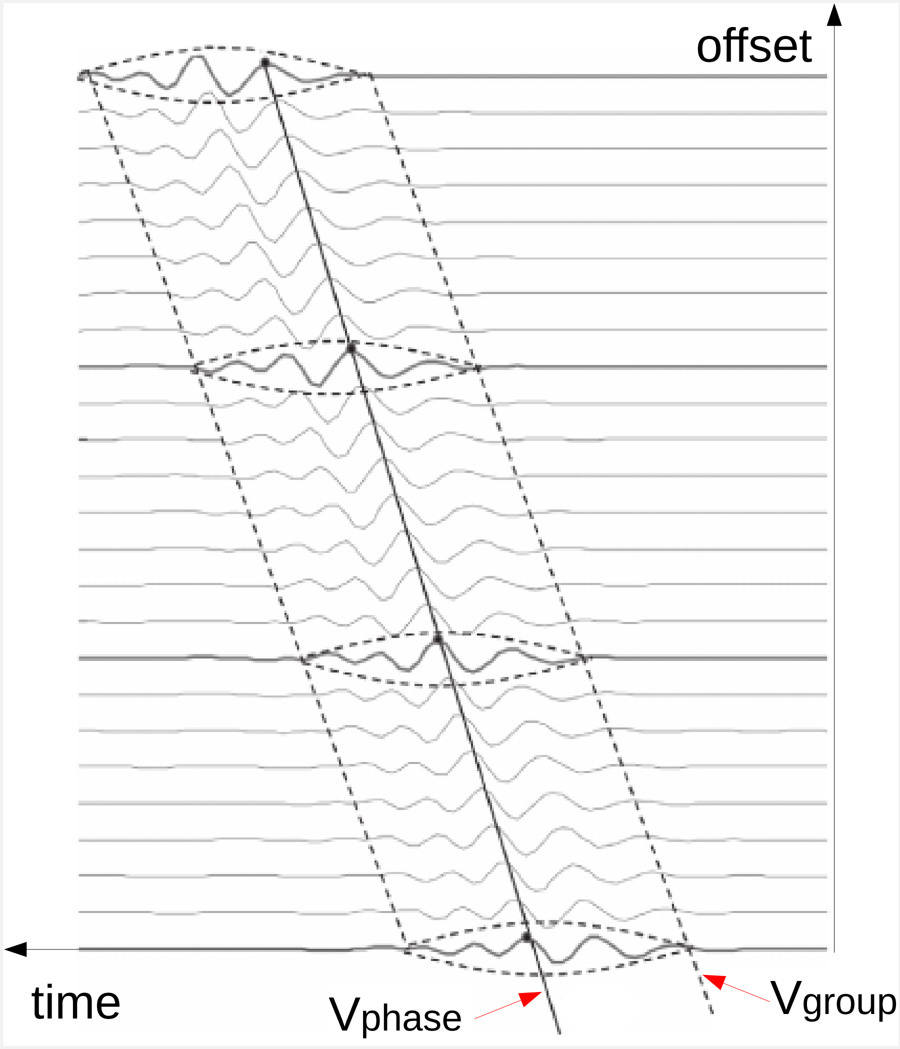
h (m)	Vp	Vs	ρ
7.5	500	200	1700
25	1350	210	1900

Last line is the half-space

∞	2000	1000	2500

ρ (kg/m^3)
Vp and Vs (m/s)
h (m)

Phase Velocity and Group Velocity



Difference between phase velocity and group velocity

Definitions

$$V_{phase} = \frac{\omega}{k}$$

$$V_{group} = \frac{d\omega}{dk}$$

$$\begin{cases} V_{phase}\, dk = d\omega \\ \dfrac{\omega}{d\omega} = \dfrac{k}{dk} \\ \dfrac{\lambda}{d\lambda} = -\dfrac{f}{df} \end{cases}$$

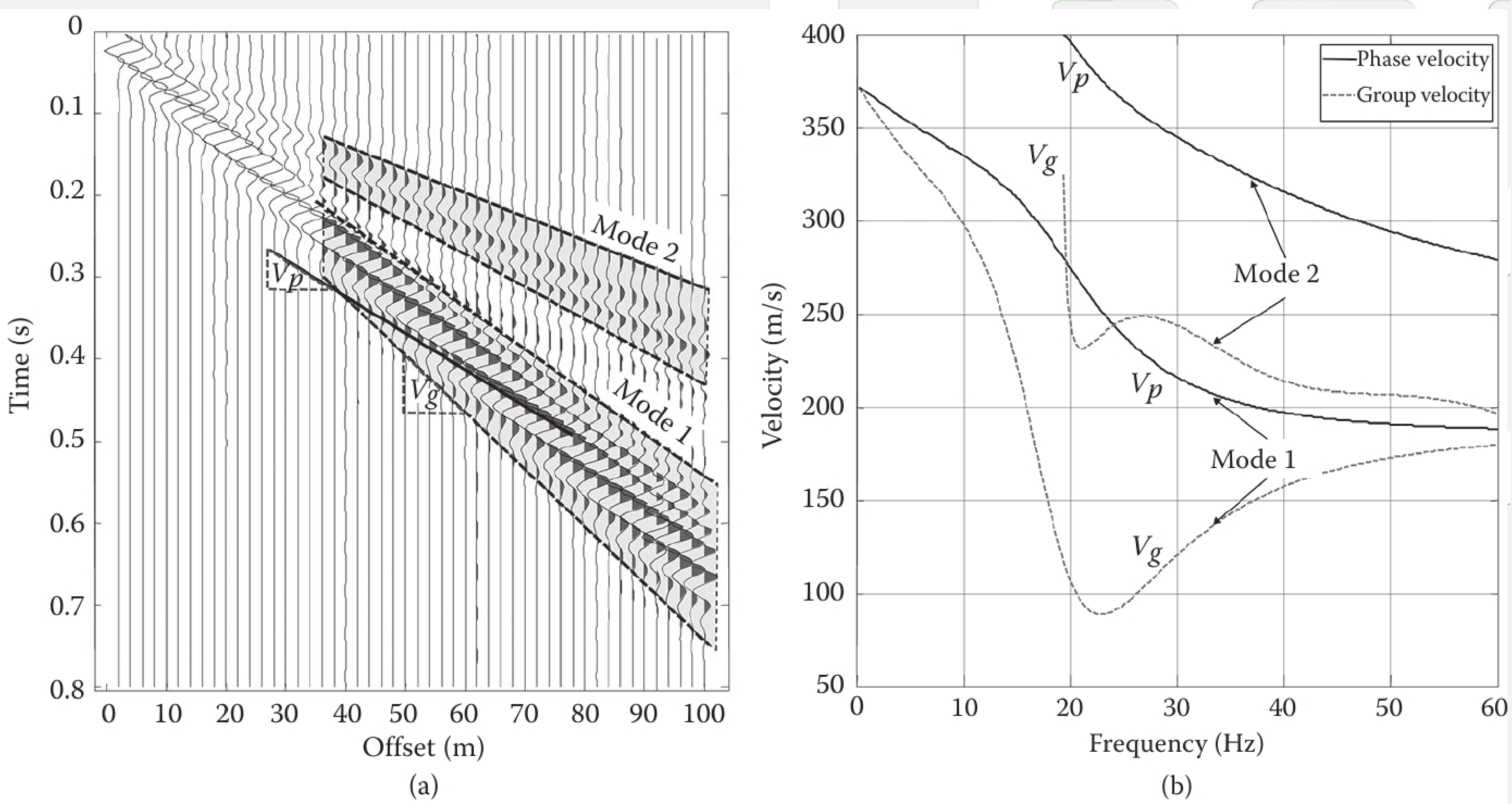
We can express the group velocity as a function of the phase velocity:

$$V_{group} = \frac{d(V_{phase}\, k)}{dk} = V_{phase} + \frac{dV_{phase}}{dk} k$$

$$V_{group} = V_{phase} + \omega \frac{dV_{phase}}{d\omega}$$

$$V_{group} = V_{phase} - \lambda \frac{dV_{phase}}{d\lambda}$$

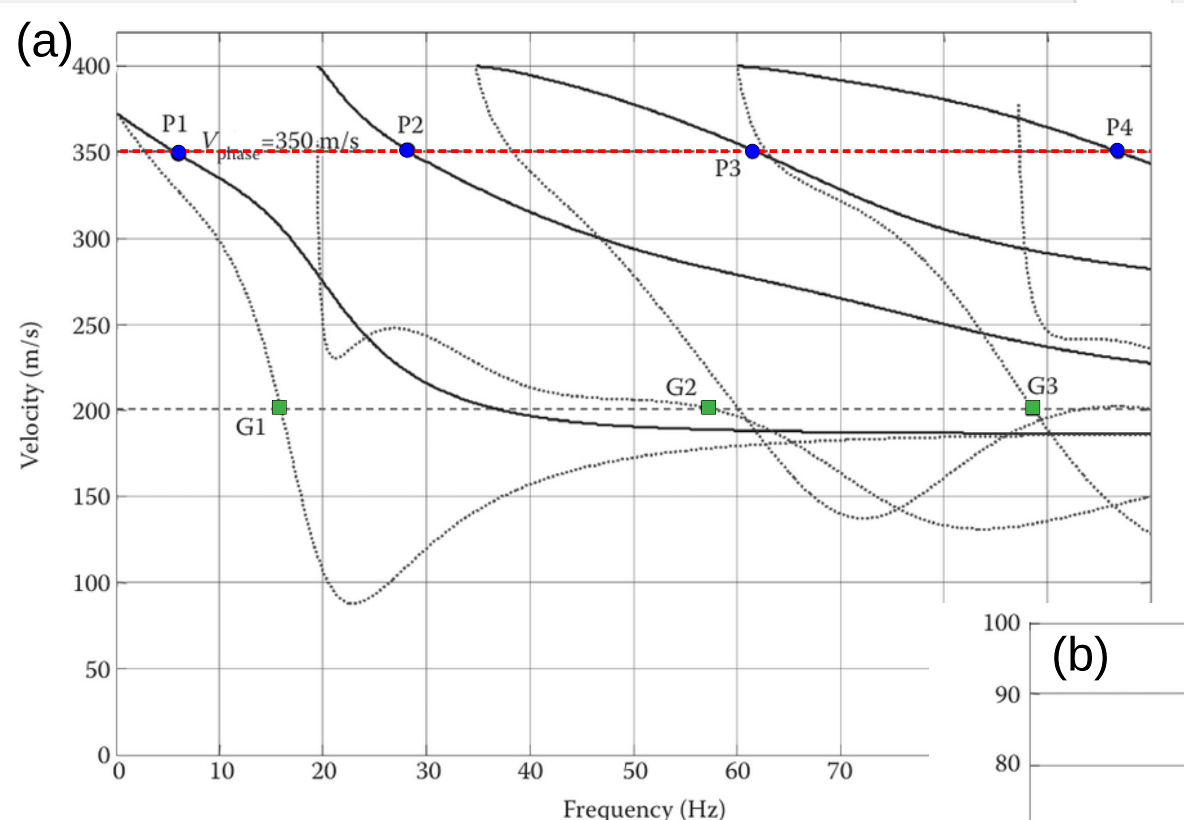
Dispersion Curves: Phase and Group Velocity



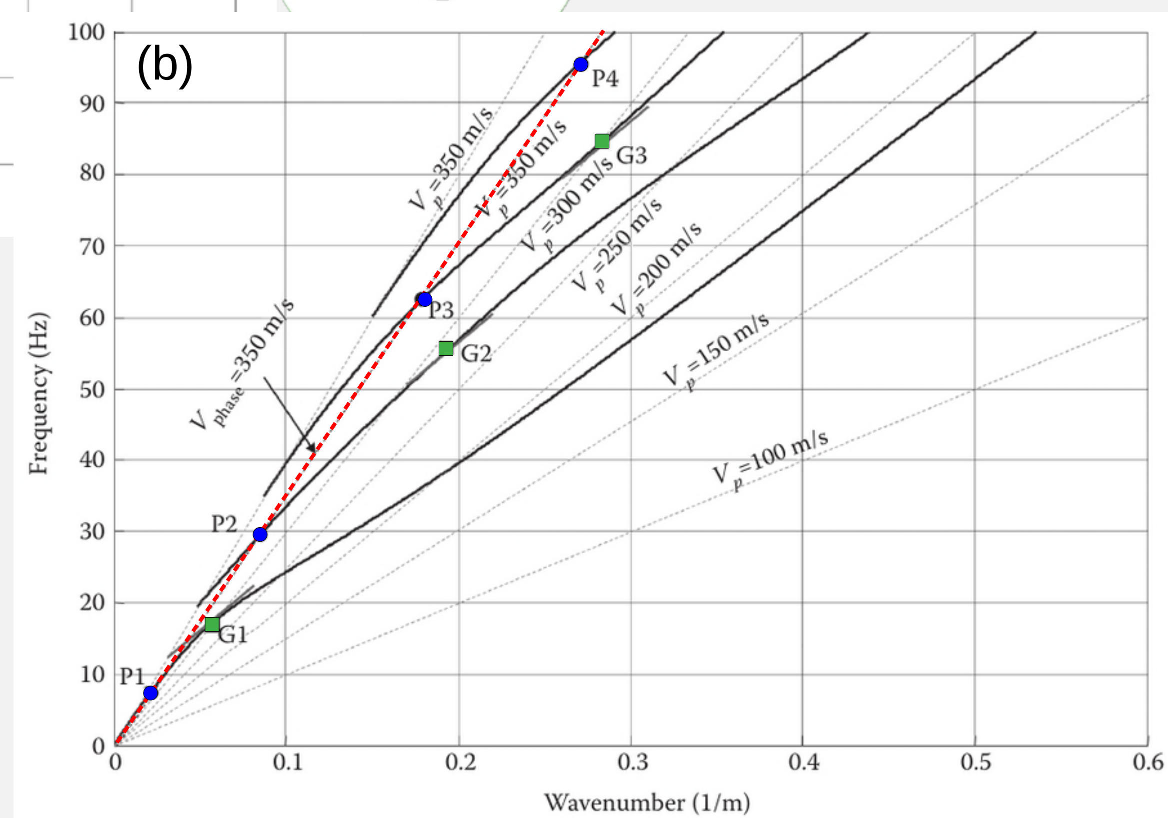
Example with synthetic data of surface waves with two dispersive modes:
(a) synthetic shot gather; (b) phase velocity and group velocity dispersion curves

Foti et al. 2015

Dispersion Curves: Phase and Group Velocity



Example of surface waves with four dispersive modes. Phase velocity and group velocity dispersion curves are represented as:
(a) velocity versus frequency and
(b) frequency versus wavenumber.
Dotted straight lines in (b) represent constant phase velocity events



Group velocity can be computed by the relation:

$$V_{group} = V_{phase} + \omega \frac{dV_{phase}}{d\,\omega}$$

Foti et al. 2015

MASW

The Multichannel Analysis of Surface Waves (MASW) is based on the analysis of the geometric dispersion of surface waves

MASW:

- measures seismic surface waves generated from various types of seismic sources
- analyzes the propagation velocities of those surface waves,
- estimates shear-wave velocity (Vs) variations below the surveyed area that is most responsible for the analyzed propagation velocity pattern of surface waves.

- Shear-wave velocity (Vs) is closely related to shear modulus.

Under most circumstances, Vs is a direct indicator of the ground strength (stiffness) and therefore commonly used to derive load-bearing capacity

- Mapping the top 20-30m [and more] of the subsurface

$$V_{s,\mathrm{eq}} = \frac{H}{\sum_{l=1}^{N} \frac{h(l)}{V_s(l)}}$$

Usually:
H=30 m

Usual Assumptions

- Horizontally layered medium (no lateral variation!)

- Only plane Rayleigh waves (far field, body waves contribution negligible)

- Fundamental mode is dominant (usually)

Pros and Cons

−

The assumed model is 1D, and so the result is one-dimensional

+

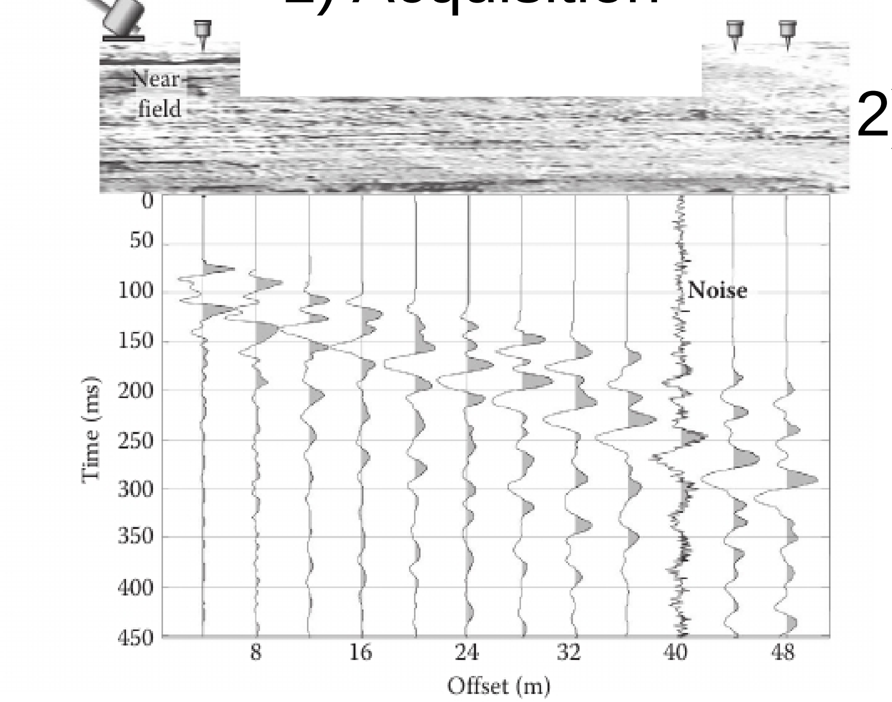
Overcome some intrinsic limitations of the refraction technique (hidden layer, velocity inversion)

Higher sensitivity to the mechanical properties of the solid skeleton in saturated materials

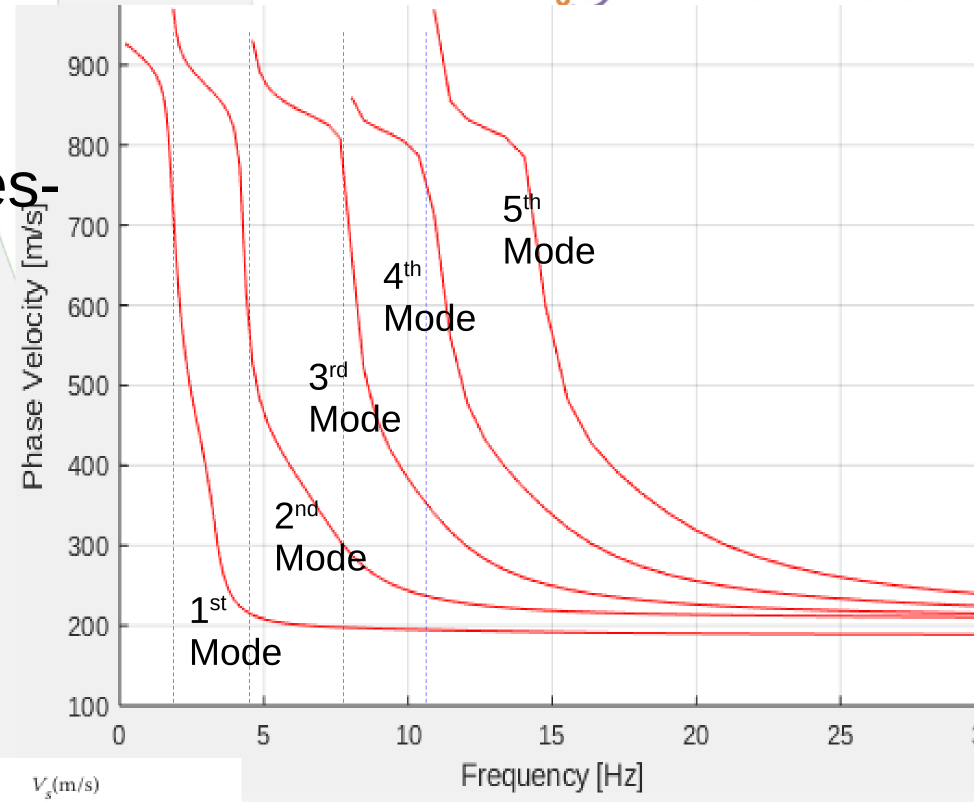
Acquisition of P-wave refraction and SW data is very similar and can be performed simultaneously

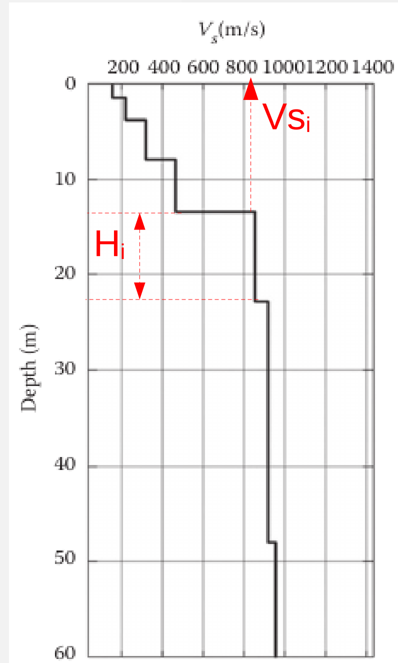
Workflow

1) Acquisition



2) Proces-sing





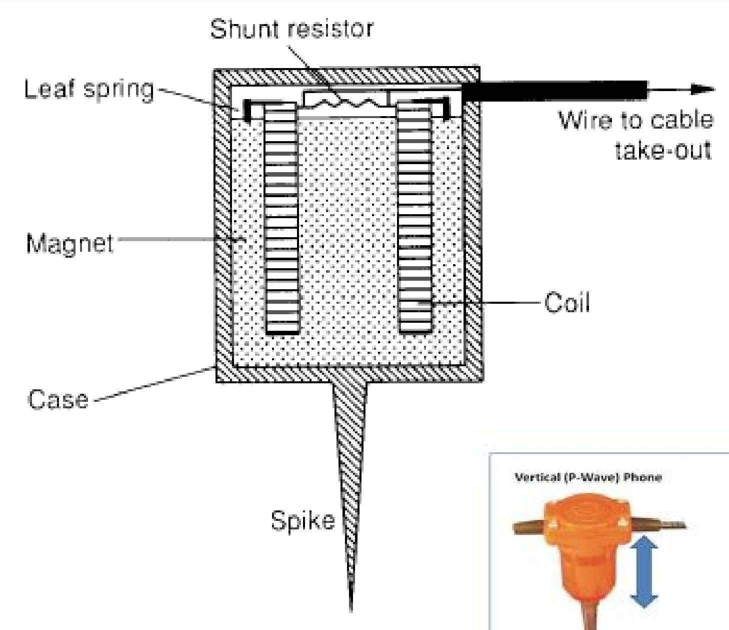
3) Inversion

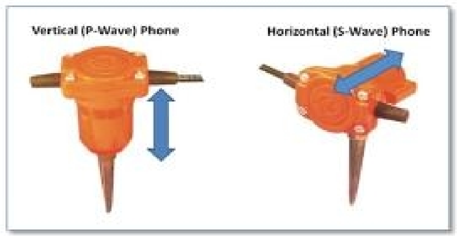
Estimated model parameters

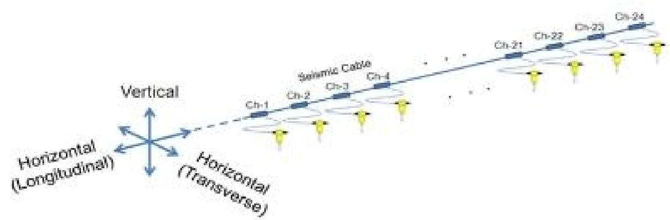
H_i, Vs_i

for each layer

Acquisition - Geophone





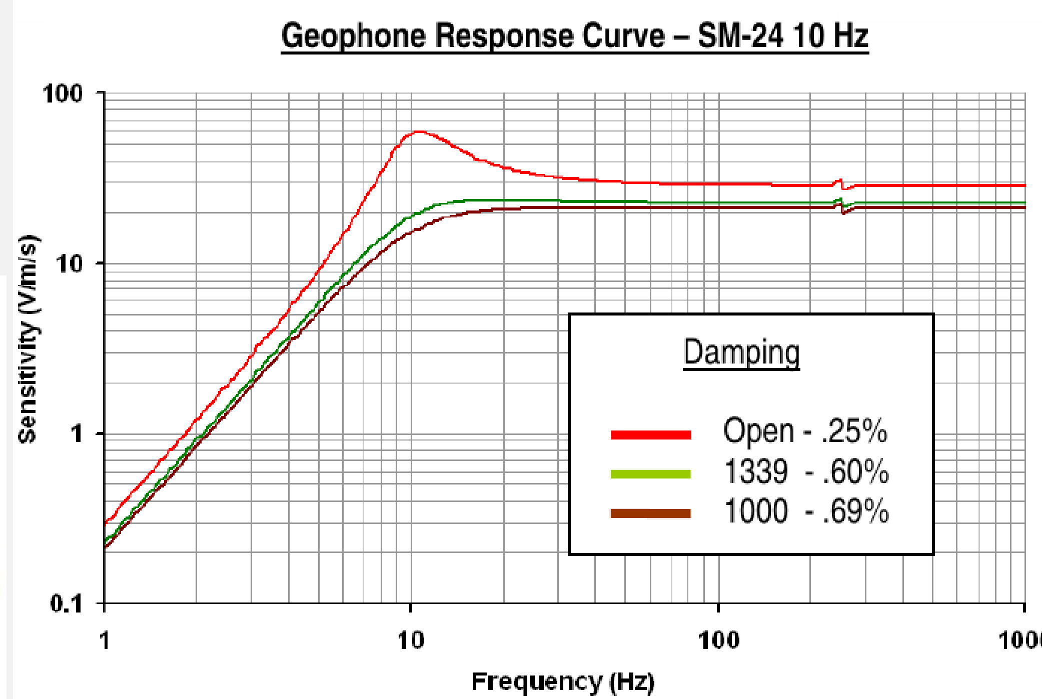


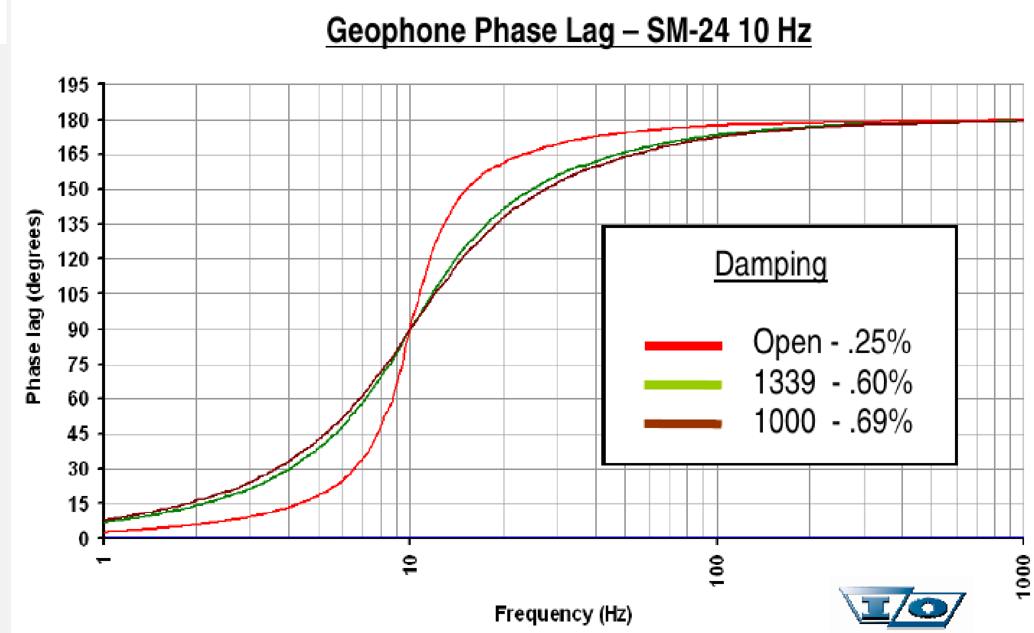
Importance of Geophone Frequency (f_G)

- f_G determines the lowest frequency (f_{min}) measurable: $f_{min} \approx f_G$.
- Then, f_{min} determines the longest wavelength (λ_{max}) measurable:
  - $\lambda_{max} = V_{max}/f_{min} \approx V_{max}/f_G$ (V_{max} = maximum phase velocity measured)
- **Therefore, f_G determines the maximum investigation depth (Z_{max}) of an MASW survey:**
  - $Z_{max} \approx ½ \lambda_{max} \approx ½ V_{max}/f_G$

Table: Geophone frequency and possible investigation depth (Zmax)

Geophone Frequency (f_G)	Max. Investigation Depth (Z_{max})
≥ 100 Hz	≤ 5 m
30 – 100 Hz	≤ 10 m
10 – 30 Hz	≤ 20 m
4 – 10 Hz	≤ 30 m
1 – 4 Hz	≤ 100 m

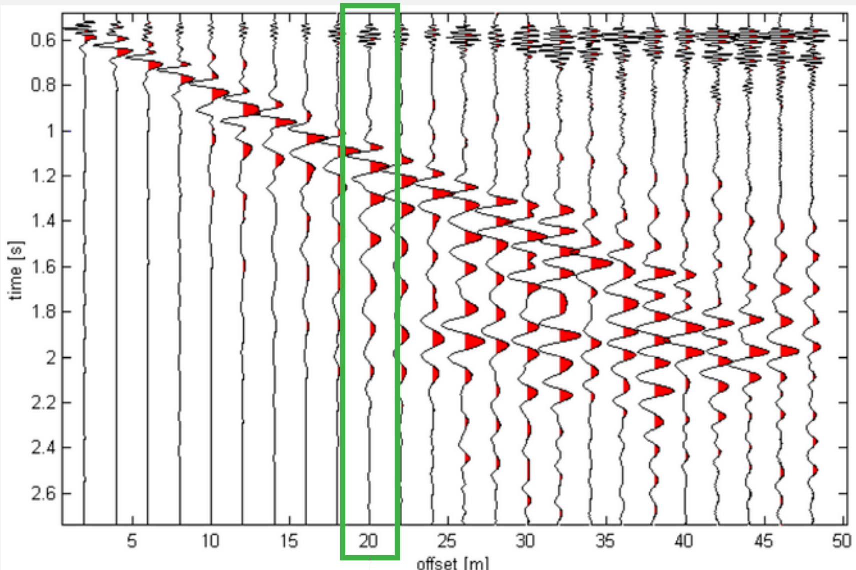




http://www.masw.com/Receivers.html

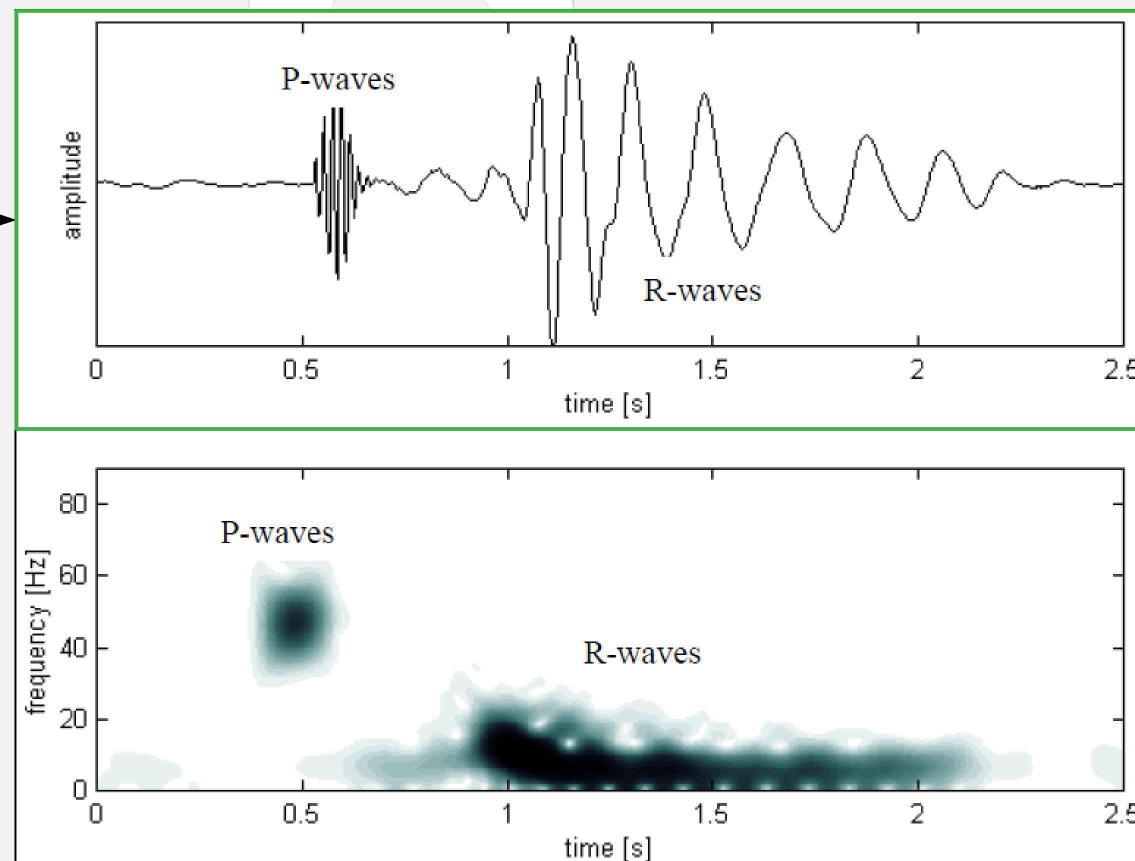
Acquisition – Body and Surface Waves

Hammer source



In surface wave inversion the body waves are considered as coherent noise

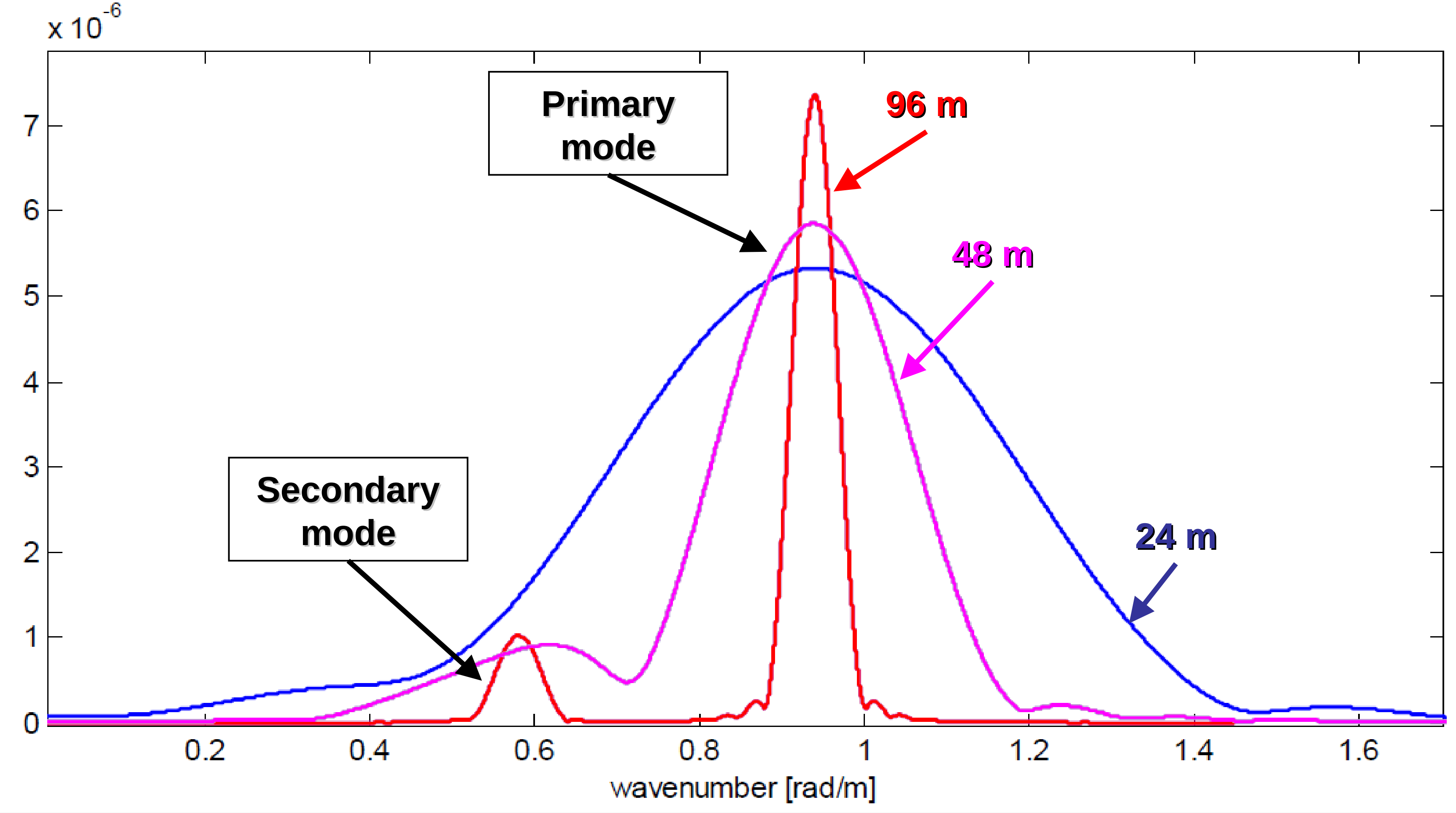
This coherent noise can be filtered out or muted before the inversion



FT energy spectrum of the trace at 20 m offset evidencing the separation in frequency and time between the P-waves and the Rayleigh waves

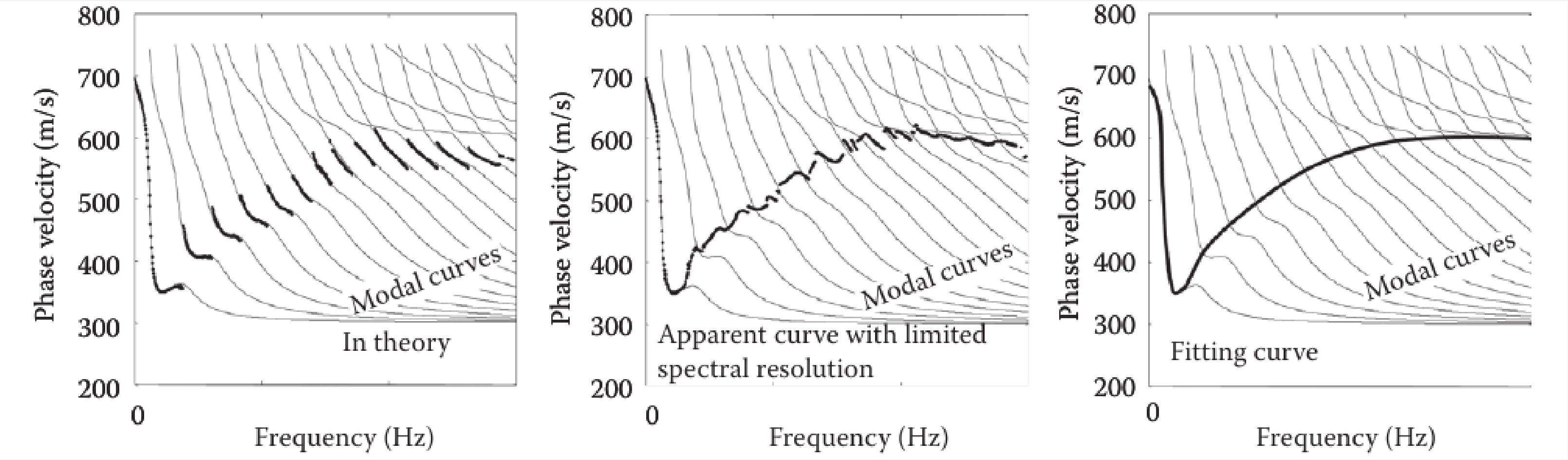
(Strobbia, 2003)

Acquisition – Array Length



Array length influences wavenumber resolution and mode separation

(Strobbia, 2003)

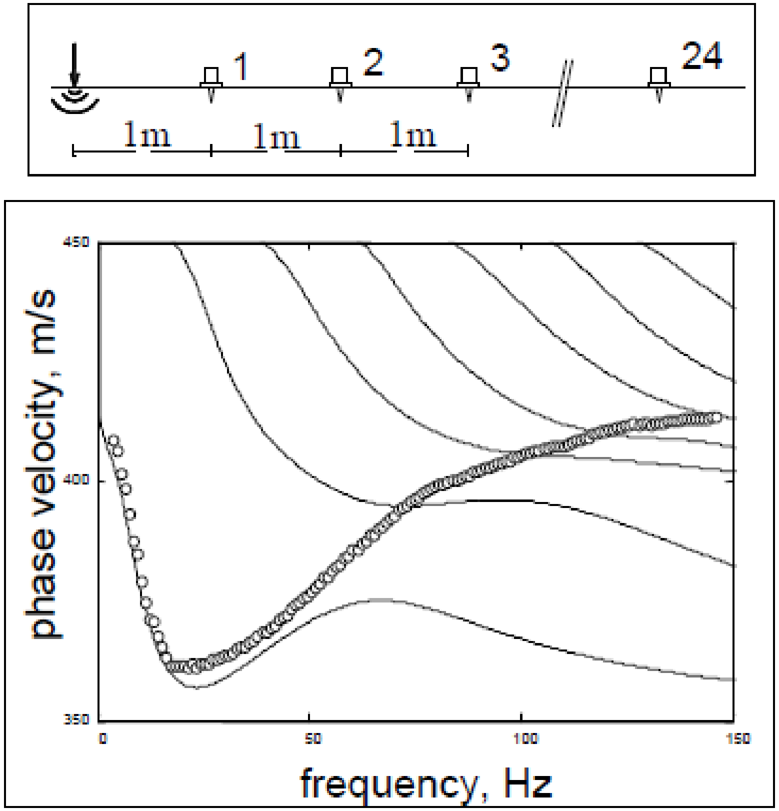


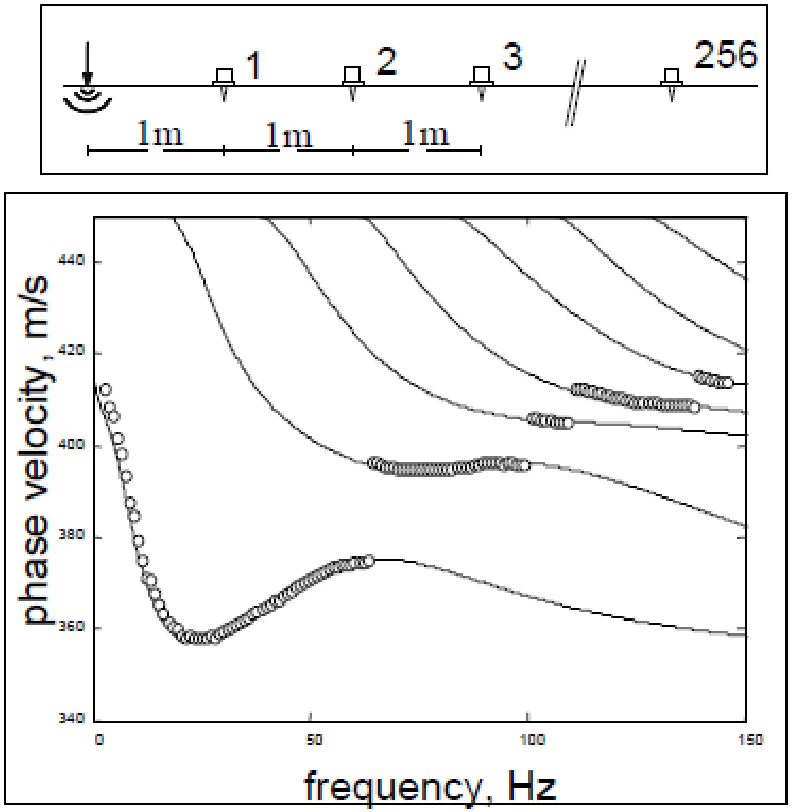
Example of modal superposition with two different arrays for the same synthetic case.

(Foti, 2015)

Acquisition – Array Length
Apparent Dispersion Curve

The apparent dispersion curve:
- is due to modal superposition
- is not an intrinsic property of the site
- but depends on the acquisition and the processing procedures





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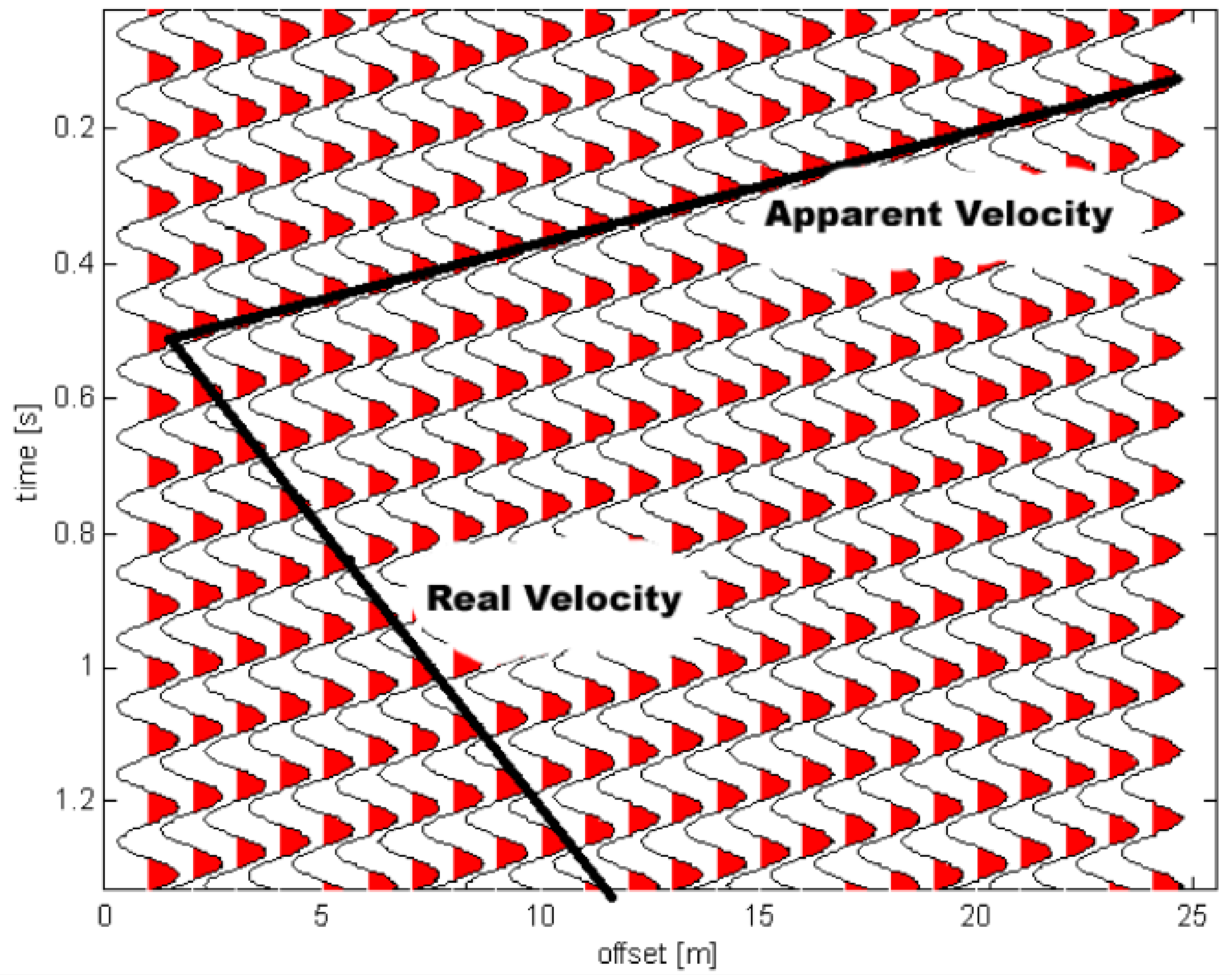
Acquisition - Receiver Spacing

Receiver spacing influences spatial aliasing and maximum wavenumber



$$k_{Nyq} = \frac{1}{2}\frac{1}{\Delta x}$$

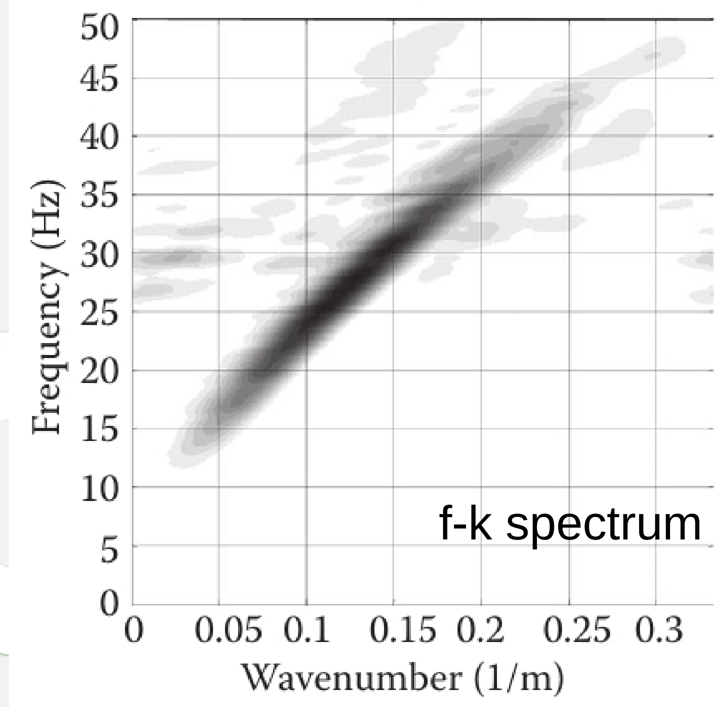
Apparent negative velocity caused by a too coarse spatial sampling

Processing - Dispersion Curves in f-k Domain

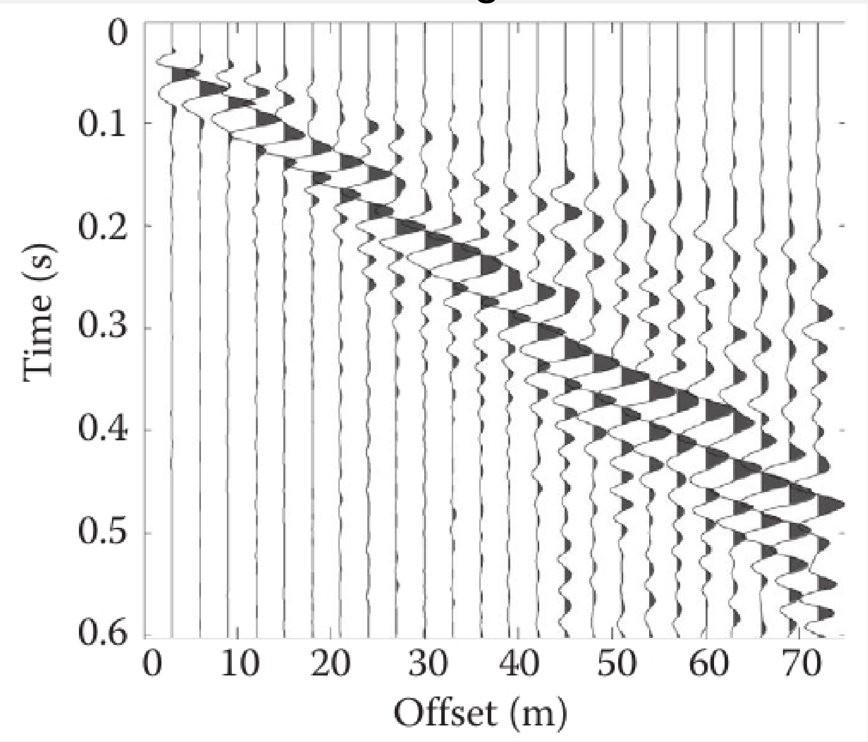
Bi-dimensional Fourier transform:

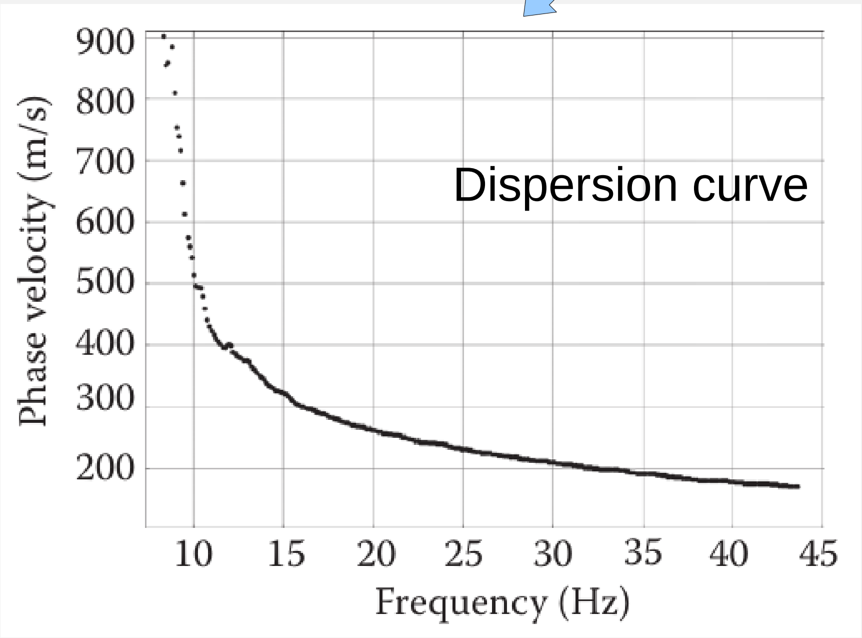
$$S(f,k)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(t,x)\,e^{-j2\pi(kx+ft)}\,dx\,dt$$

Picking of maxima in the f-k spectrum allows to build the experimental dispersion curve



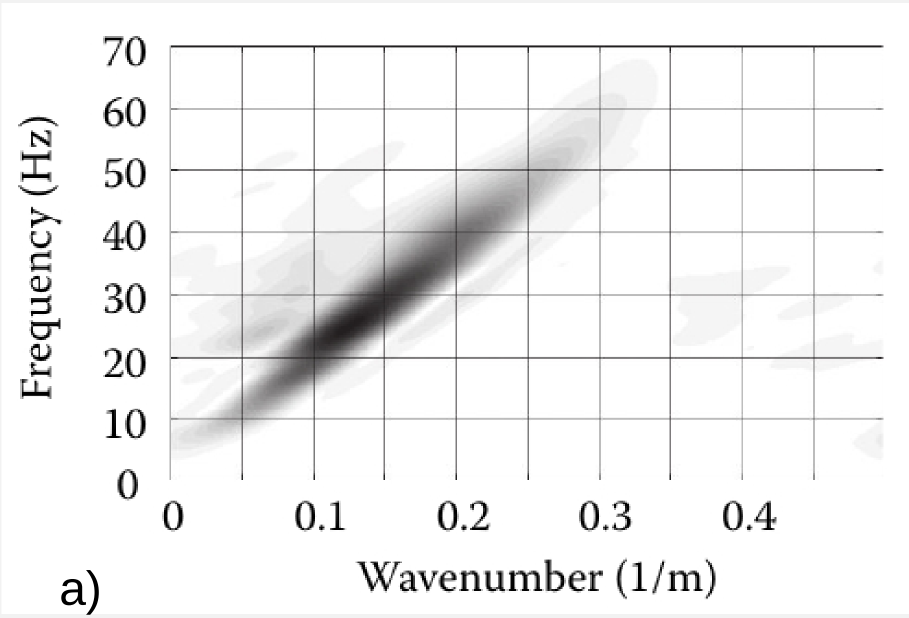
Seismogram

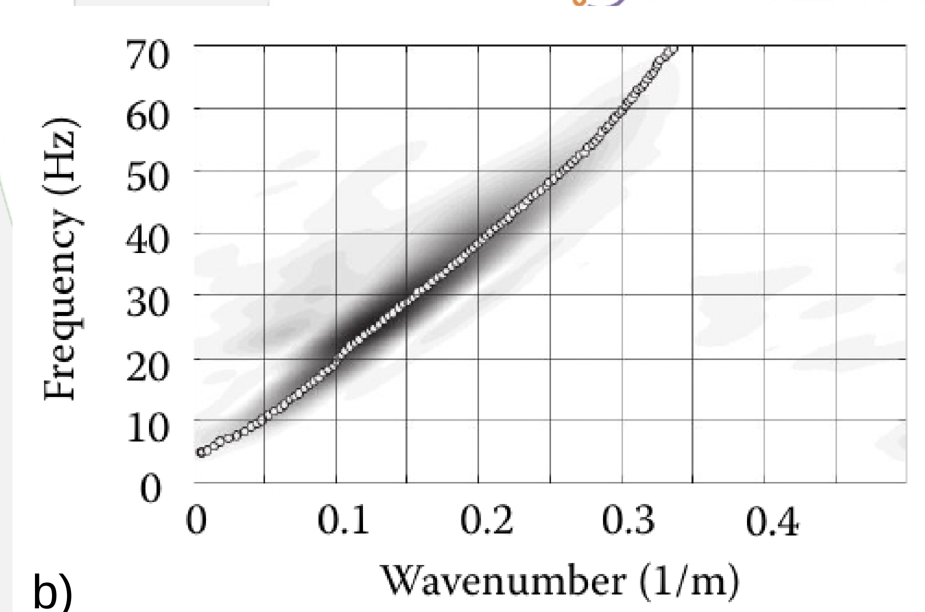




Foti et al. 2015

Processing - Dispersion Curves in f-k Domain



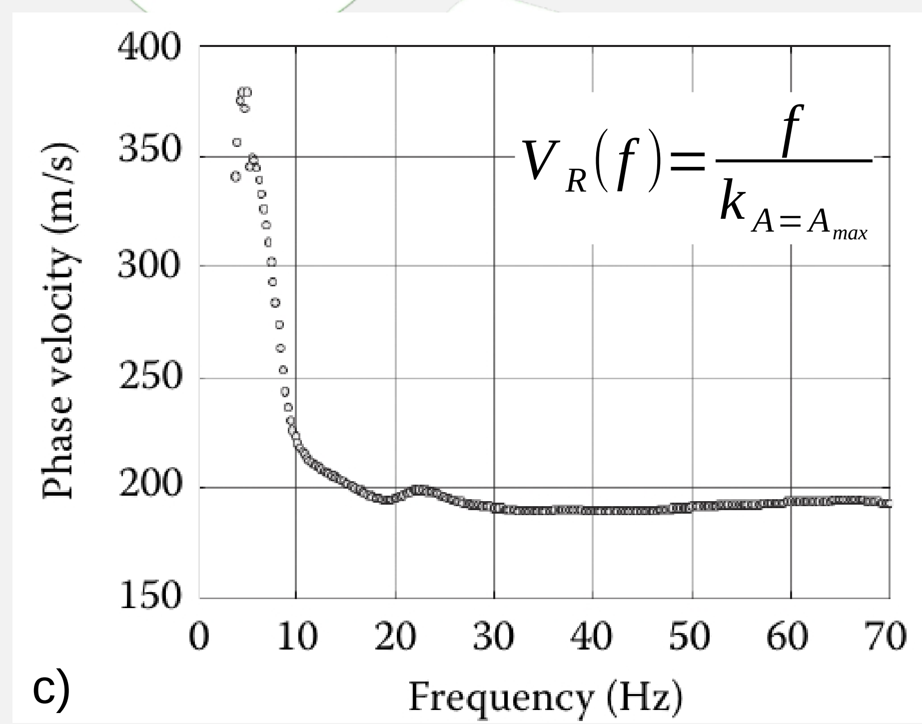


Example of dispersion analysis in the frequency domain using the 2D Fourier transform:

(a) f–k spectrum;

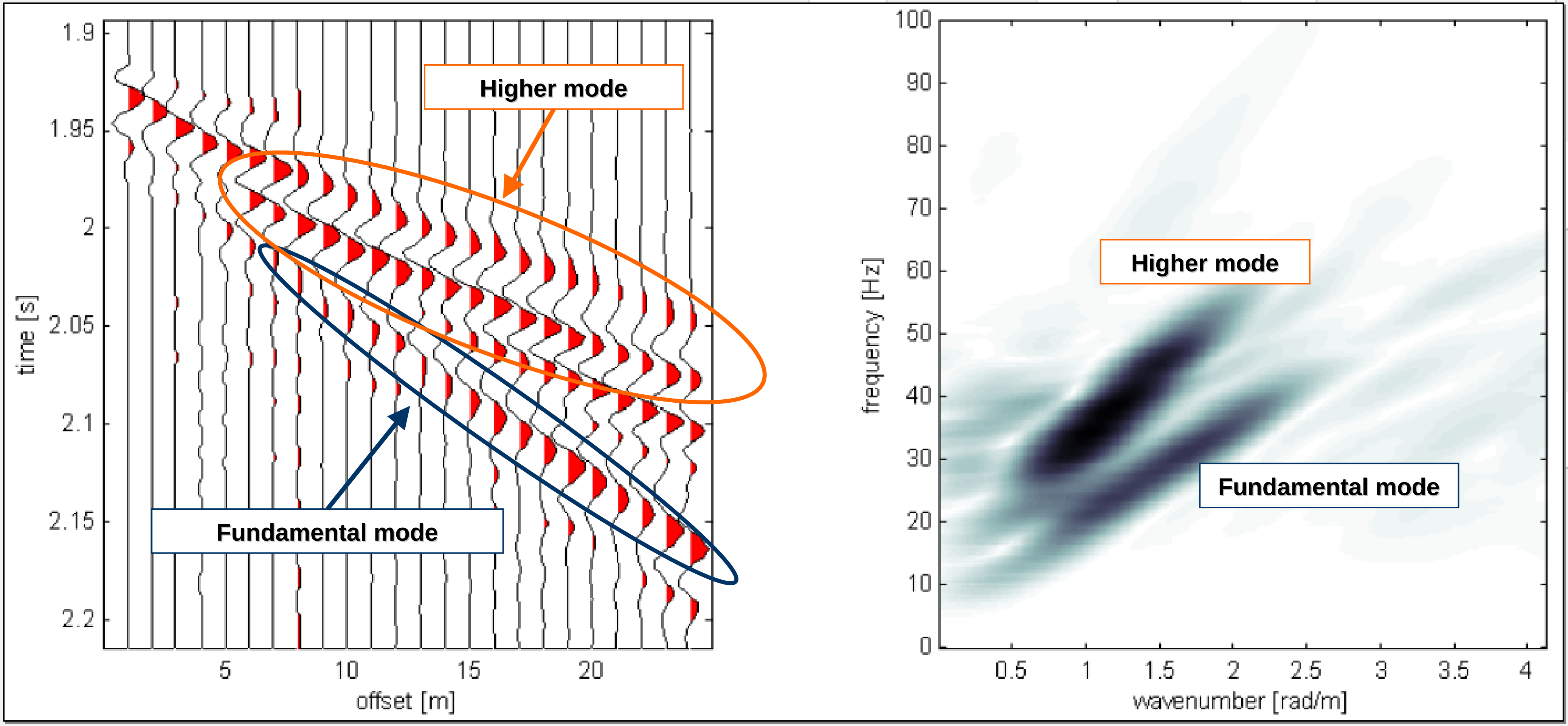
(b) picking of maxima in the f–k spectrum;

(c) experimental dispersion curve



Foti et al. 2015

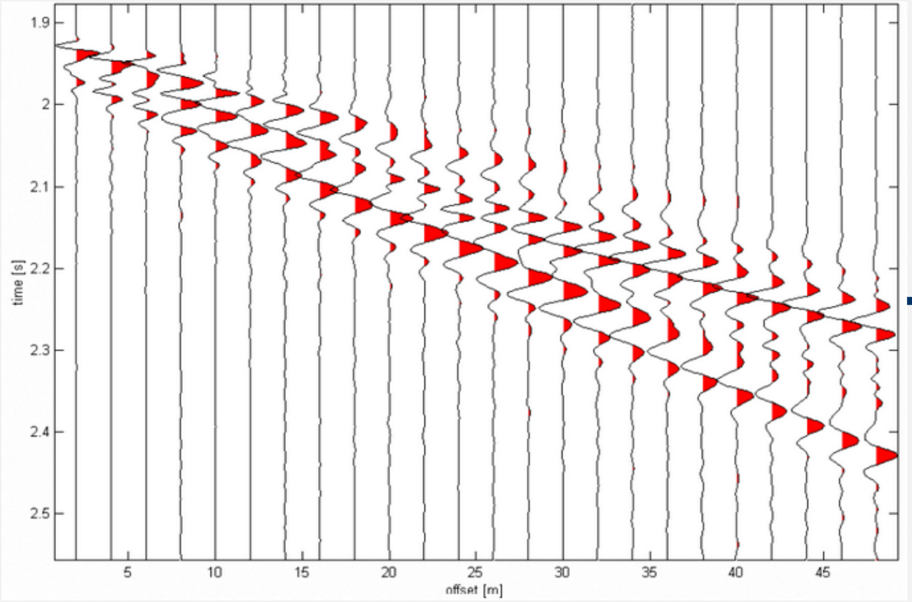
Processing - Dispersion Curves in f-k Domain
Higher Modes

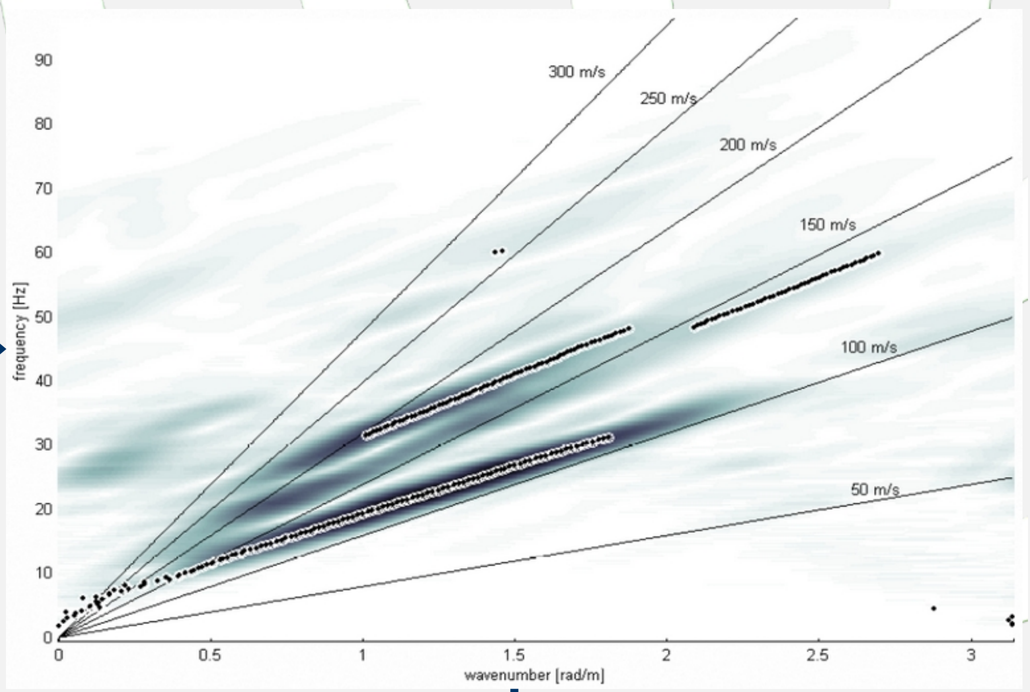


(Strobbia, 2003)

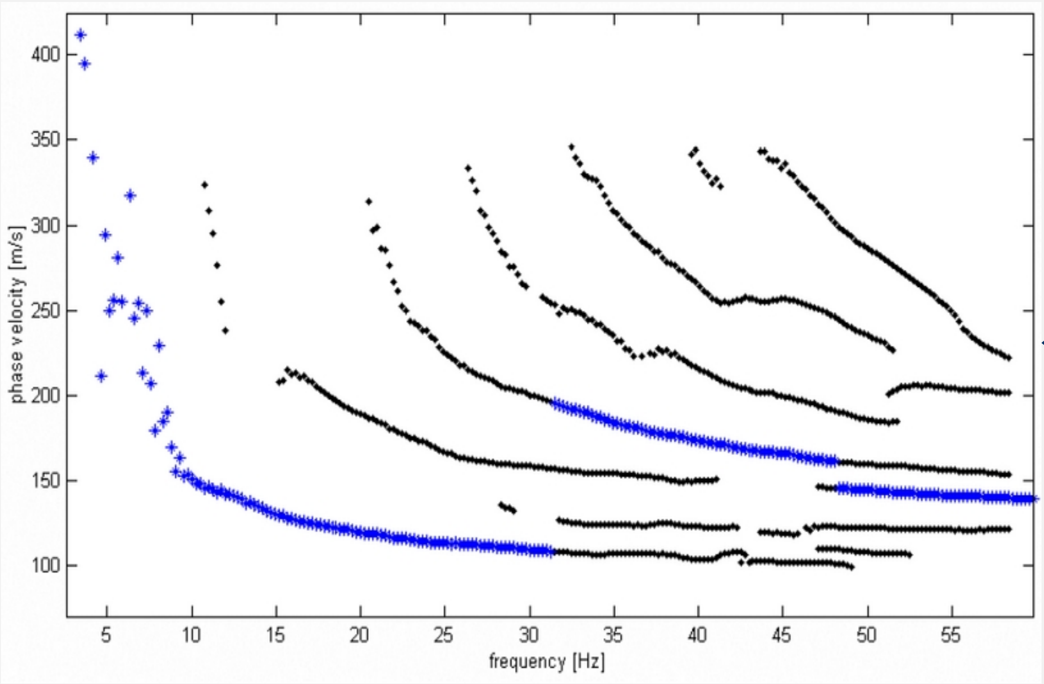
Different modes can be separated in the f-k domain due to their different phase velocity and processing applied if required.

Processing - Dispersion Curves in f-k Domain
Picking Higher Modes(?)





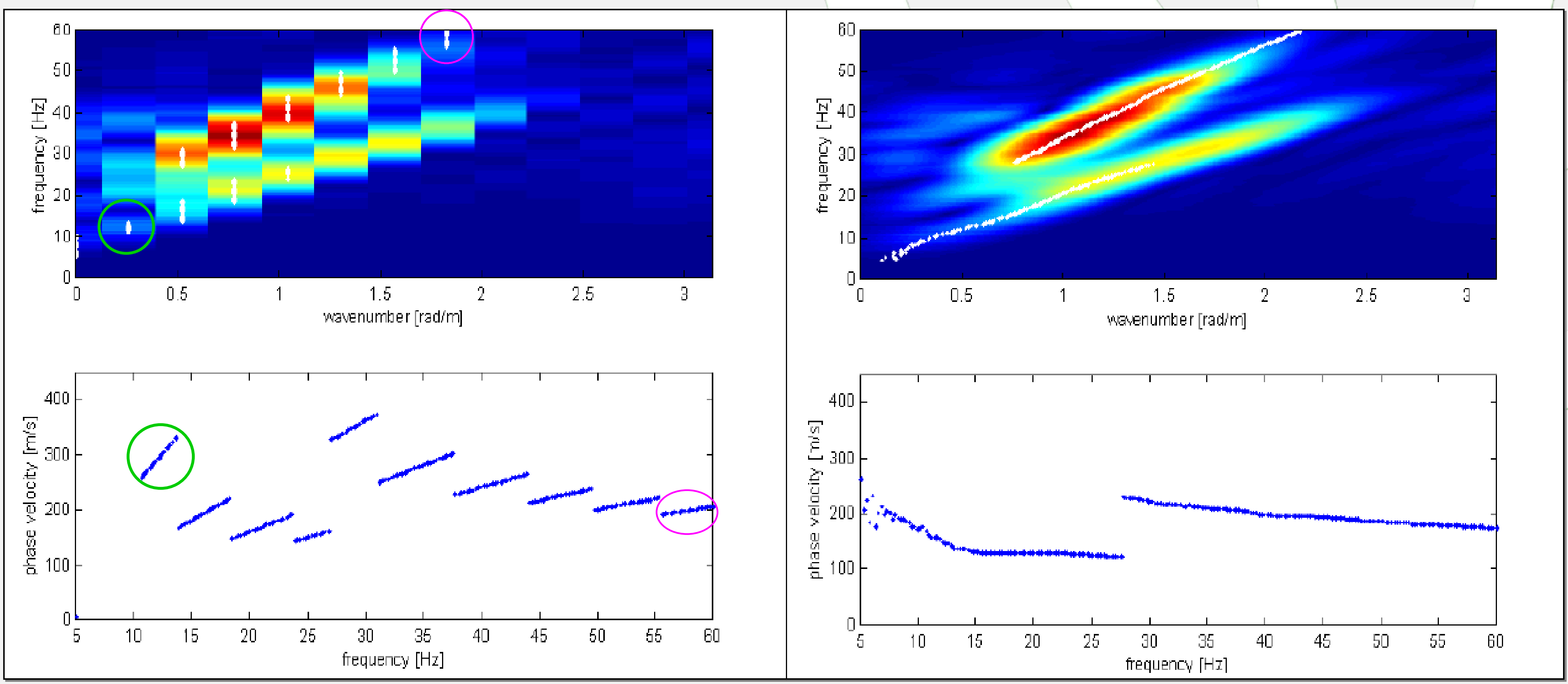
(Foti, 2004)



This example highlights the possibility to pick in the f-k domain different portions of the dispersion curve relative to different modes

Processing - Dispersion Curves in f-k Domain
Spatial Zero Padding

Without | With



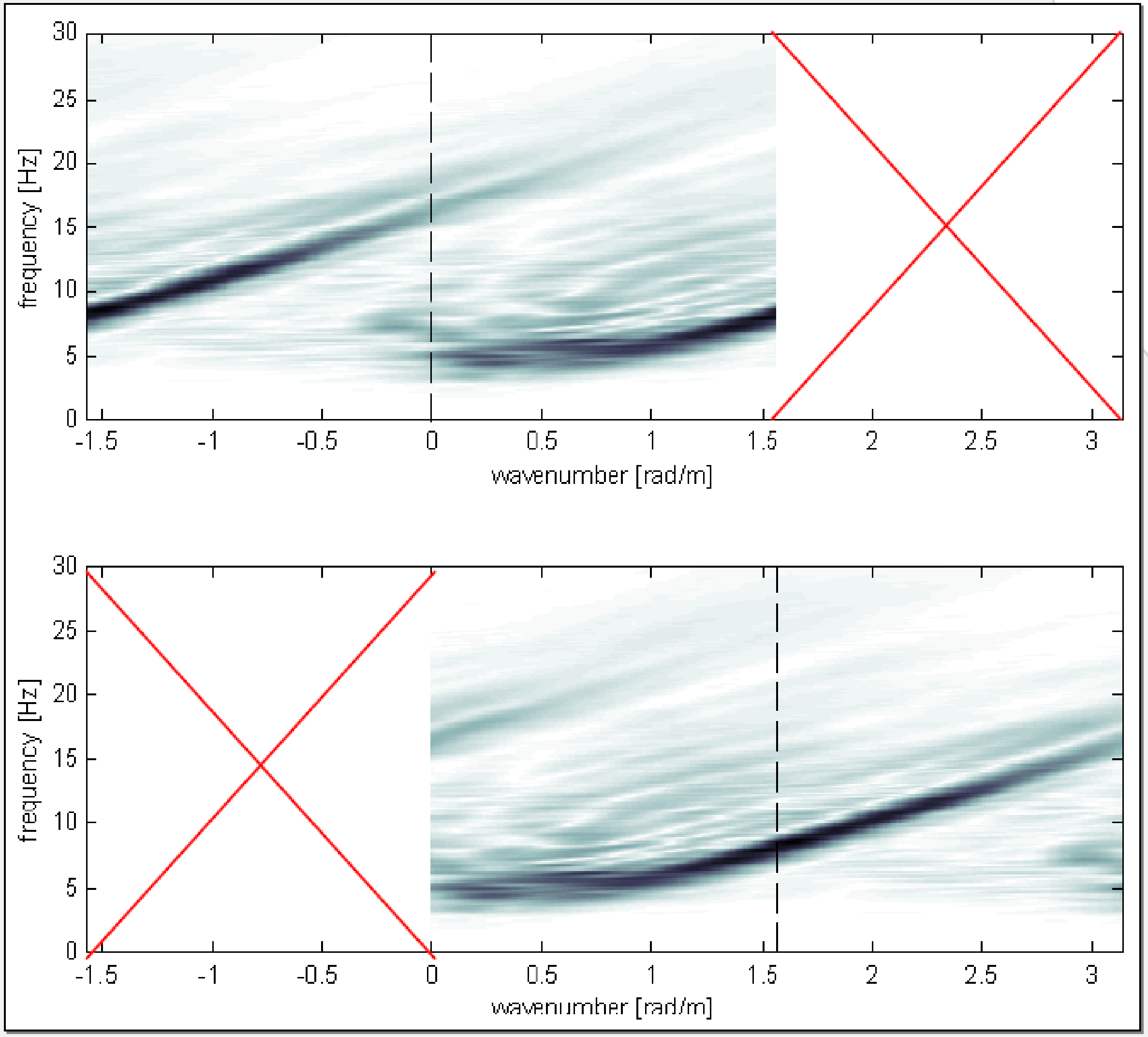
(Strobbia, 2003)

$$\Delta k = \frac{1}{N \cdot \Delta x}$$

$$\Delta k^* = \frac{1}{(N + zp) \cdot \Delta x}$$

Processing - Dispersion Curves in f-k Domain
Spatial Aliasing Recovering

In some cases spatial aliasing can be recovered unwrapping by 2*K_N one quadrant



(Strobbia, 2003)

In the example shown the left quadrant is moved to the right of the K_N wavenumber, in this way the picking of the dispersion curve can be extended up to 2*K_N

$$k_{Max}=\frac{1}{2}\frac{1}{\Delta x}\Rightarrow\lambda_{\min}=2\,\Delta x$$

$$k_{Max}=\frac{1}{\Delta x}\Rightarrow\lambda_{\min}=\Delta x$$

Vphase – Frequency Panel:
Linear Radon Transform + Fourier Transform

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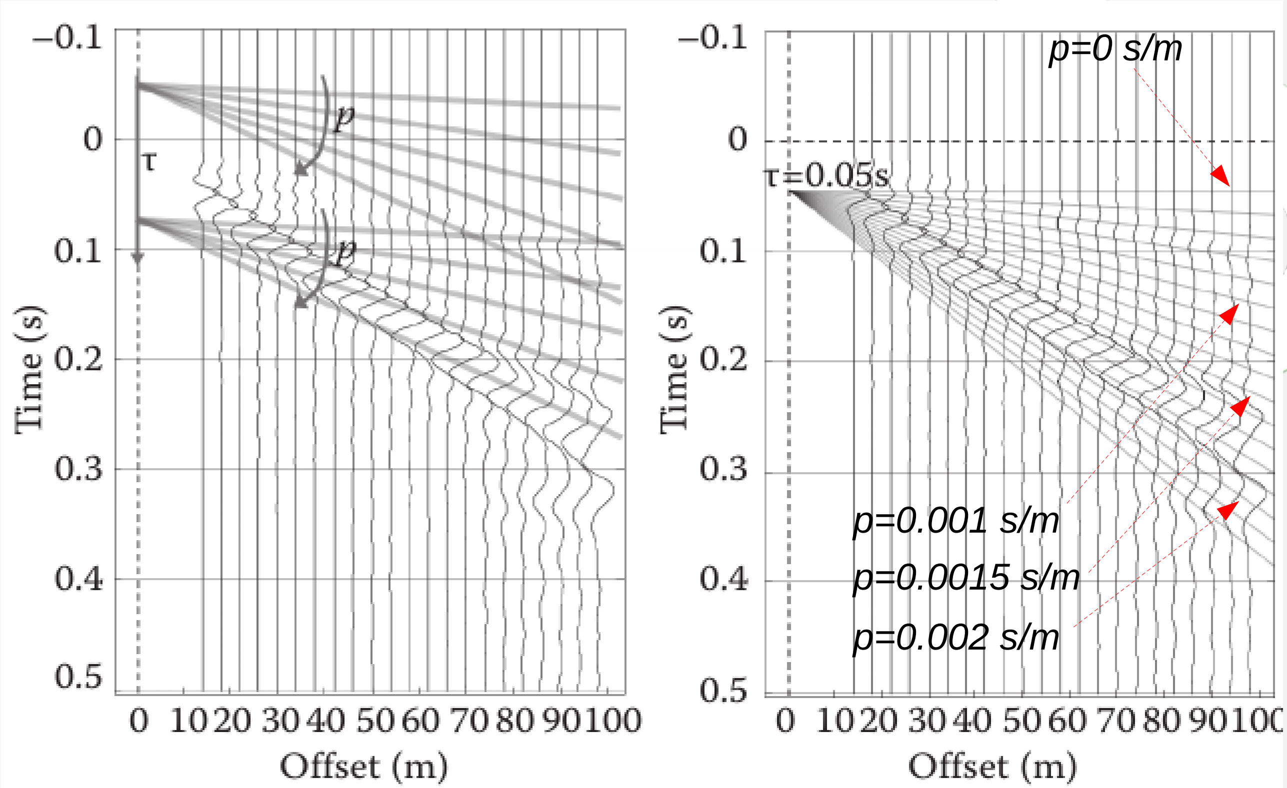
τ–p transform: $$u(\tau, p) = \int_{-\infty}^{\infty} d(\tau + px, x)\, dx$$



Two conditions must be satisfied to avoid alias in the τ–p transform:

1) $$\Delta p \leq \frac{1}{f_{max}\, x_{max}}$$

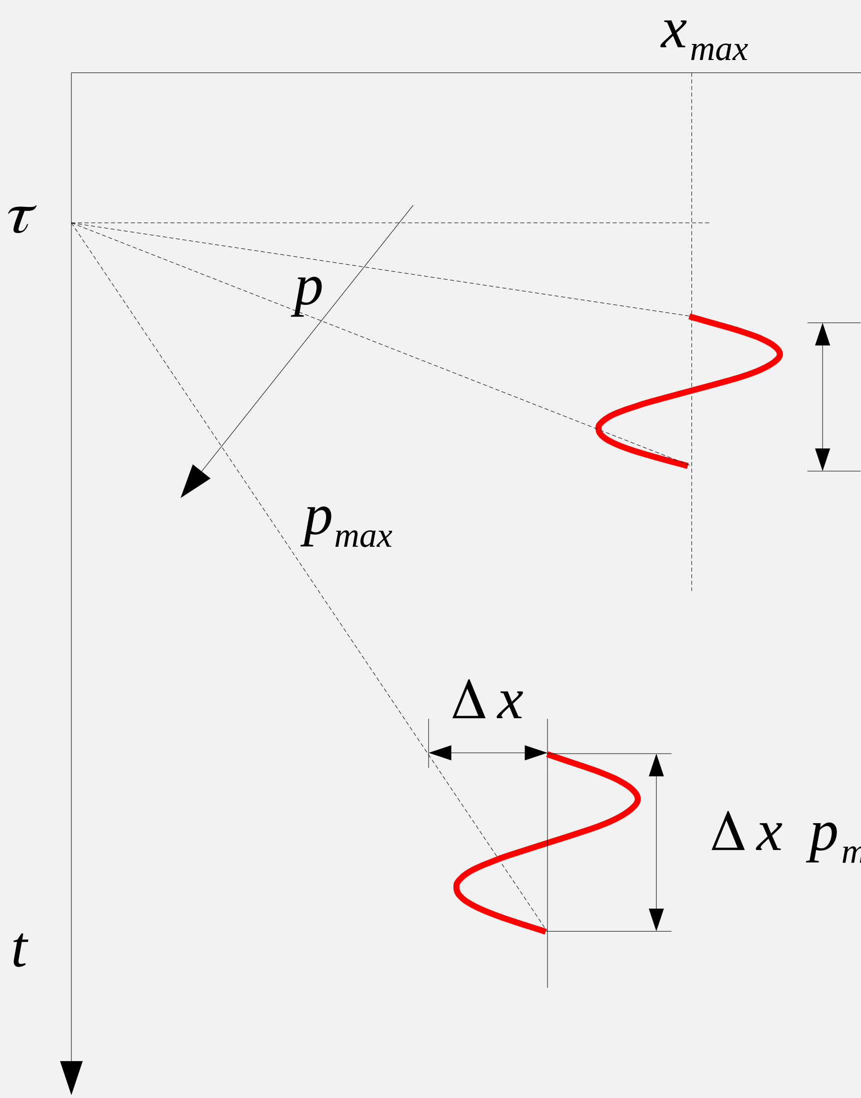
2) $$p_{max} \leq \frac{1}{f_{max}\, \Delta x}$$

Schematic representation of the principle of τ–p transform, or linear Radon transform

The data in time-offset are summed along straight lines, with intercept τ and slope p.

Linear Radon Transform: Δp and p_{max}

τ–p transform: $$u(\tau, p) = \int_{-\infty}^{\infty} d(\tau + px, x)\, dx$$



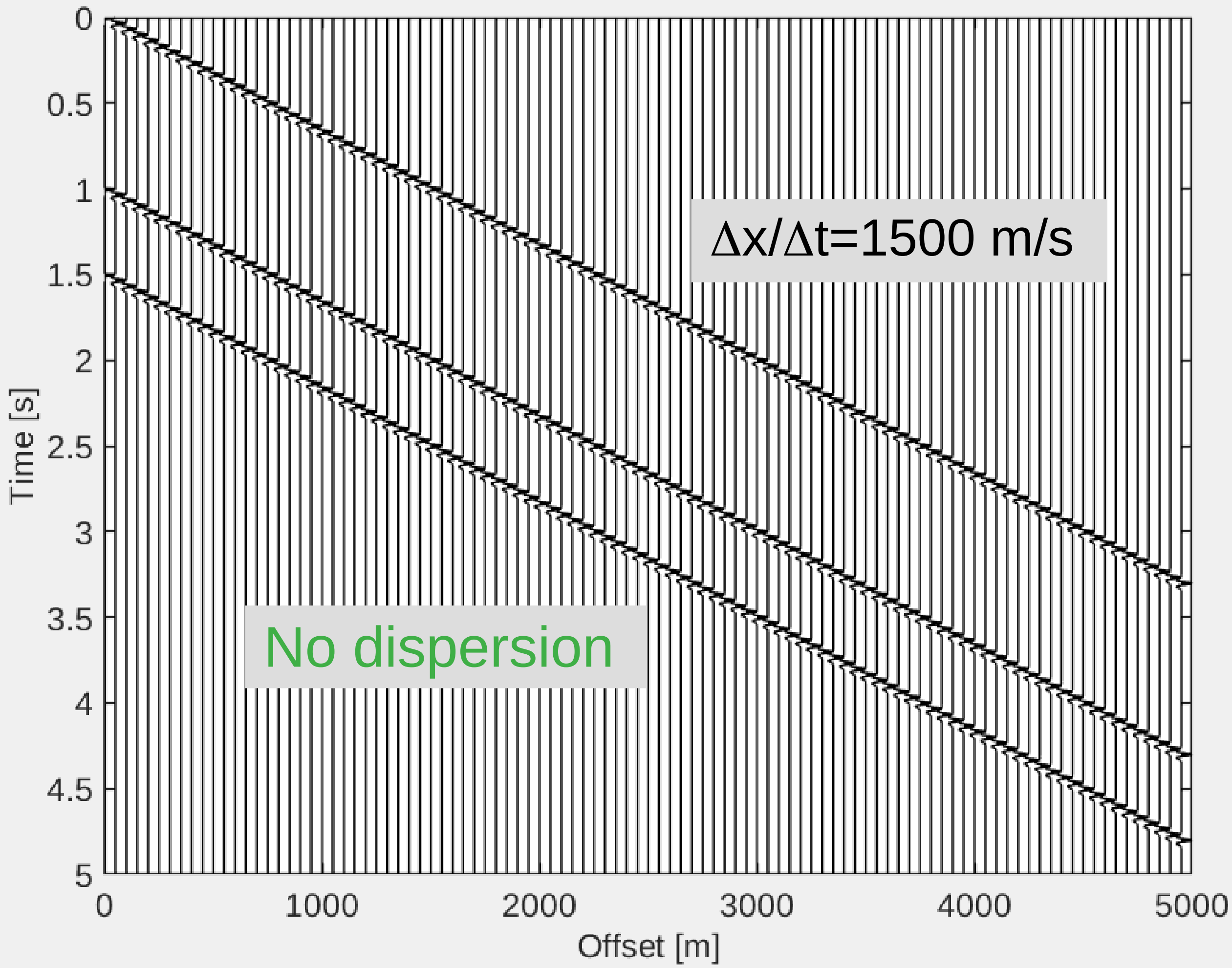
$$\Delta p \; x_{max} \leq T_{min} = \frac{1}{f_{max}} \quad \Rightarrow \quad \Delta p \leq \frac{1}{f_{max}\; x_{max}} \qquad (1)$$

$$\Delta x \; p_{max} \leq T_{min} = \frac{1}{f_{max}} \quad \Rightarrow \quad p_{max} \leq \frac{1}{f_{max}\; \Delta x} \qquad (2)$$

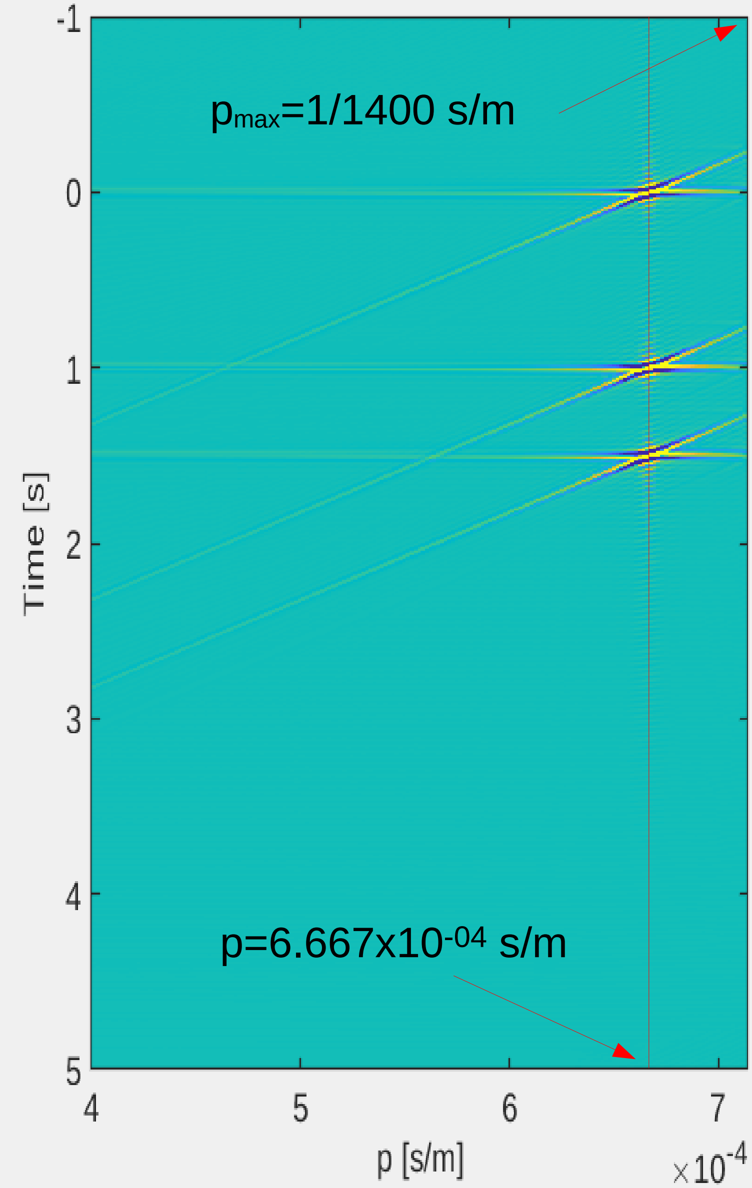
Vphase – Frequency Panel: Linear Radon Transform + Fourier Transform

Example of a seismogram with three linear events and no dispersion

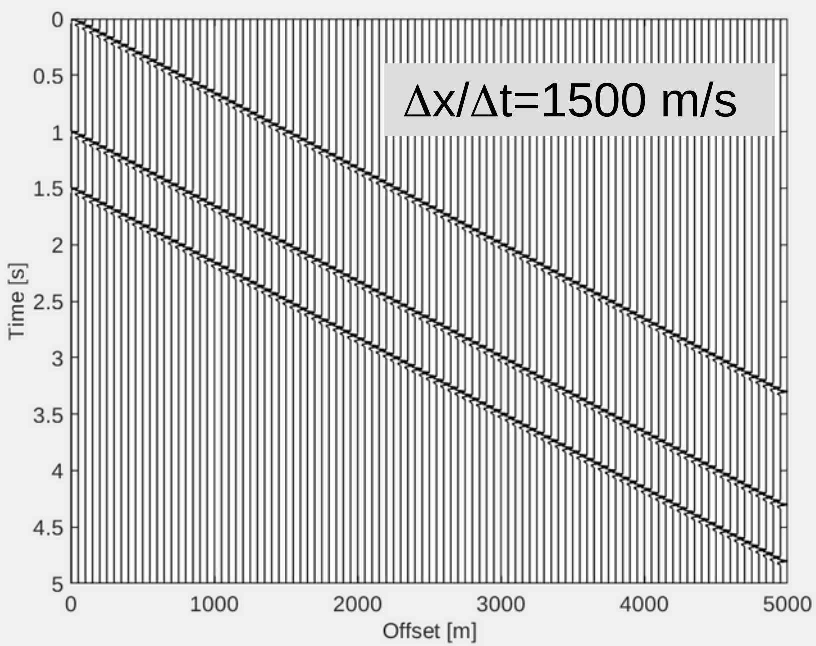
V=1500 m/s → p=6.667x10^{-04} s/m



τ–p transform of three linear events



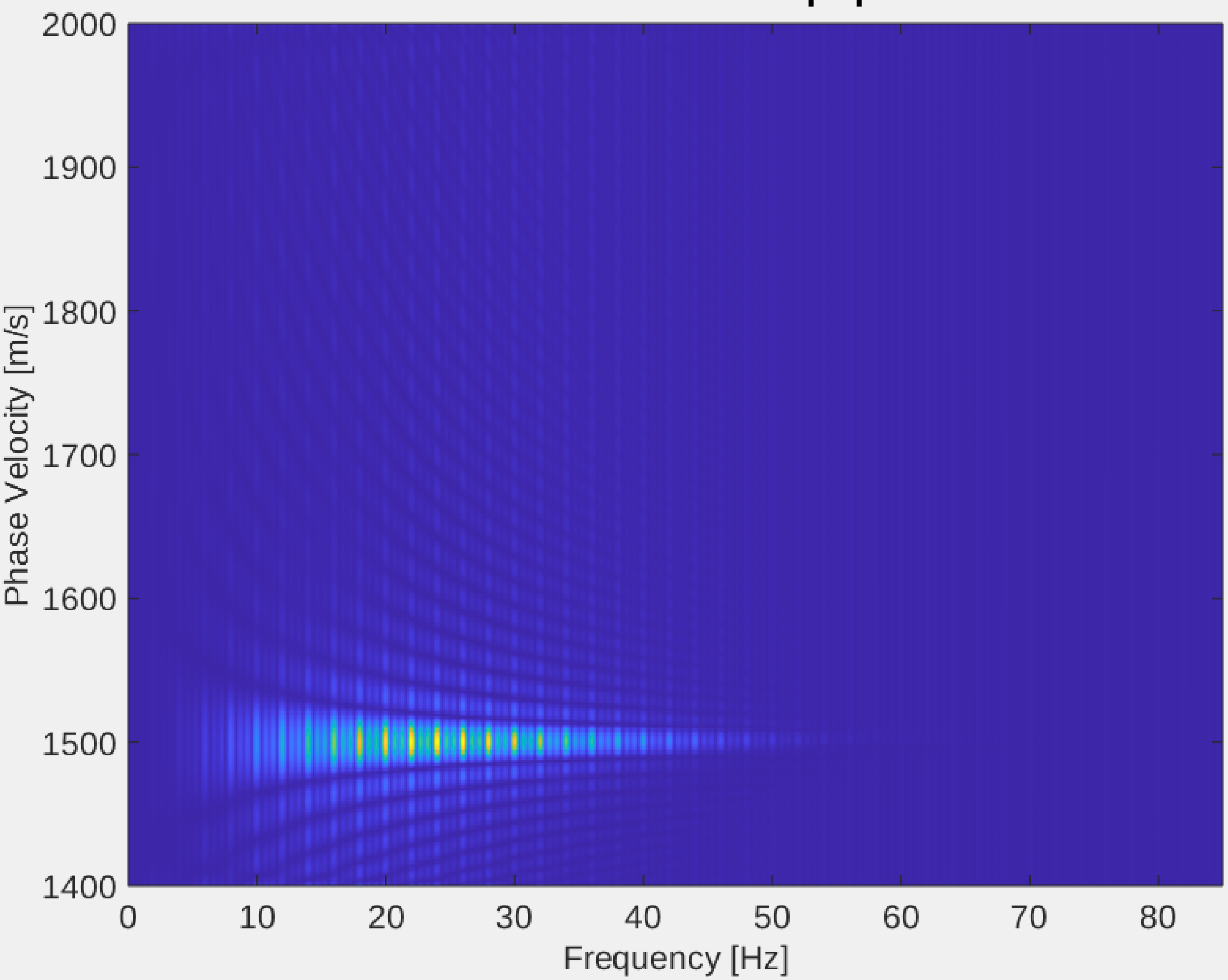
Vphase – Frequency Panel:
Linear Radon Transform + Fourier Transform



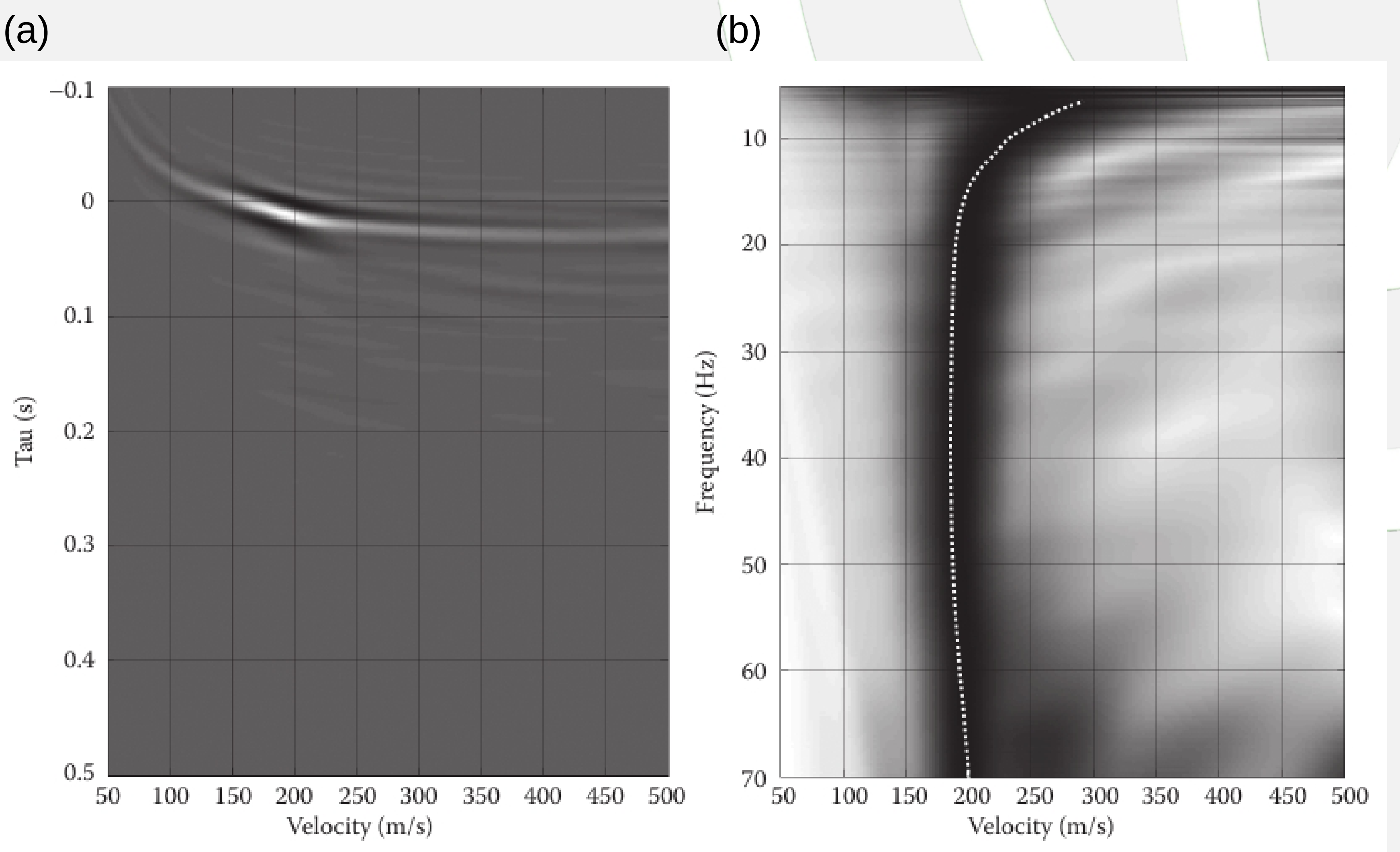
No dispersion

$$V_R(f) = \frac{1}{p_{A=A_{max}}}$$

FFT transform of the τ–p panel



Vphase – Frequency Panel: Linear Radon Transform + Fourier Transform



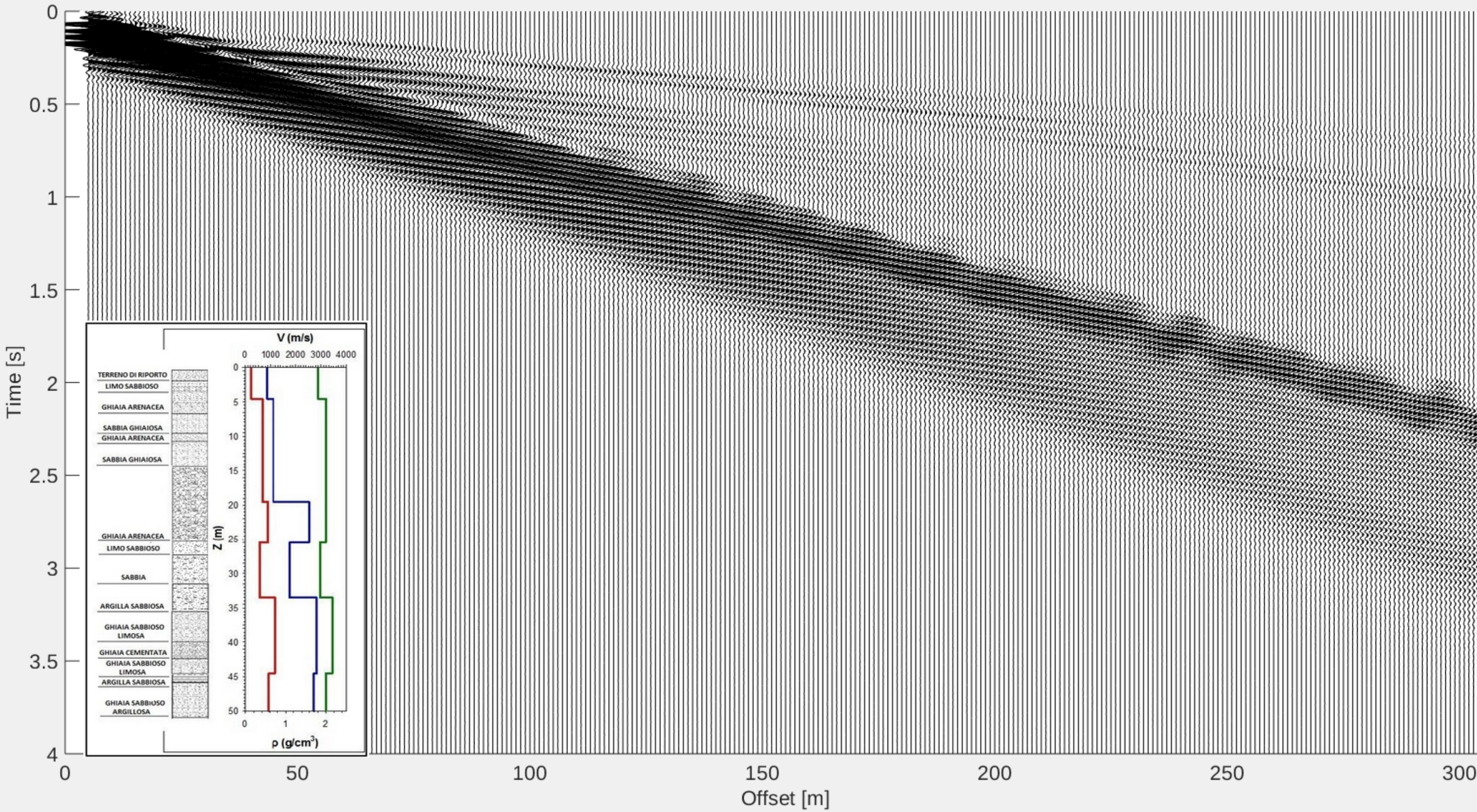
Examples of (a) τ−v and (b) frequency–velocity panels for a dataset

Foti et al. 2015

Complex case: Barga

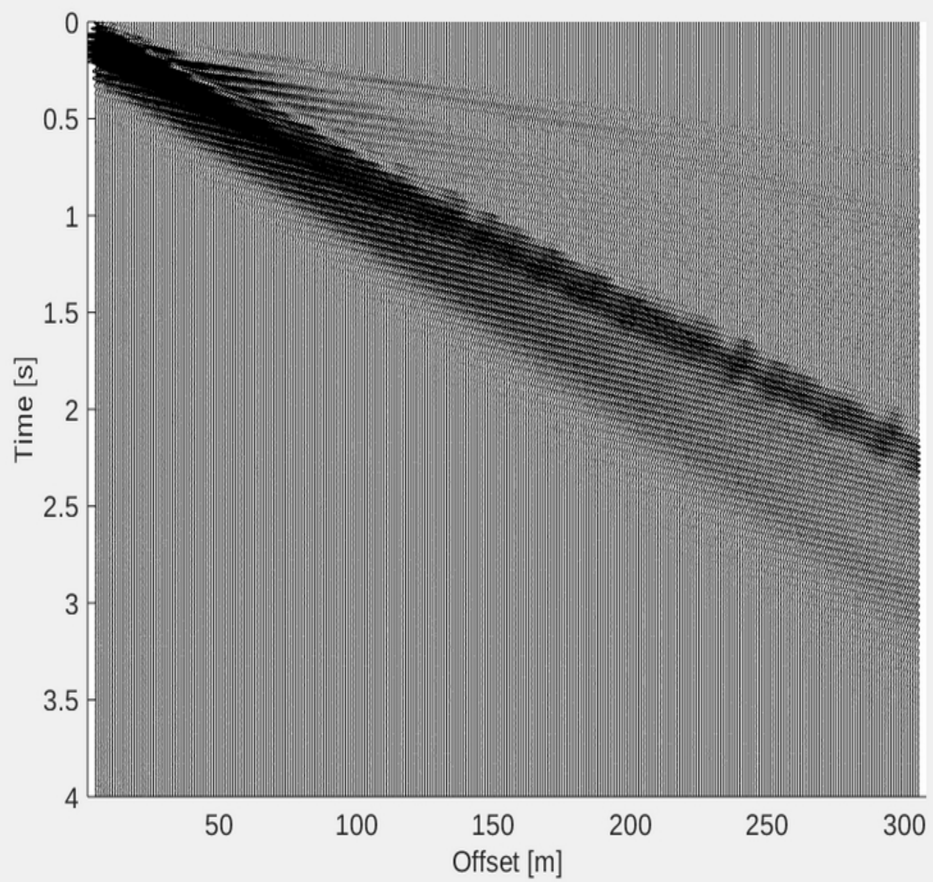
Synthetic Seismogram (Reflectivity method)

```
# Number of layers  6
# One line per layer:
# Thickness(m), Vp (m/s), Vs (m/s)  and density (kg/m³)
  4.5      875   235  1800
15.0     1115    680  2000
    6    2515   885  2000
    8    1790   565  1850
   11   2830  1195  2150
# Last line is the half-space,
    0    2725    915  2000
```

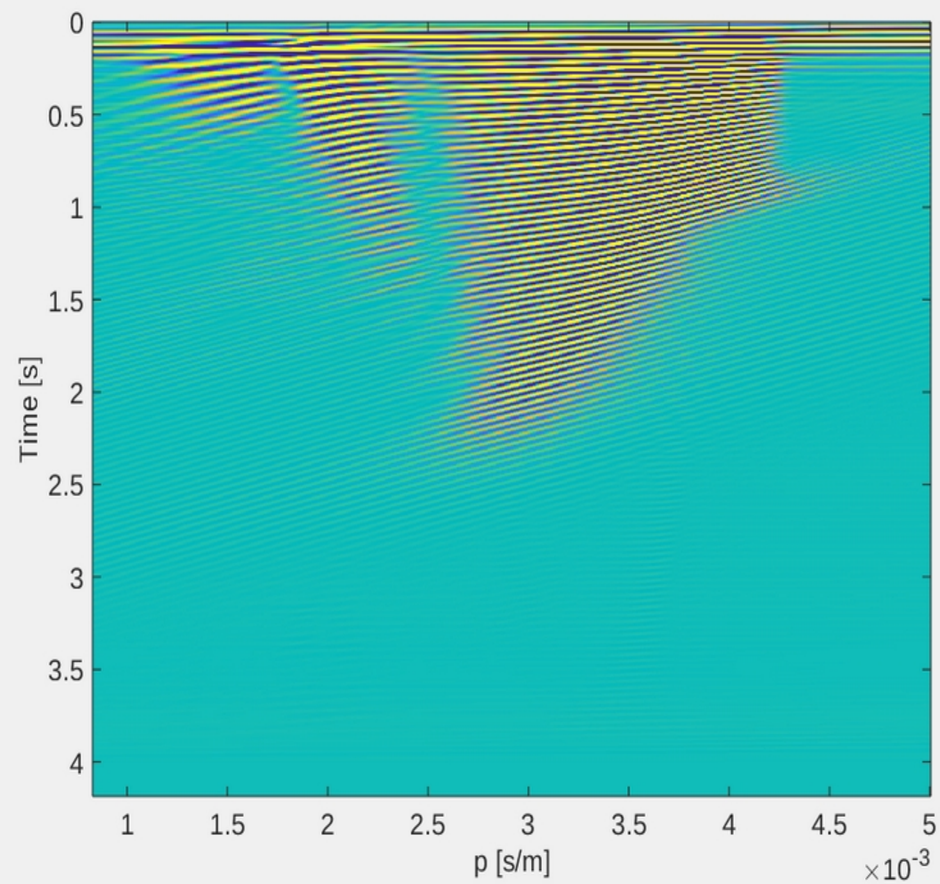


Data – τ-p Panel: Barga

Synthetic Seismogram (Reflectivity method)

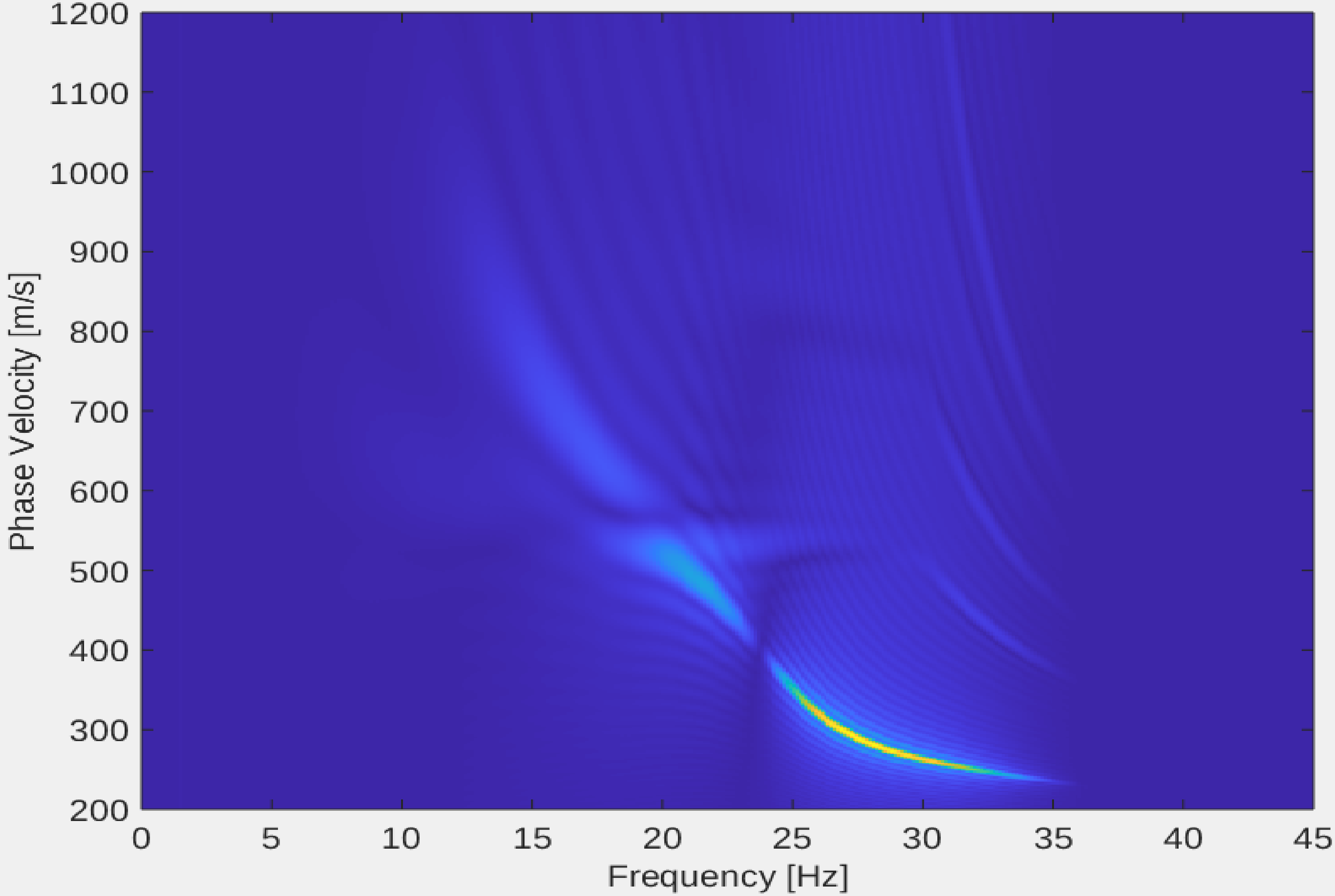


τ-p transform

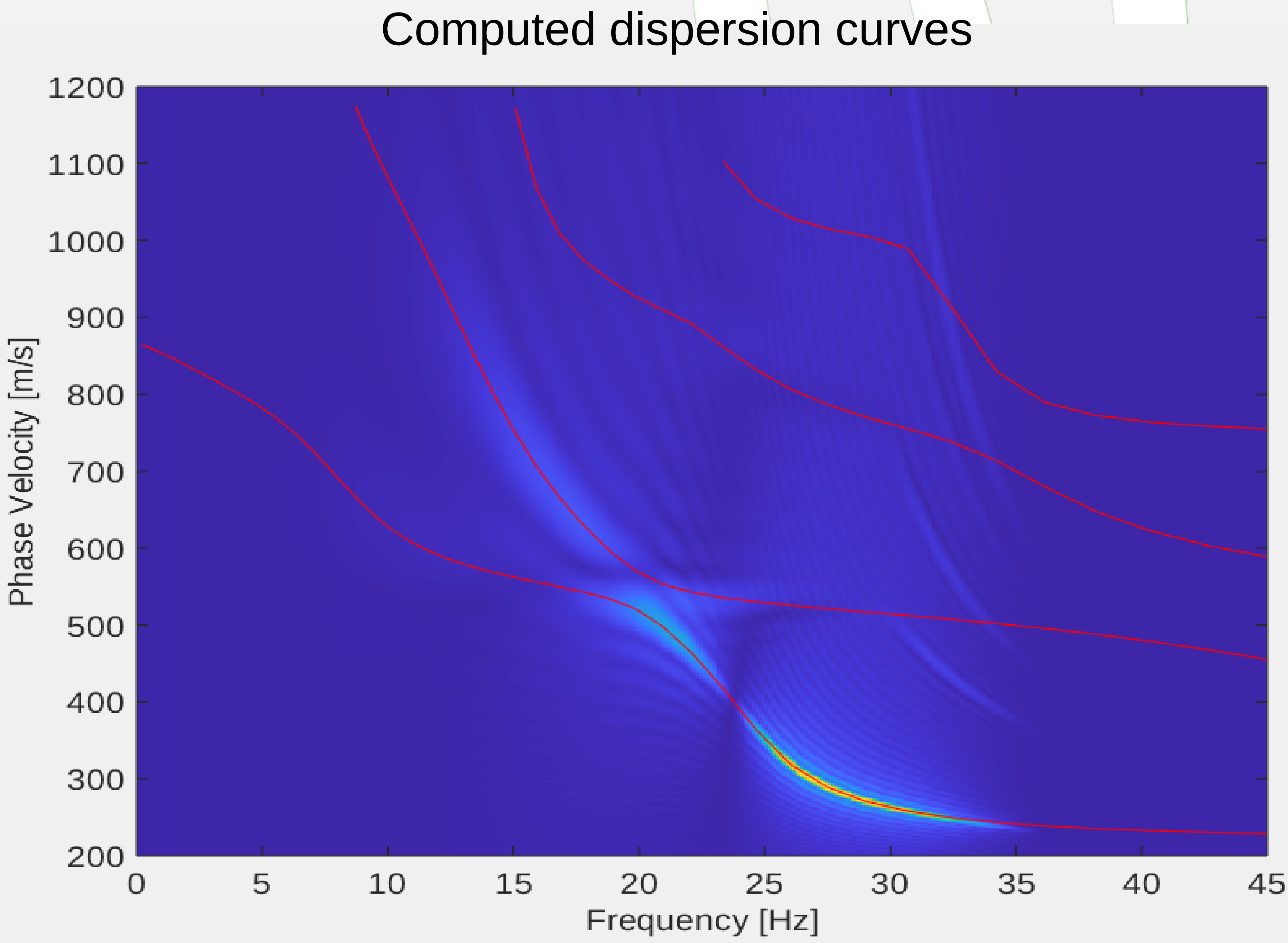


Vphase – Frequency Panel: Barga

Where do you pick the dispersion curves?



Vphase – Frequency Panel: Barga

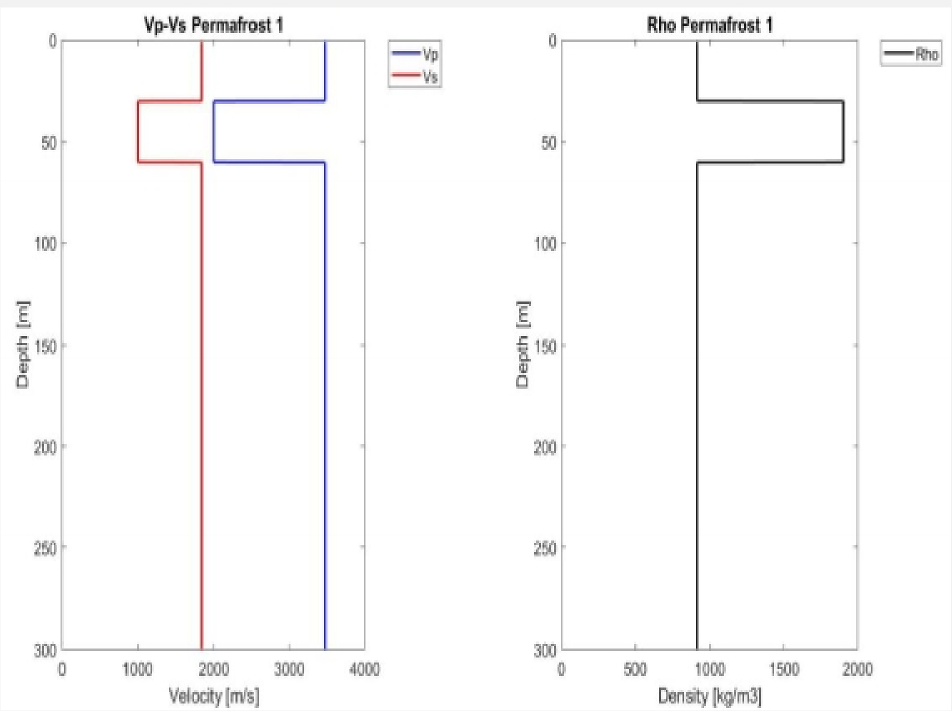


Permafrost Example

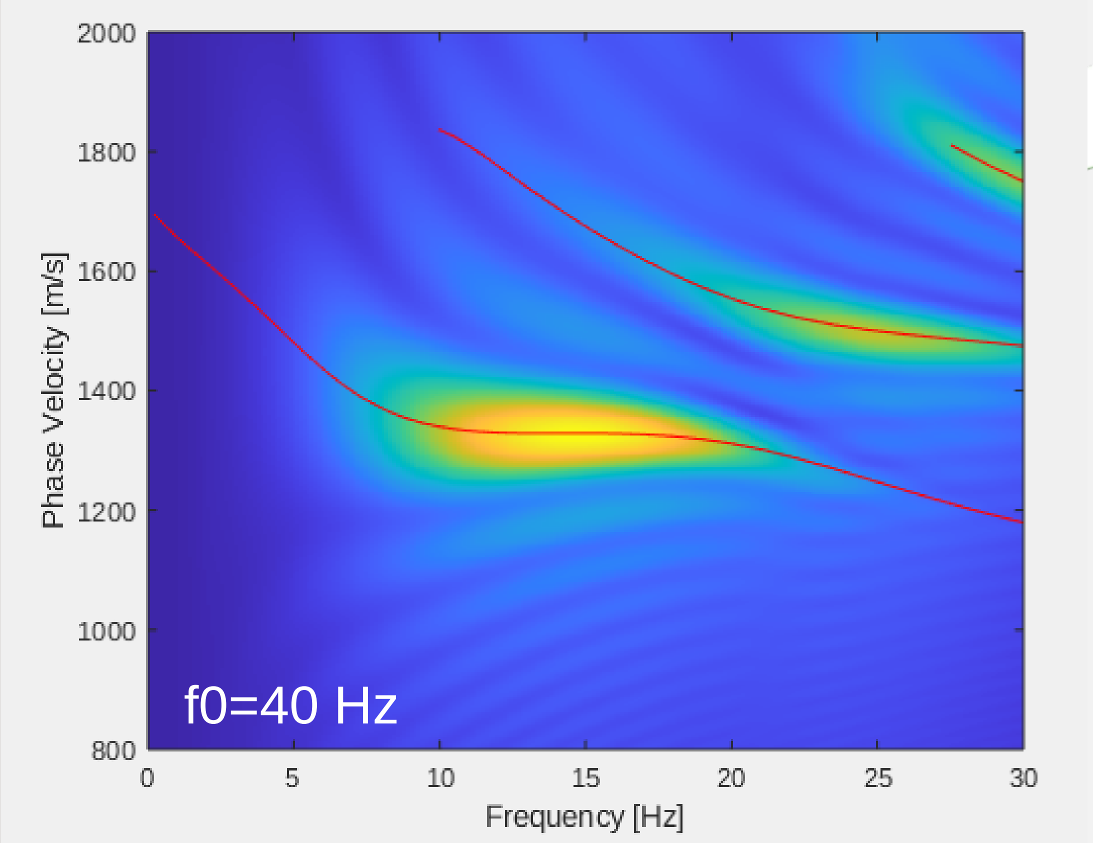
Model

Lithology	Depth (m)	Vp (m/s)	Vs (m/s)	Densità (kg/m³)	Vp/Vs
Permafrost	0-30	3466	1839	917	~1.8
Gravel	30-60	2000	1000	1900	2
Permafrost	60-200	3466	1839	917	~1.8

Permafrost: scheme of the Vp and Vs velocity model (left) and of the model density (right)



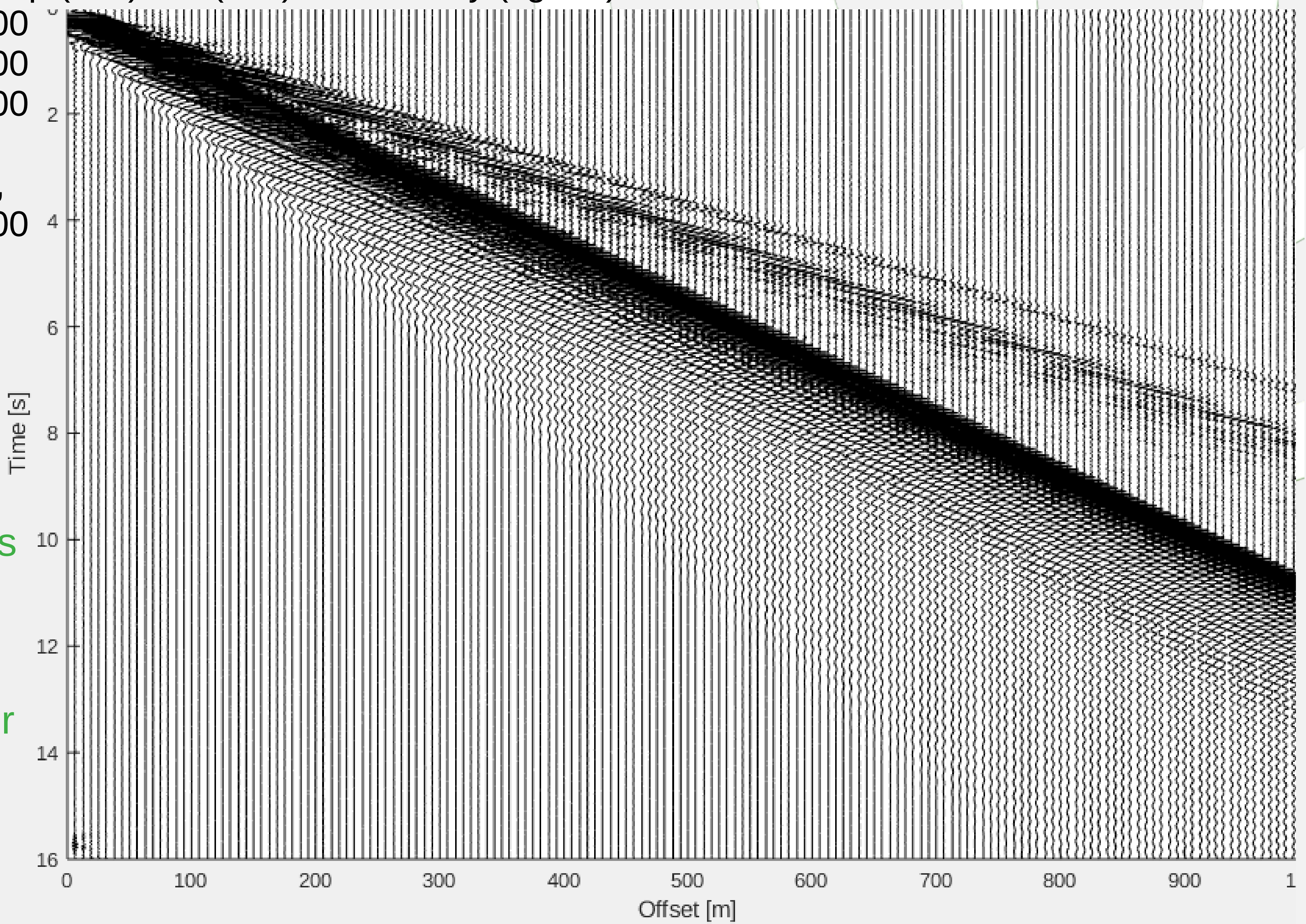
Dispersion curves (1st, 2nd and 3rd modes) and Vphase – Frequency data



4-Layer Model

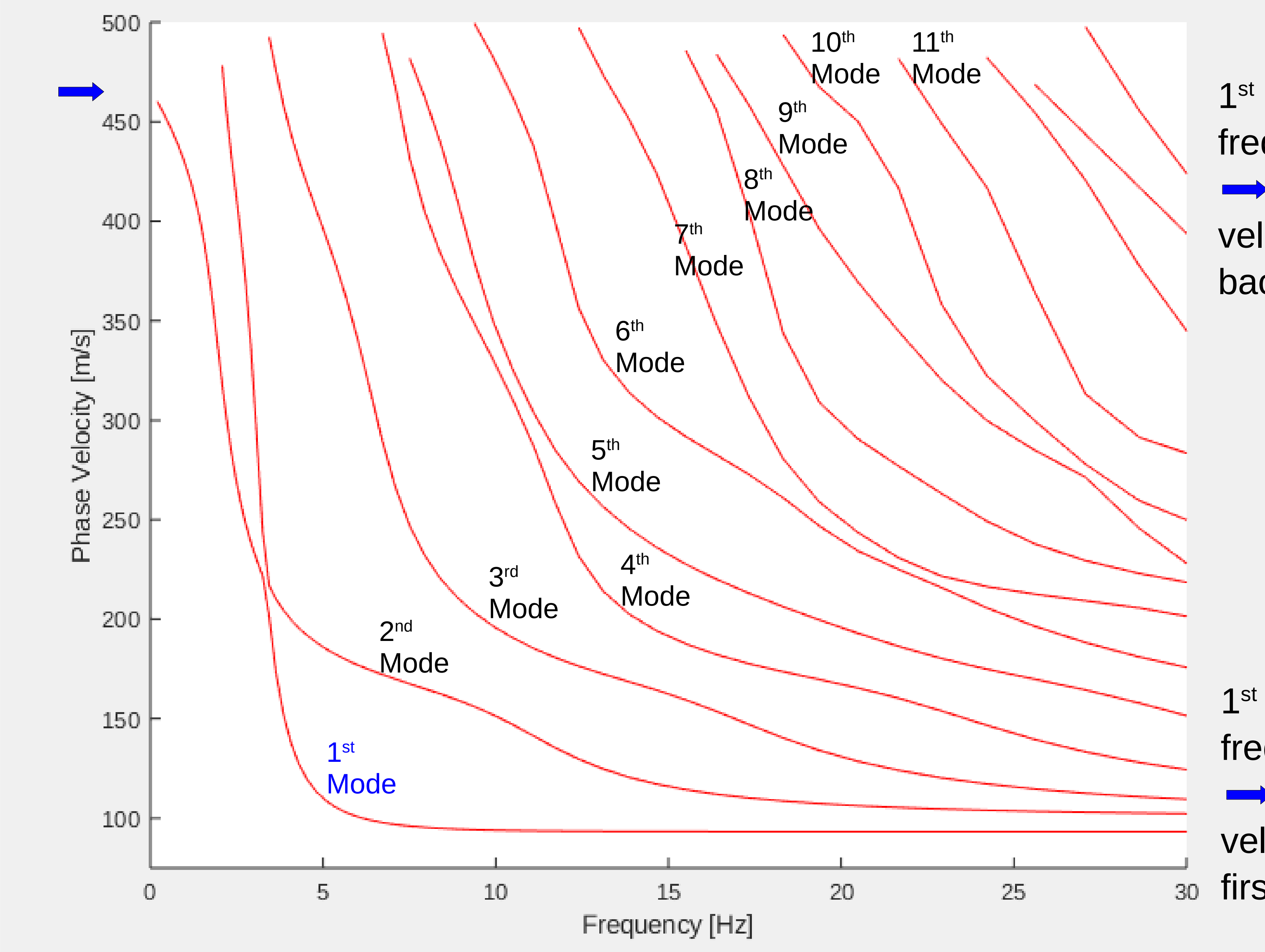
Number of layers
4
One line per layer:
Thickness(m), Vp (m/s), Vs (m/s) and density (kg/m3)
10 200 100 1900
10 400 200 1900
10 600 300 1900
Last line is
the half-space,
0 1000 500 1900
(Foti, 2015)

Δg = 6.25 m
Δt = 0.004 s
f0 Ricker = 20Hz



Note that:
- 160 channels
- 16 s
- 1000 m
are unusual parameters for near surface surveys

Dispersion Curves – 4-Layers Model



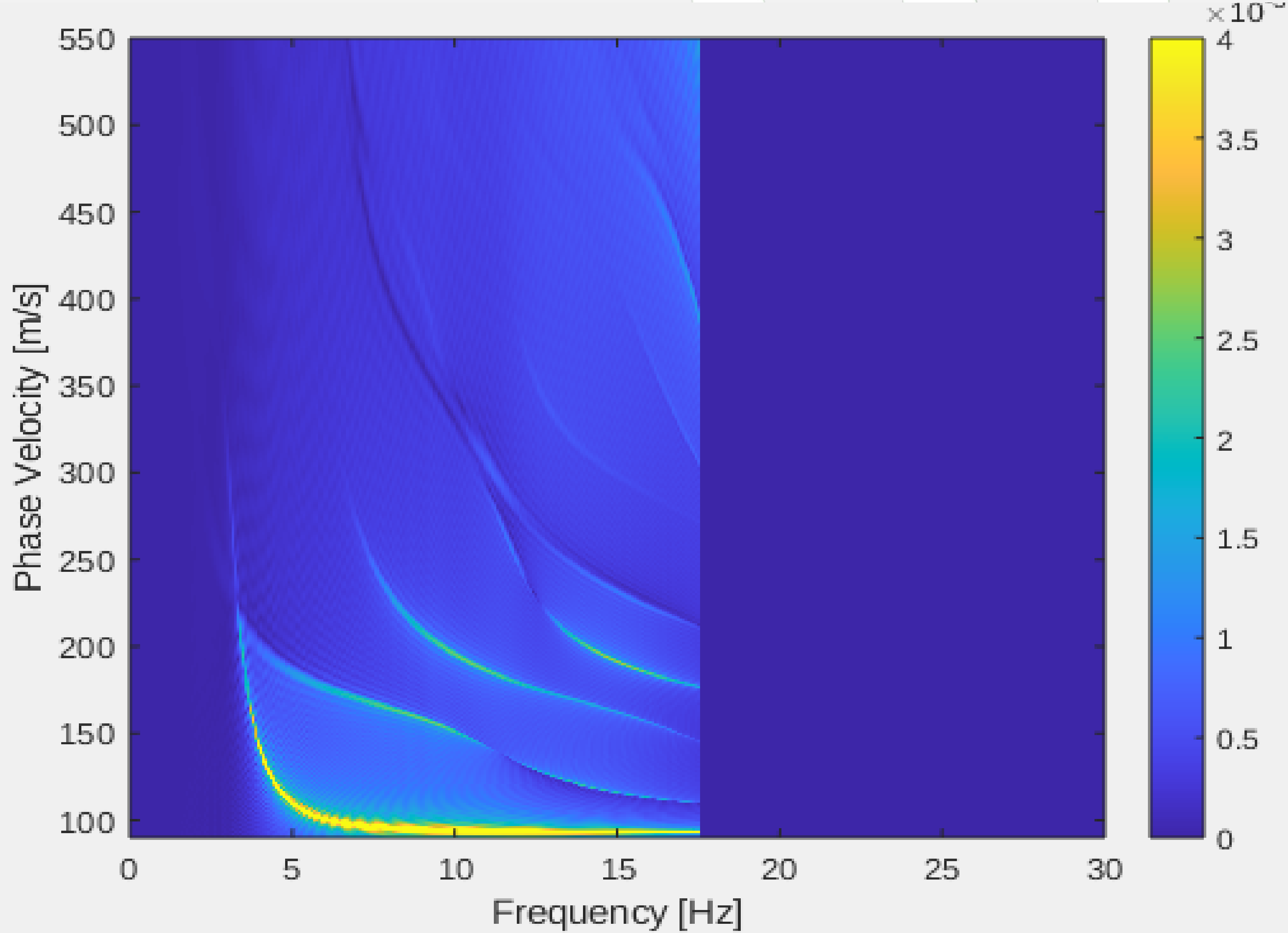
1st Mode, low frequencies → Rayleigh velocity of the background

1st Mode, high frequencies → Rayleigh velocity of the first layer

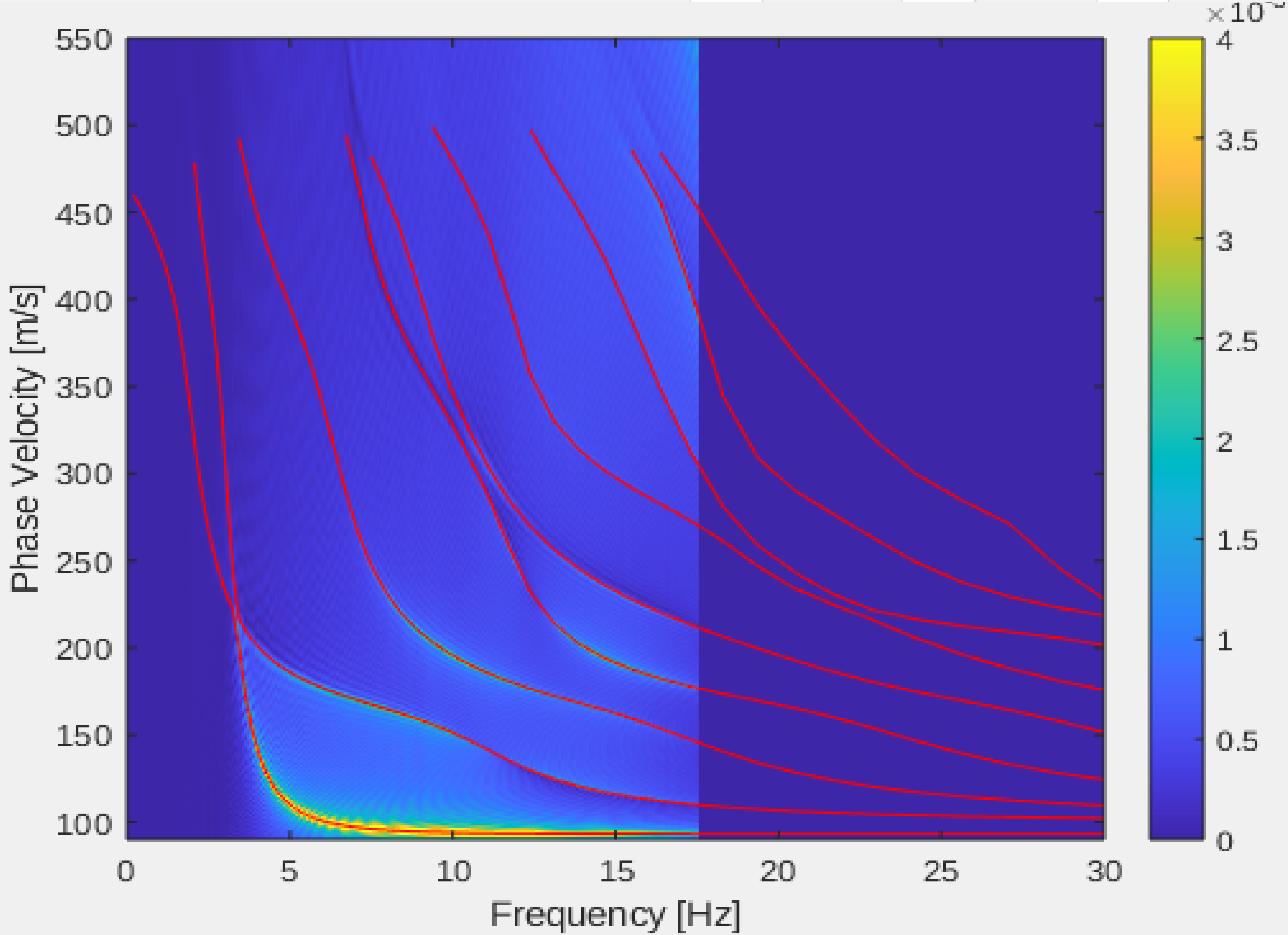
Dispersion curves are computed by means of the gpdc software

See: http://www.geopsy.org

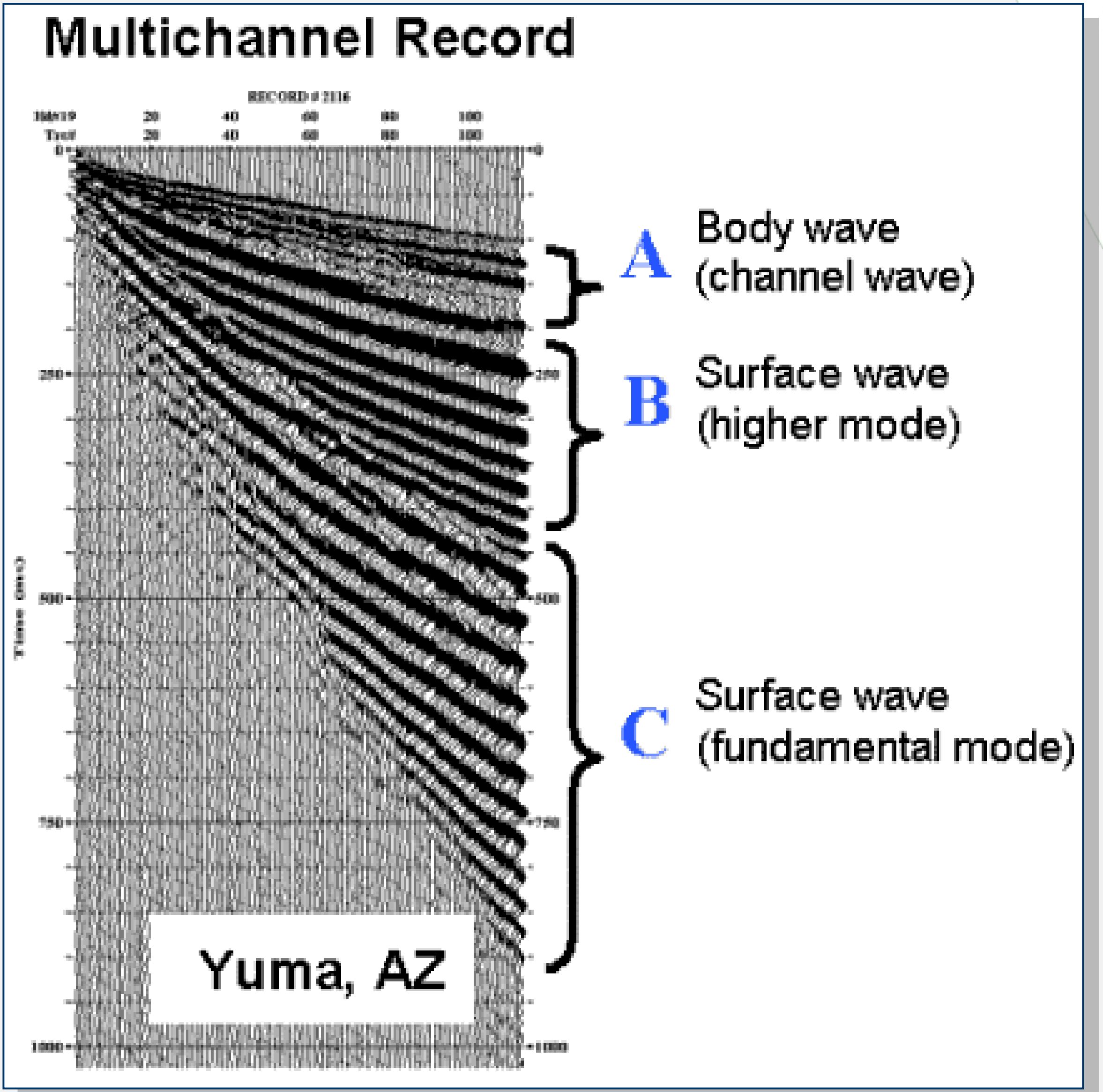
Vphase – Frequency Display

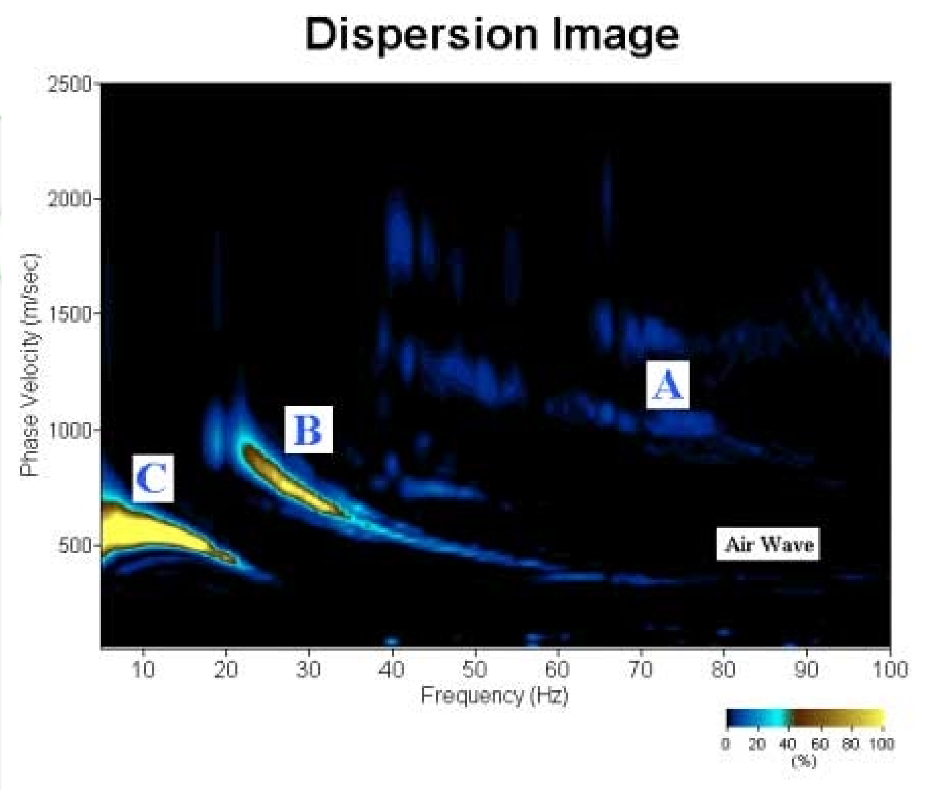


Vphase – Frequency Display + Dispersion Curves



Example of Higher Modes



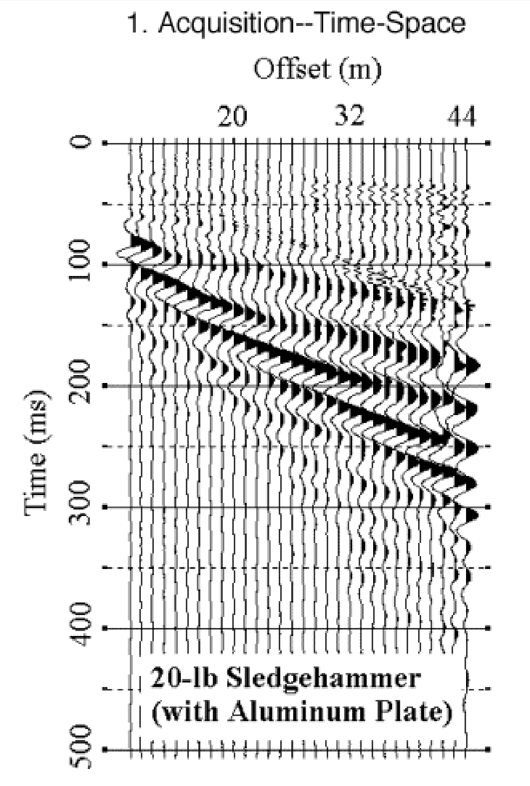


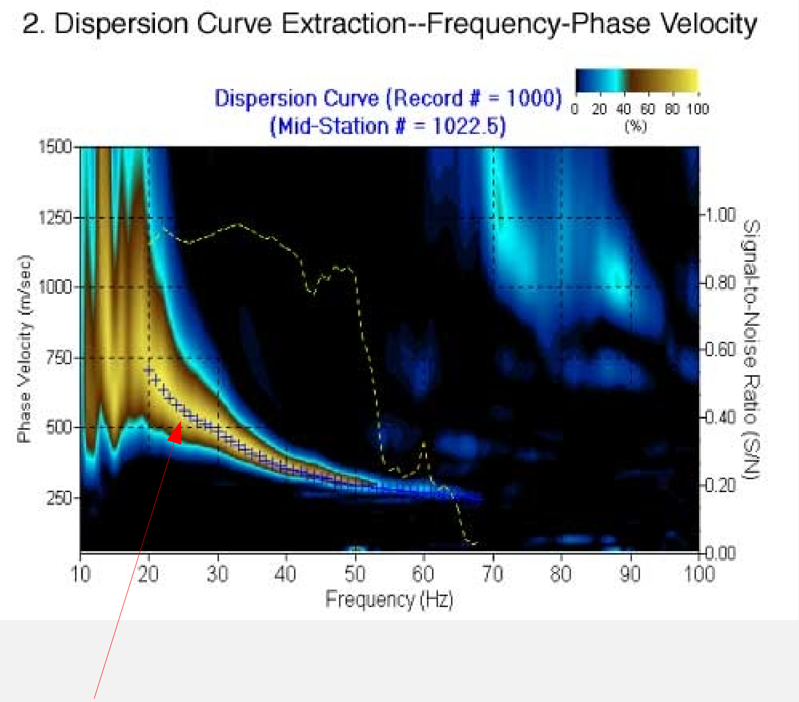
(Kansas Geological Survey)

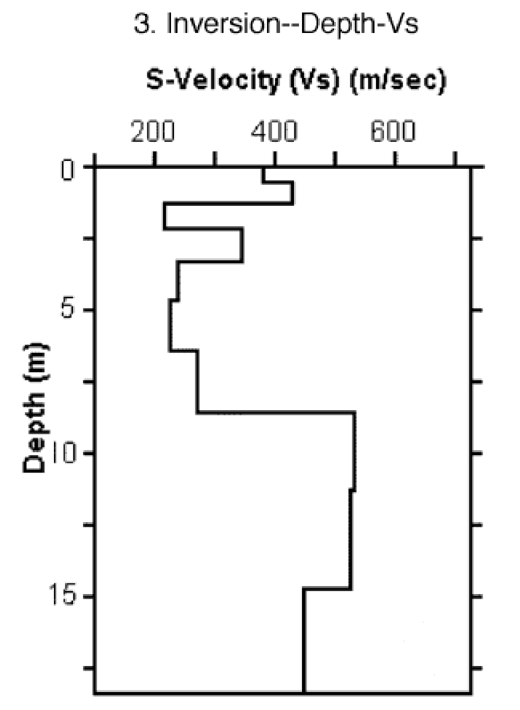
Inversion of Dispersion Curves - Example

The entire procedure for MASW usually consists of four steps (Miller et al., 1999):

- Acquiring multichannel records (or shot gathers)
- Estimating the fundamental-mode dispersion curves (one curve from each record)
- Inverting these curves to obtain 1-D (depth) Vs profiles (one profile from one curve),
- Assembling multiple 1-D results into 2-D or 3-D images







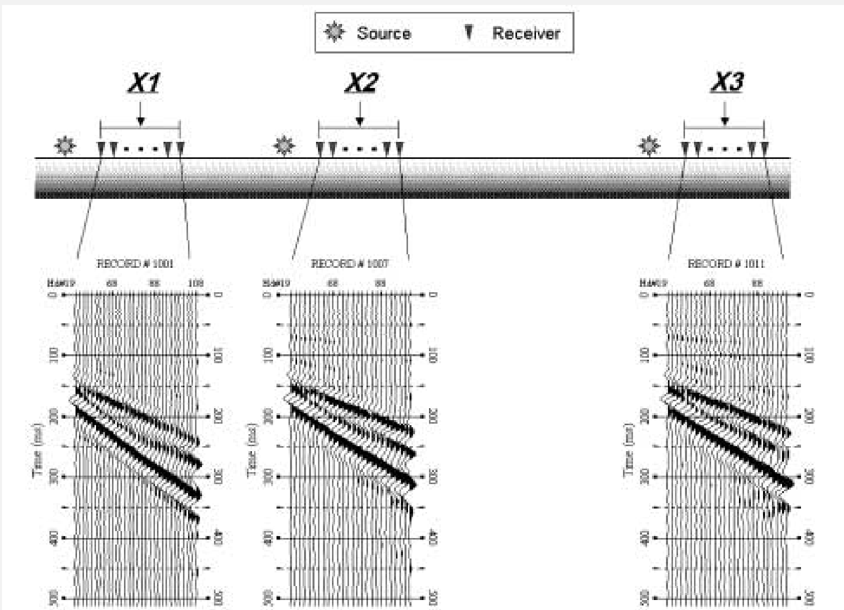
Picking of the fundamental mode

(Kansas Geological Survey)

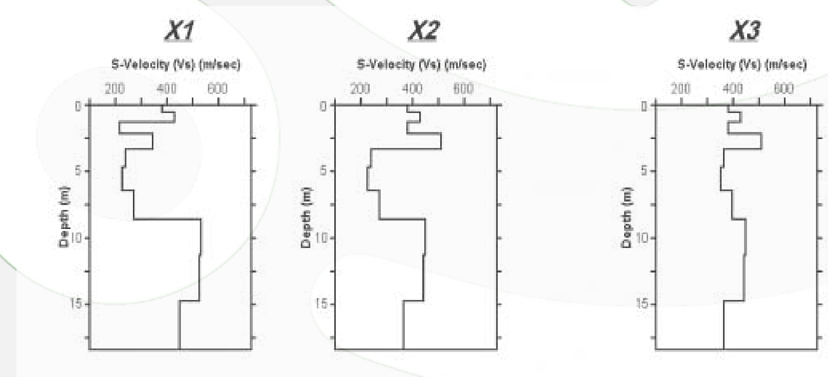
Inversion of Dispersion Curves - Example

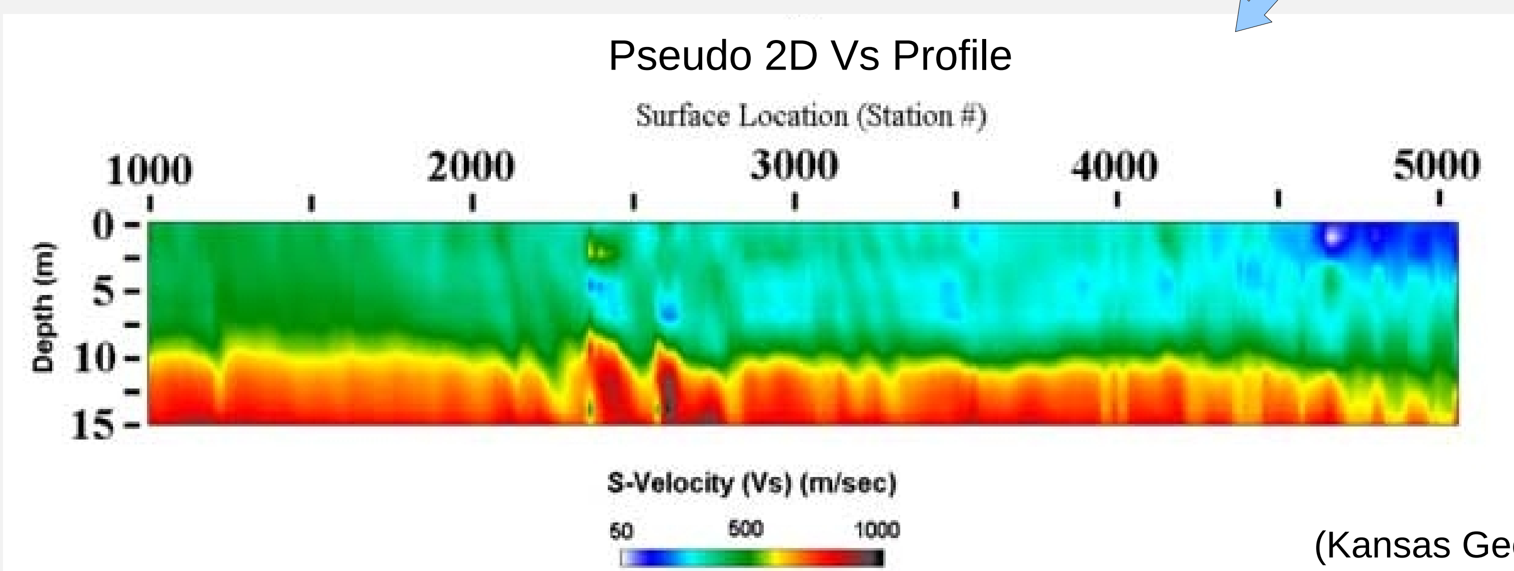
By placing each 1-D Vs profile at a surface location corresponding to the middle of the receiver line X1, X2, X3,… , a 2-D (surface and depth) Vs map is constructed through an appropriate interpolation scheme

Multichannel Record



1D Vs Profile





(Kansas Geological Survey)

References

Miller, R. D., J. Xia, C. B. Park, and J. M. Ivanov, 1999, Multichannel analysis of surface waves to map bedrock: The Leading Edge, 18, 1392-1396.

Foti S., C. G. Lai, G. J. Rix, C. Strobbia, 2105, Surface Wave Methods for Near-Surface Site Characterization. 1st Edition, 2015. CRC Press

F. E. Richart, J. R. Hall, R. D. Woods, 1970, Vibrations of Soils and Foundations Prentice-Hall, 1970,

Strobbia C., 2003, Surface Wave Methods Acquisition, processing and inversion. PhD Thesis Politecnico di Torino

Kansas Geological Survey: http://www.kgs.ku.edu/software/surfseis/masw.html

http://www.geopsy.org

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THANKS!

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