

## Seismic Refraction

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**IR0000032 – ITINERIS, Italian Integrated Environmental Research Infrastructures System**  
(D.D. n. 130/2022 - CUP B53C22002150006) Funded by EU - Next Generation EU PNRR-  
Mission 4 “Education and Research” - Component 2: “From research to business” - Investment  
3.1: “Fund for the realisation of an integrated system of research and innovation infrastructures”



# Seismic Refraction

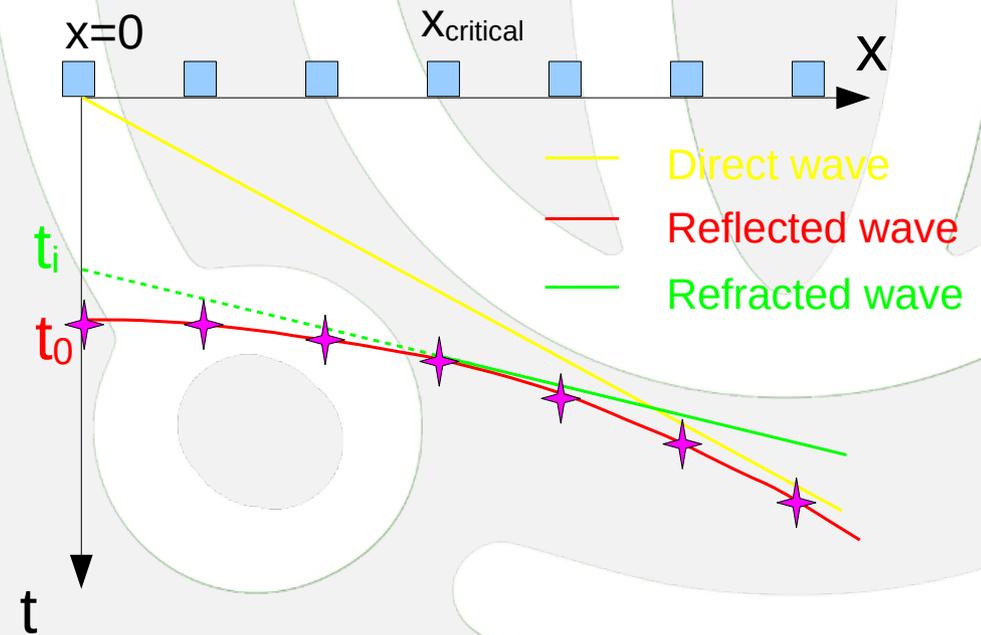
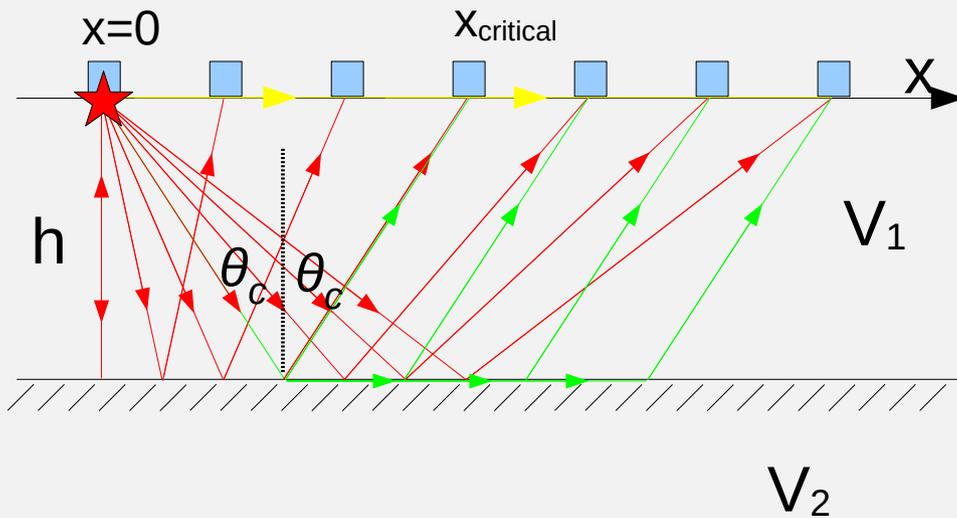


Crustal studies  
(ex: Moho)

Geotechnical and engineering studies  
(ex: depth to bedrock,  
survey at dam sites,  
weathered rocks investigation,  
site investigation for waste disposal [also toxic waste],  
Presence of faults in the bedrock,  
rippability)

Seismic exploration  
(ex: refraction statics computation)

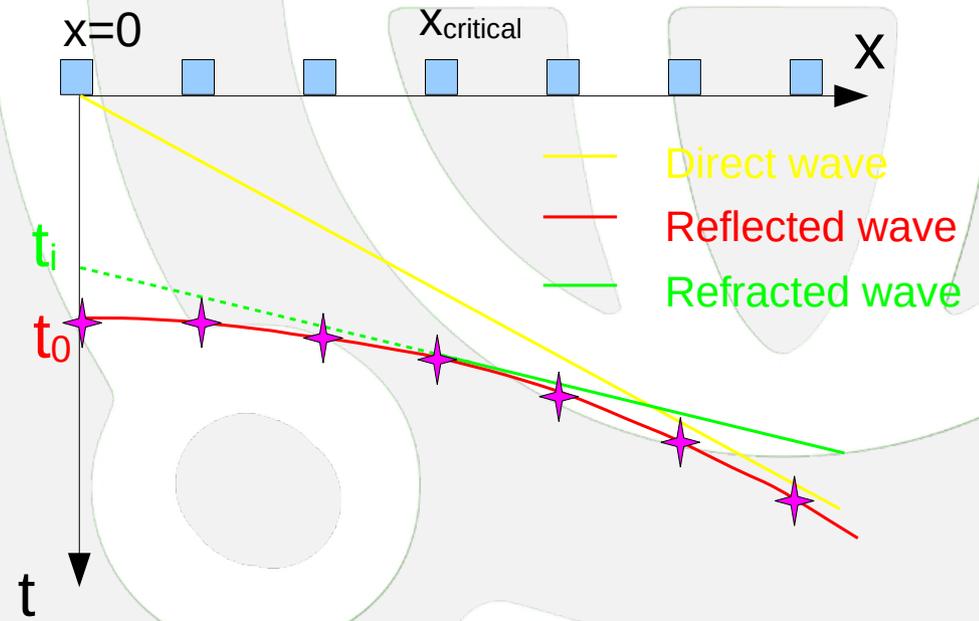
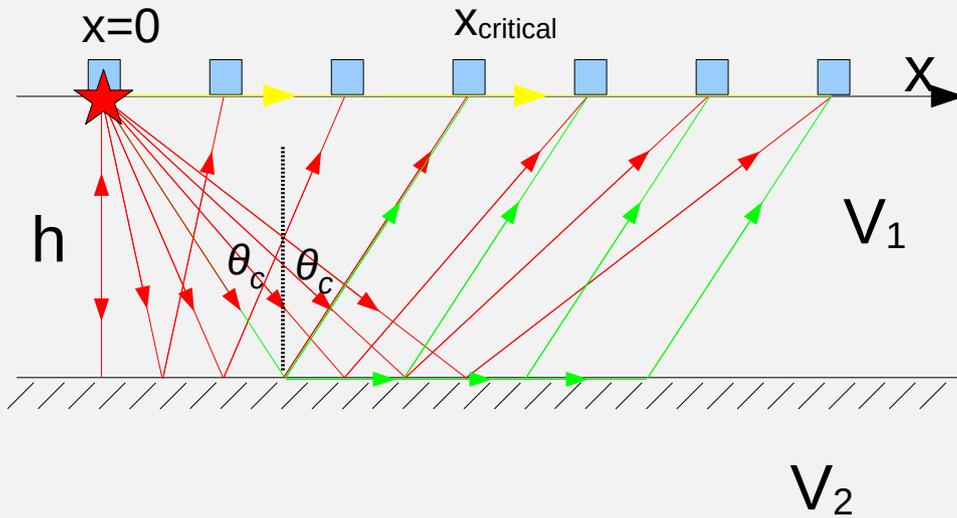
# Single interface model



Source is excited in  $x=0$ . The receivers record:

- 1) direct waves
- 2) reflected waves from the interface
- 3) if  $V_2 > V_1$  and for angles  $\theta_c \geq \arcsin(V_1/V_2)$  the head waves

# Single interface model

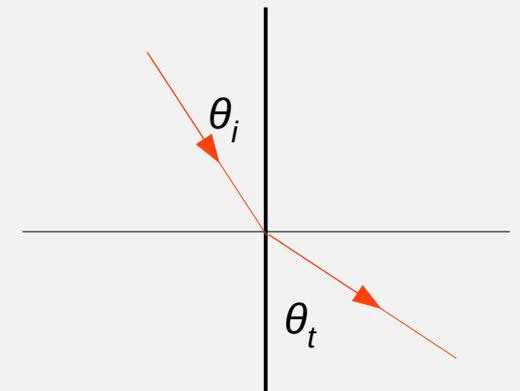


The angle  $\theta_c$  at which the head wave is generated is called the **critical angle** and is given by the Snell law when the incident angle  $\theta_i$  is such that  $\theta_t = 90^\circ$

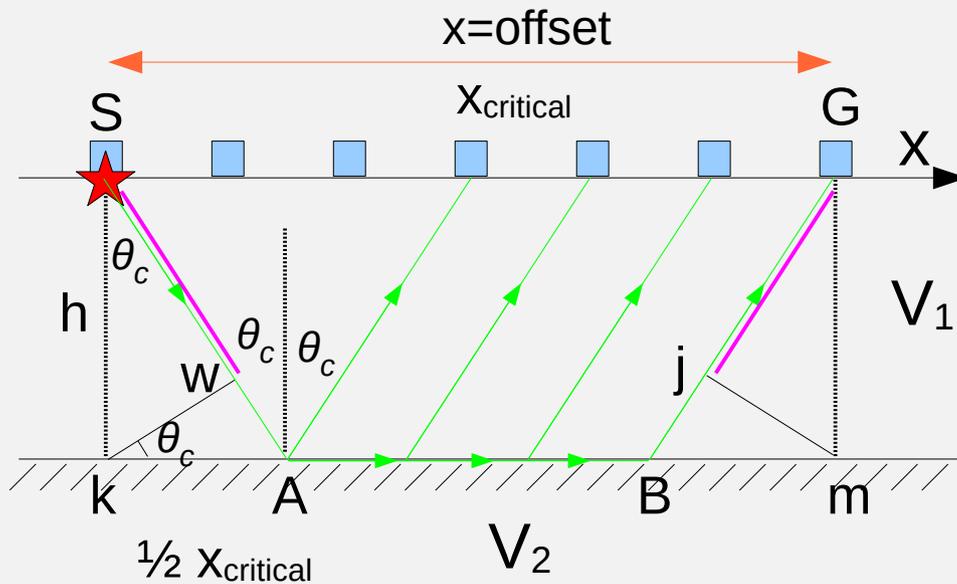
$$\frac{\sin \vartheta_i}{\sin \vartheta_t} = \frac{V_1}{V_2}$$



$$\sin \vartheta_c = \frac{V_1}{V_2}$$



# Computation of the refraction time-offset curve



Refracted ray path: SABG  
 Time  $T_{SABG}$  can be divided in three components:

$$T_{SABG} = T_{SA} + T_{AB} + T_{BG}$$

$$T_{SA} = T_{Sw} + T_{wA} \rightarrow \begin{cases} T_{Sw} = Sw/V_1 = h \cos \vartheta_c / V_1 \\ T_{wA} = kA \sin \vartheta_c / V_1 = kA (V_1/V_2) / V_1 = kA/V_2 = T_{kA} \end{cases}$$

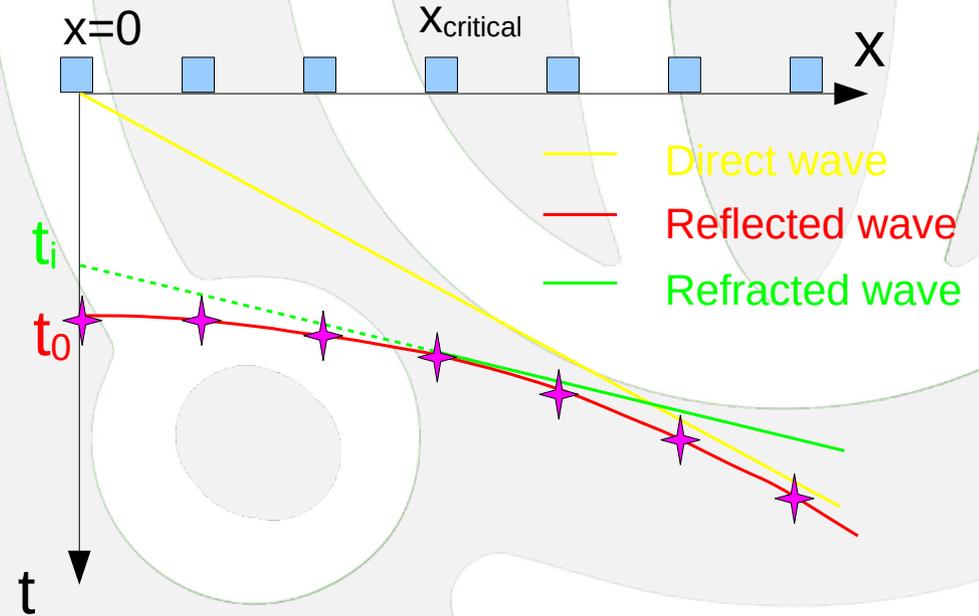
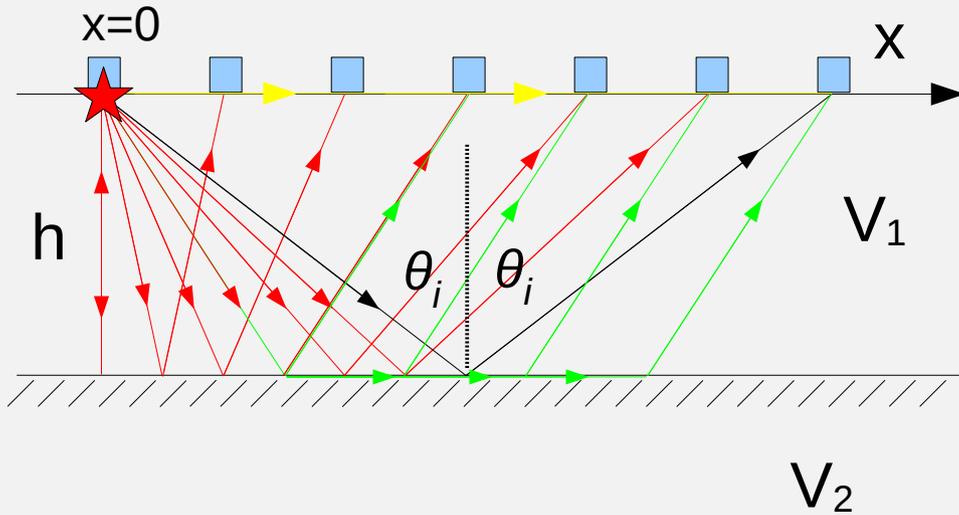
Analogously:  $T_{jB} = T_{Bm}$

Time-offset curve:

$$T_{SG} = \underbrace{T_{Sw}}_{V_1} + \underbrace{T_{jG}}_{V_2} + T_{km} \rightarrow T_{SG} = T_{Refr} = 2h \cos \vartheta_c / V_1 + x / V_2$$

N.B.: Only for  $x > X_{critical}$

# Computation of the reflection time-offset curve

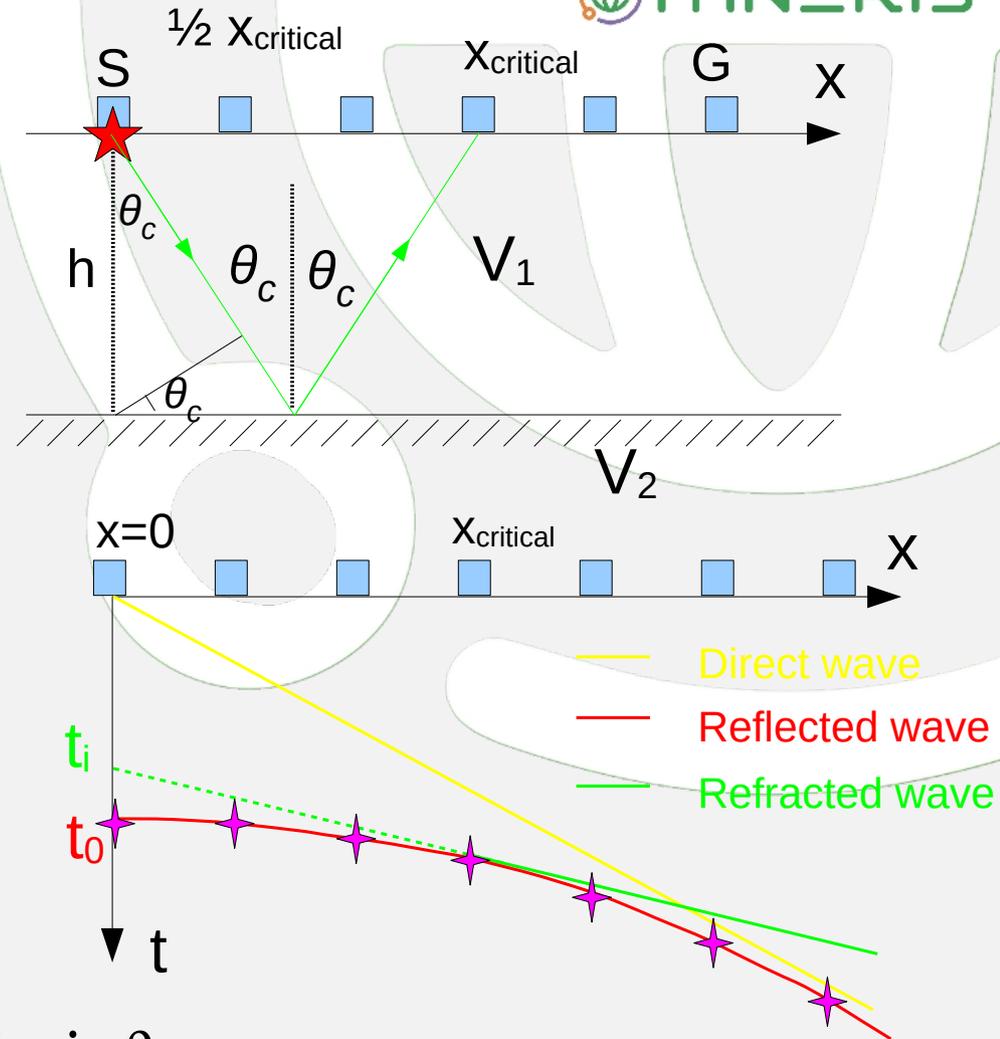


$$T_{Refl} = \frac{2}{V_1} \frac{h}{\cos \vartheta_i} = \frac{2}{V_1} \sqrt{\left(\frac{x}{2}\right)^2 + h^2} = \sqrt{\frac{4h^2}{V_1^2} + \frac{x^2}{V_1^2}} = \sqrt{T_0^2 + \frac{x^2}{V_1^2}}$$

Hyperbola of reflected arrivals with asymptote:  $T(x) = \frac{x}{V_1}$

# Time-offset curves at critical distance

At the critical distance, the hyperbola of the reflected arrivals is tangent to the line representing the refracted arrivals

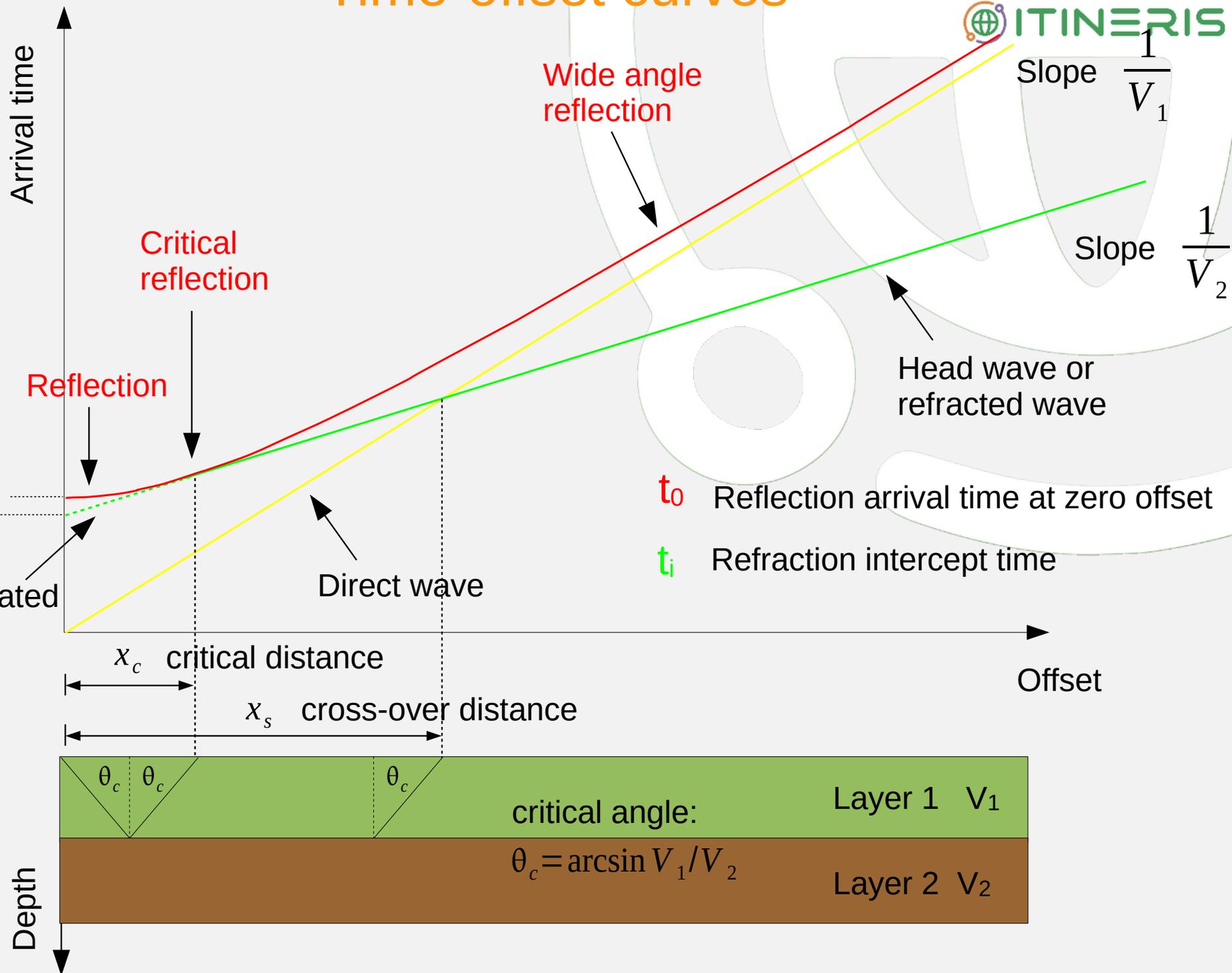


$$T_{Refl} = \frac{2}{V_1} \sqrt{\left(\frac{x^2}{4}\right) + h^2}$$

$$1) \quad T_{Refr} \Big|_{x_c} = T_{Refl} \Big|_{x_c}$$

$$2) \quad \frac{dT_{refl}}{dx} \Big|_{x_c} = \frac{2}{V_1} \frac{\frac{x_c}{2}}{2\sqrt{\frac{x_c^2}{4} + h^2}} = \frac{1}{V_1} \frac{\frac{h \sin \vartheta_c}{\cos \vartheta_c}}{h} = \frac{\sin \vartheta_c}{V_1} = \frac{1}{V_2} = \frac{dT_{refr}}{dx}$$

# Time-offset curves



# Cross-over distance

$$T_{Dir} = T_{Refr} \quad \rightarrow$$

$$\frac{x_s}{V_1} = \frac{2h \cos \vartheta_c}{V_1} + \frac{x_s}{V_2}$$

$$\rightarrow x_s \left( \frac{1}{V_1} - \frac{1}{V_2} \right) = \frac{2h}{V_1} \sqrt{(1 - \sin^2 \vartheta_c)} = \frac{2h}{V_1} \sqrt{1 - \frac{V_1^2}{V_2^2}}$$

$$x_s = \frac{2h}{V_1} \sqrt{1 - \frac{V_1^2}{V_2^2}} \frac{V_1 V_2}{V_2 - V_1} = \frac{2h}{V_1} \sqrt{\frac{V_2^2 - V_1^2}{V_2^2}} \frac{V_1 V_2}{V_2 - V_1}$$

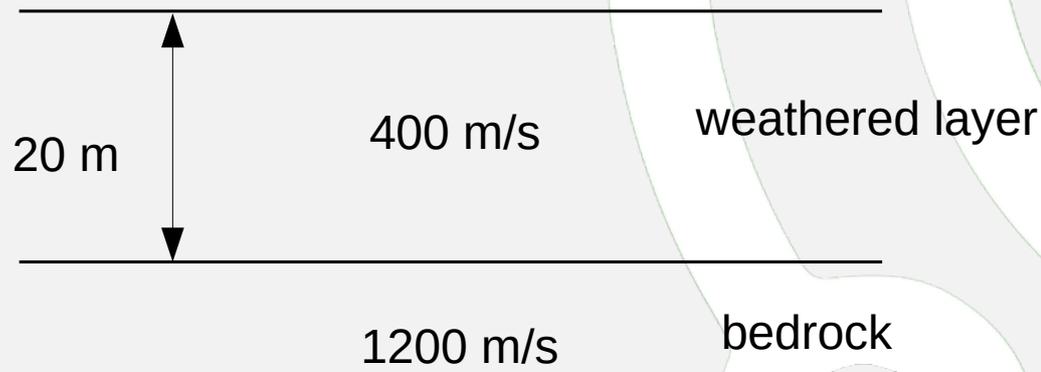
$$x_s = 2h \sqrt{\frac{V_2 + V_1}{V_2 - V_1}} \quad \rightarrow \quad 2h < x_s < \infty$$

Multiplying by  $V_2 + V_1$  and gathering the factor  $V_2^2$   $\rightarrow x_s = 2h \frac{1 + \sin \vartheta_c}{\cos \vartheta_c}$

Because  $x_c = 2h \frac{\sin \vartheta_c}{\cos \vartheta_c}$   $\rightarrow$

$x_s$  is always bigger than  $x_c$

# Example



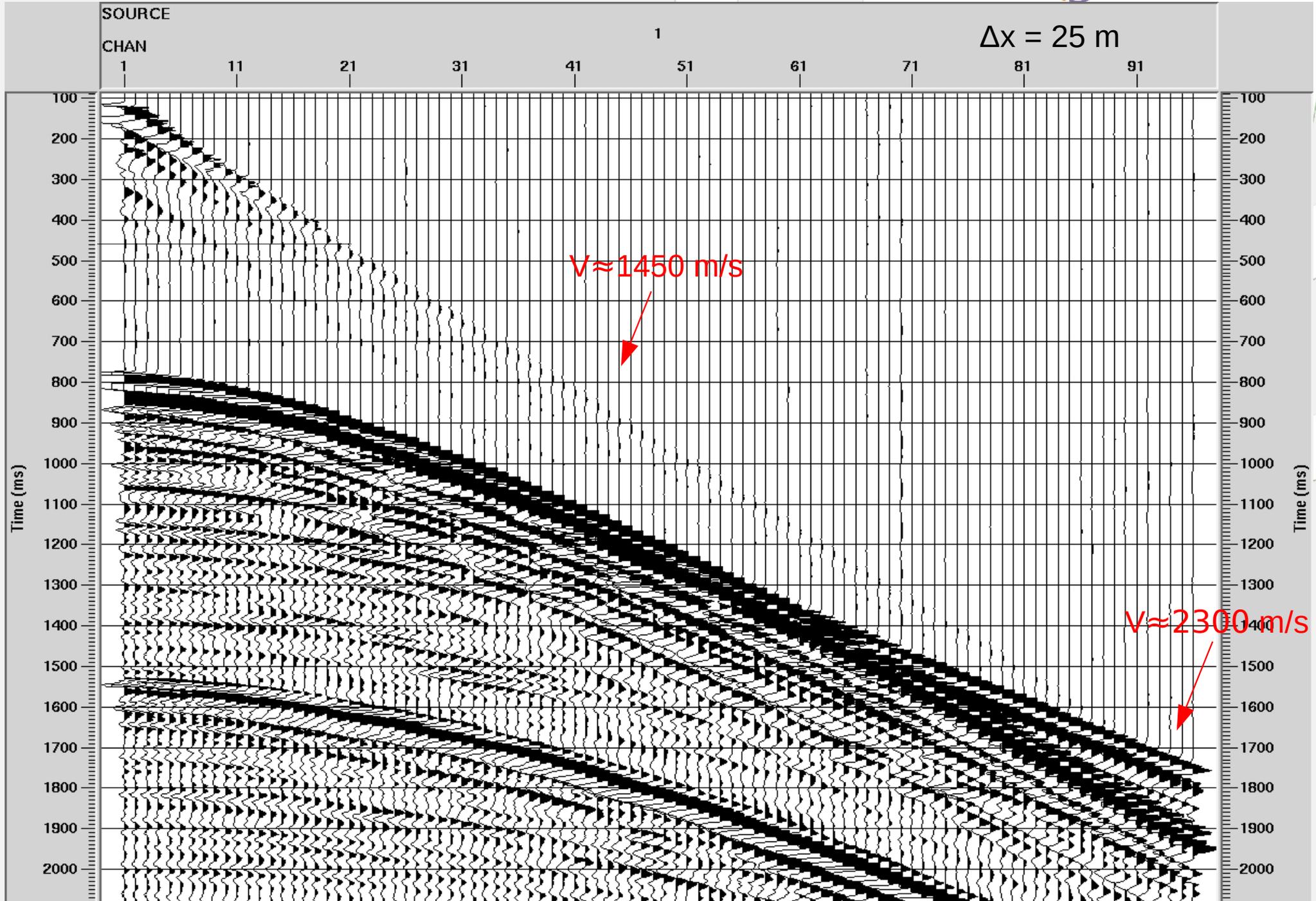
$$\sin \vartheta_c = V_1/V_2 = 1/3 \Rightarrow \vartheta_c = 19.5^\circ$$

$$\tan \vartheta_c = 0.35 \Rightarrow x_c = 2h \tan \vartheta_c \simeq 14 \text{ m}$$

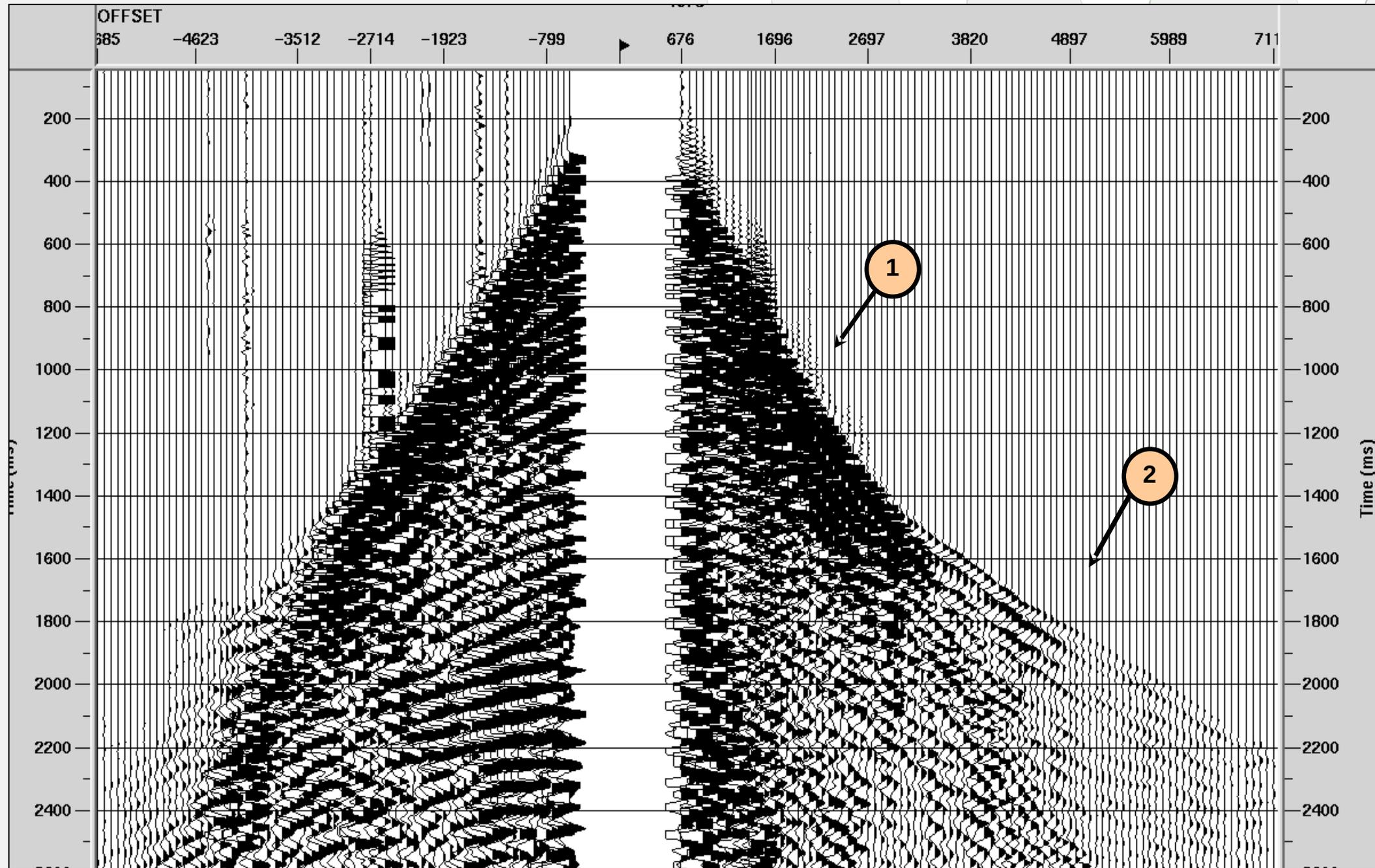
$$x_s = 2h \sqrt{\frac{V_2 + V_1}{V_2 - V_1}} \simeq 56.5 \text{ m}$$

For offsets  $> 56.5$  m, head waves are recorded before the direct waves

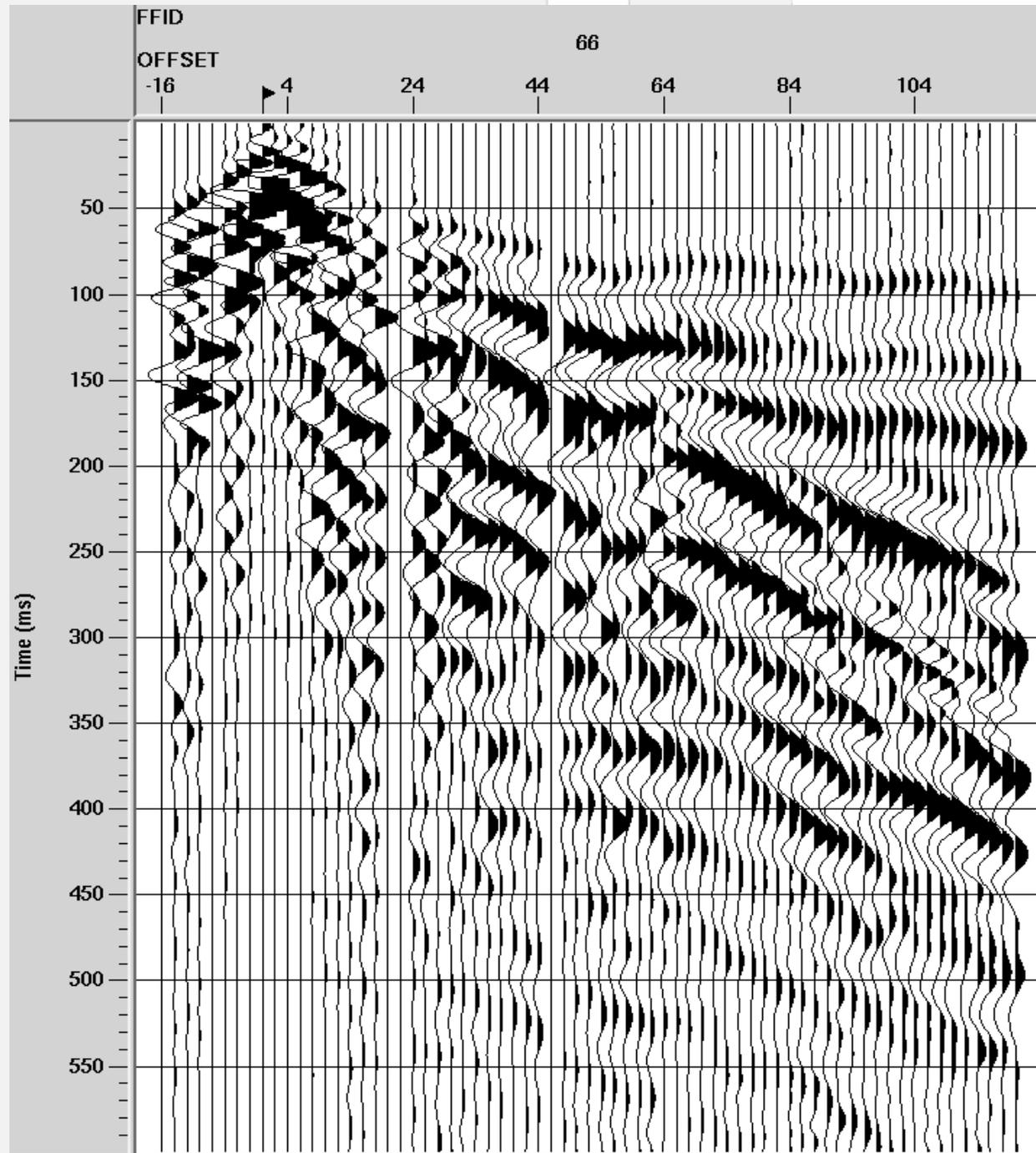
# Example of refraction – marine data



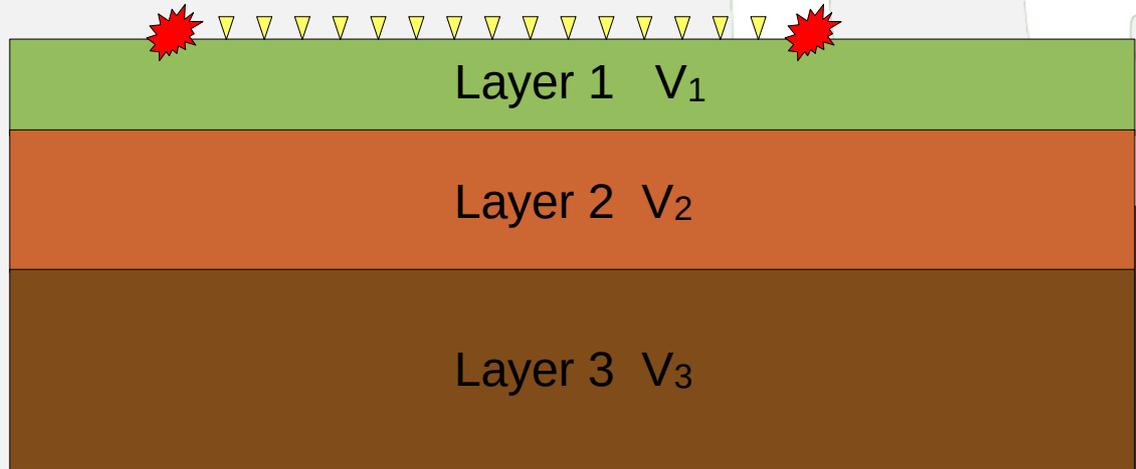
# Example of refraction – land data



# Example of refraction – land data



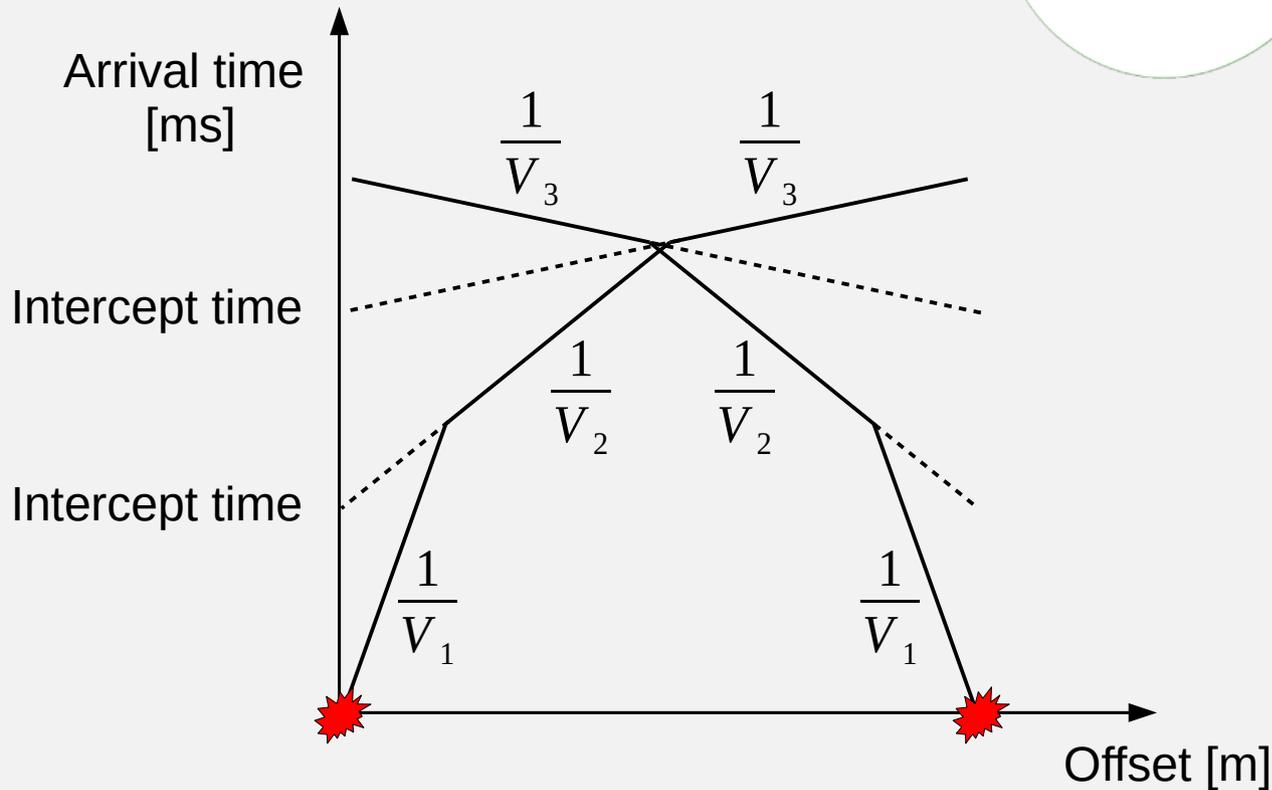
# Refraction from a 3 layer model



3 layer model

$$V_1 < V_2 < V_3$$

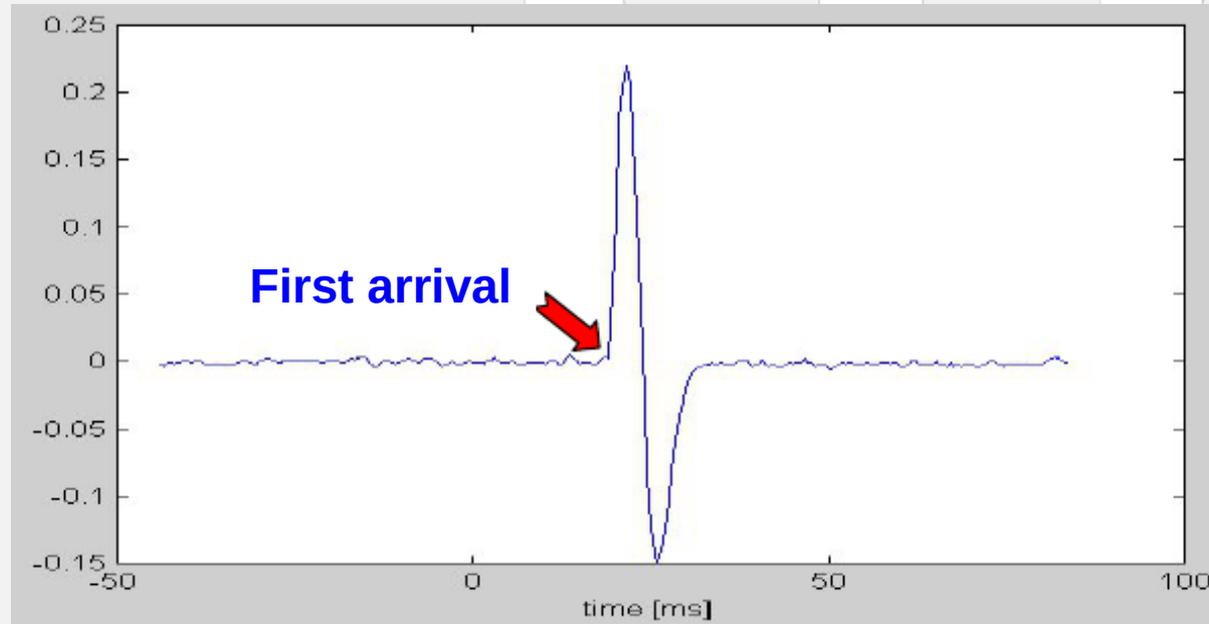
Depth



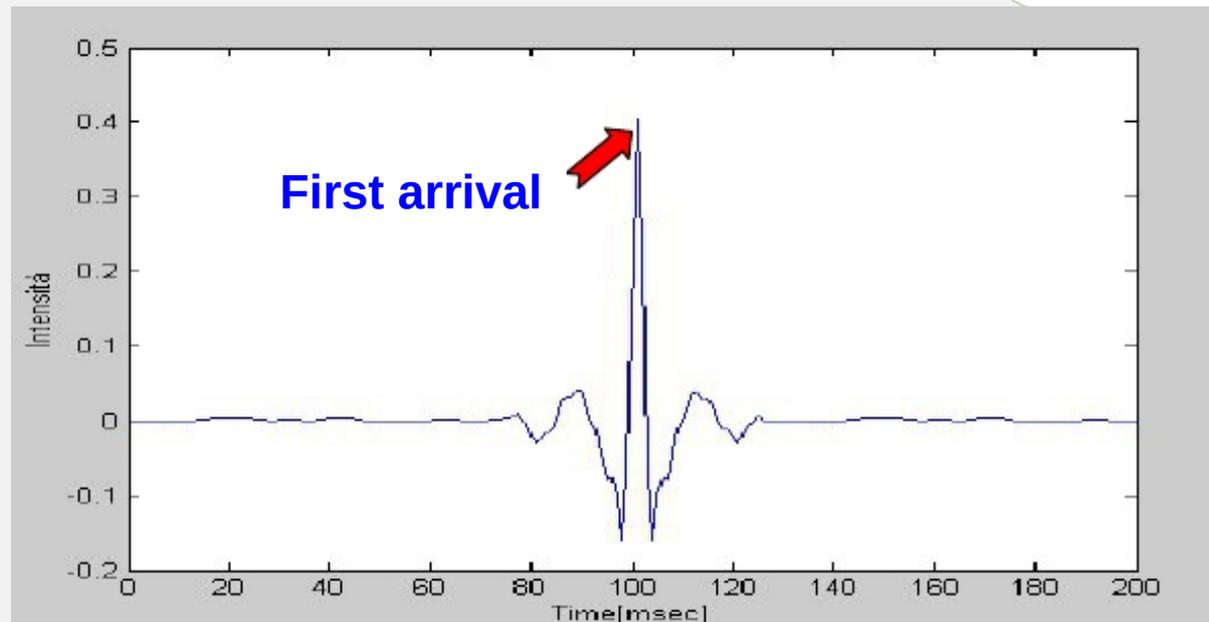
Time-distance plots

# Picking the first arrival

Signal generated by an impulsive source



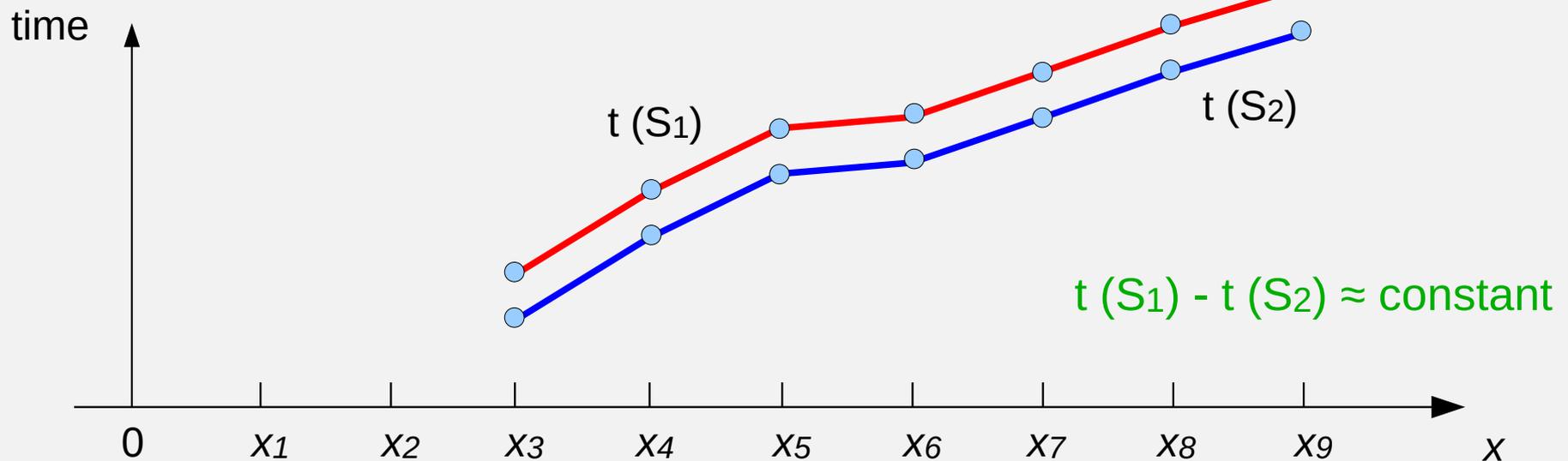
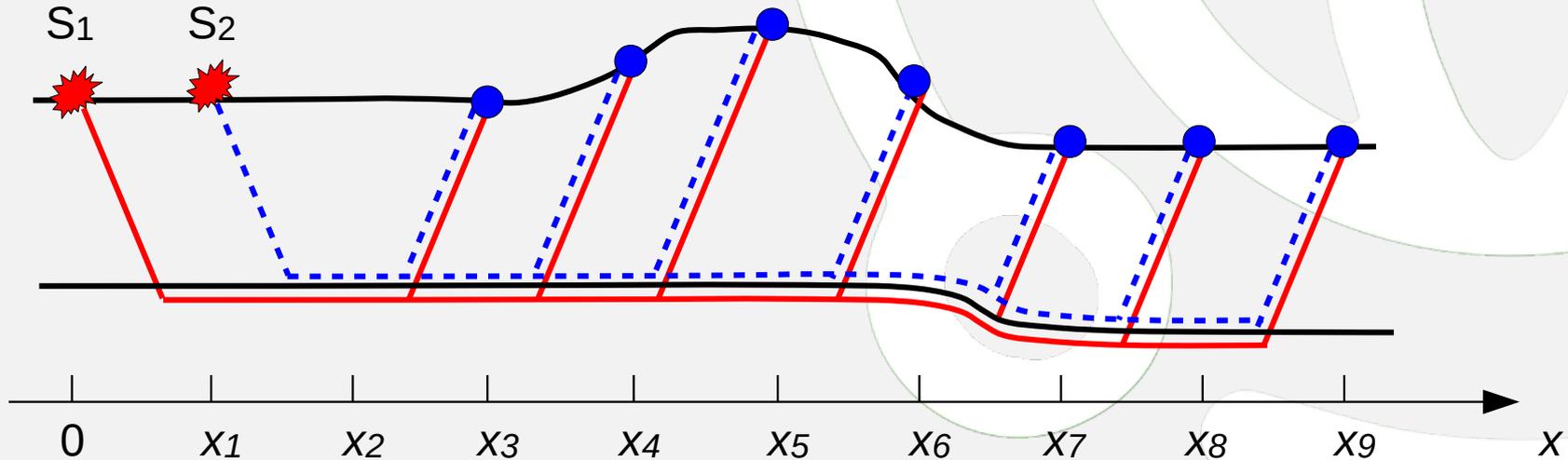
Signal generated by a Vibroseis after cross-correlation



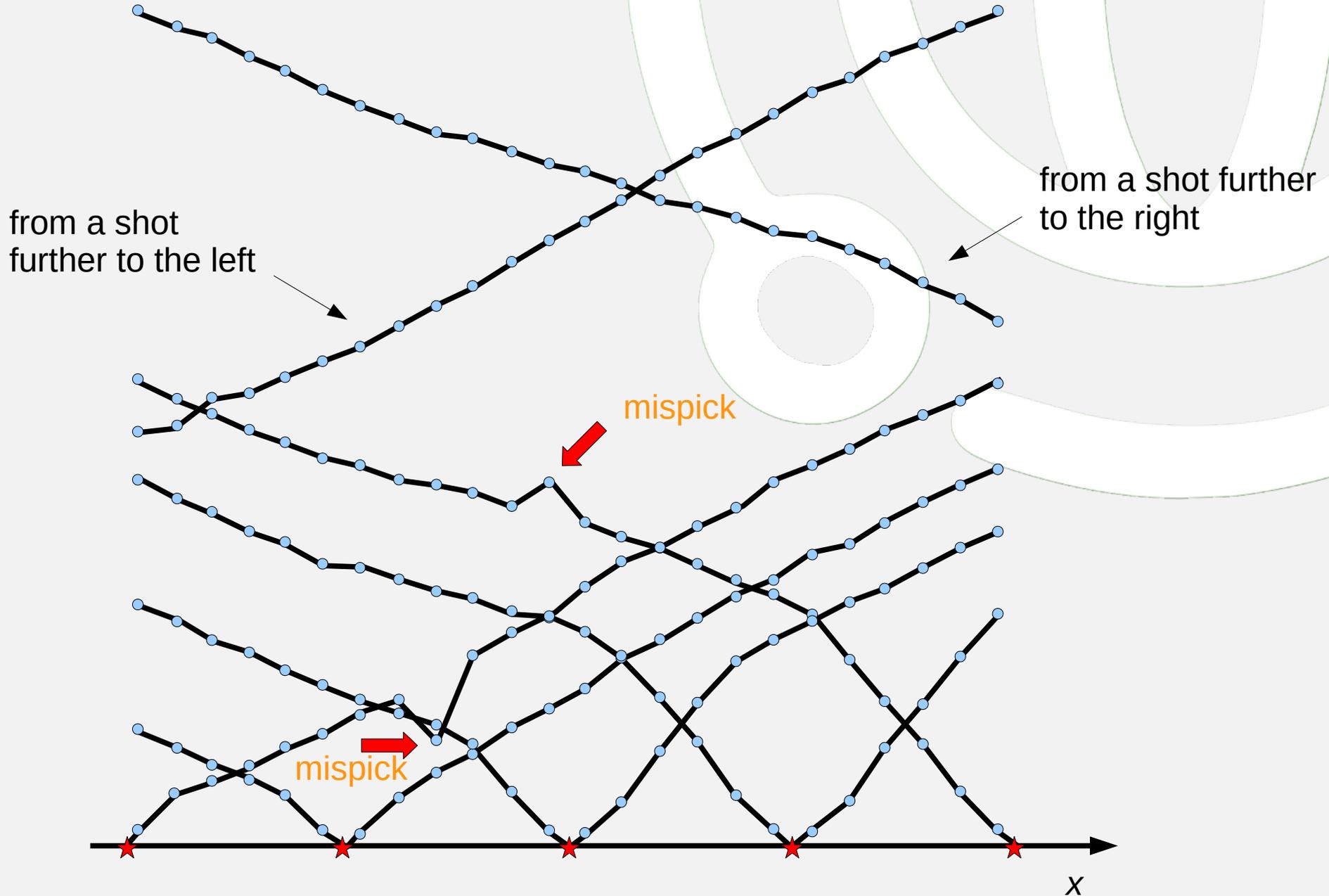
# Parallelism of travel time curves



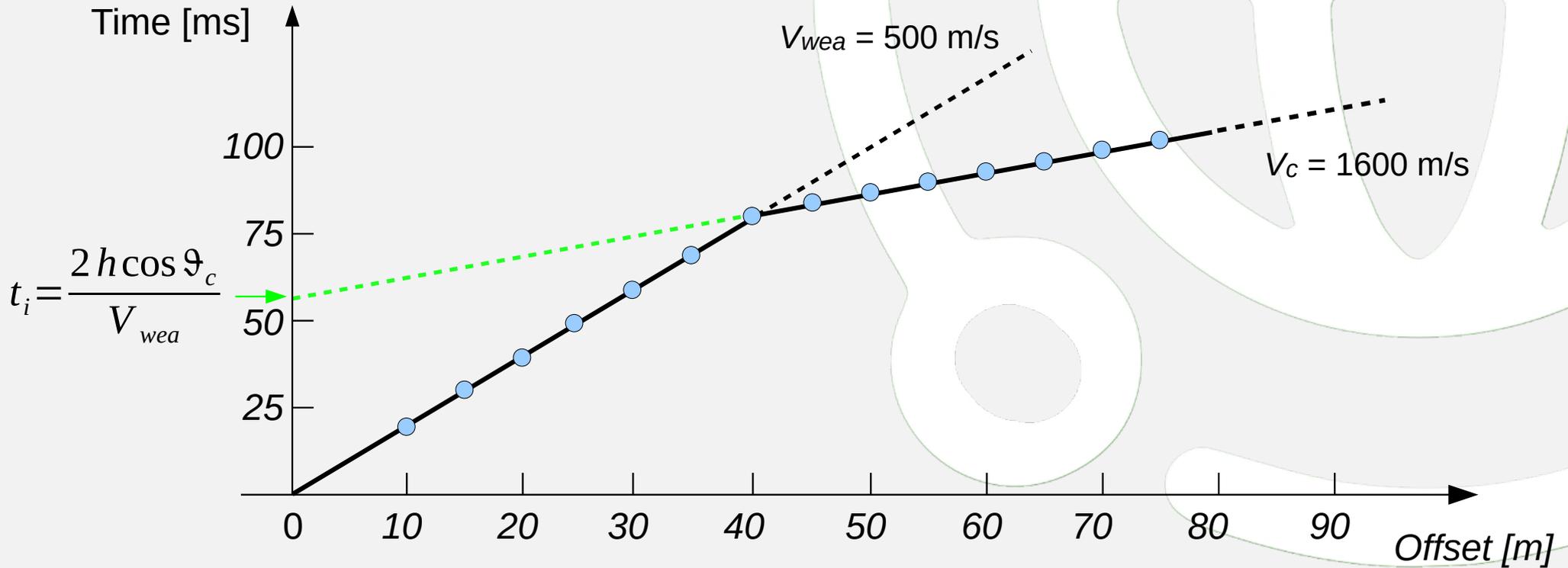
Time-distance graphs from head-waves generated from closely spaced shots are parallel



# First arrival quality control



# Intercept time method

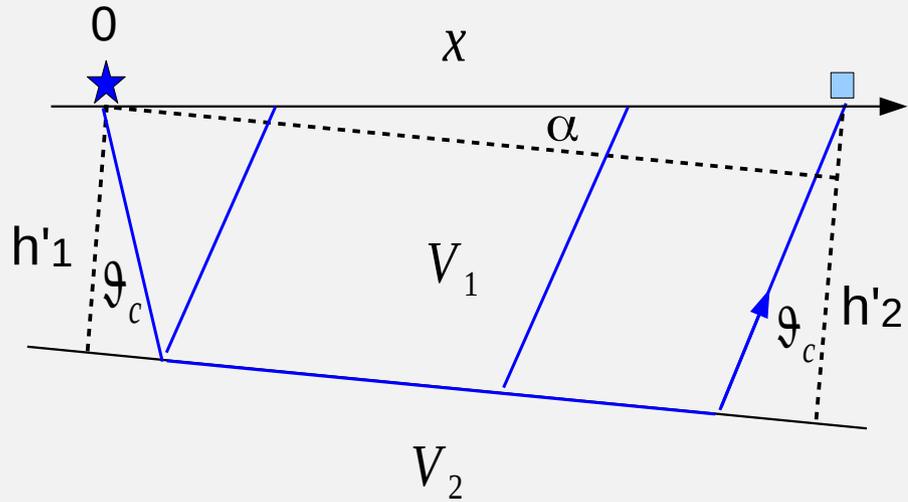


Procedure:

- $V_{wea}$  ⇒ from direct arrivals line
  - $V_c$  ⇒ from refracted arrivals line
  - $t_i$  ⇒ from linear regression of refracted arrivals
- }  $\vartheta_c$

➔ 
$$h = \frac{t_i}{2} \frac{V_{wea}}{\cos \vartheta_c}$$

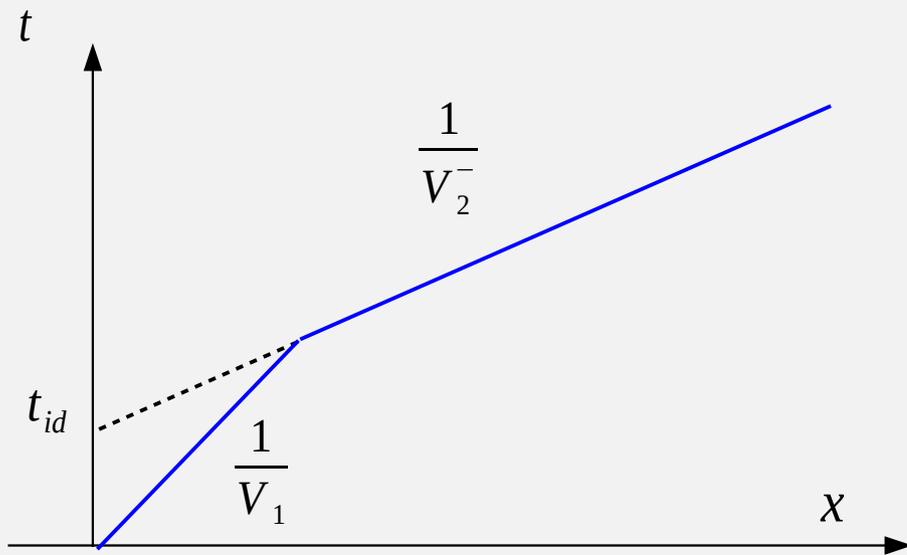
# Dipping interface time-distance curve



$$t = \frac{h_1' \cos \vartheta_c}{V_1} + \frac{h_2' \cos \vartheta_c}{V_1} + \frac{x \cos \alpha}{V_2}$$

$$h_2' = x \sin \alpha + h_1'$$

$$t = \frac{2h_1' \cos \vartheta_c}{V_1} + \frac{x \sin \alpha \cos \vartheta_c}{V_1} + \frac{x \cos \alpha \sin \vartheta_c}{V_1}$$

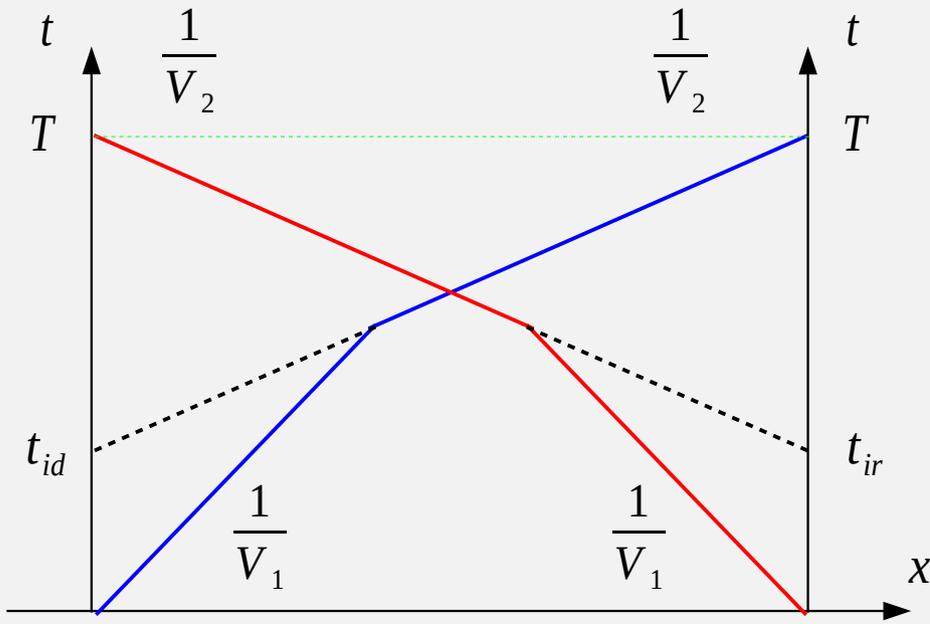
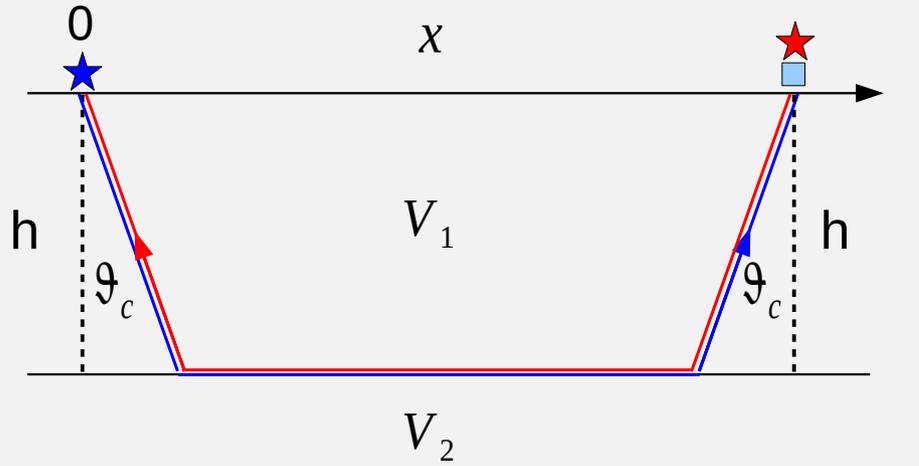


$$t = \frac{2h_1' \cos \vartheta_c}{V_1} + \frac{x \sin(\vartheta_c + \alpha)}{V_1}$$

$$\frac{1}{V_2} = \frac{\sin(\vartheta_c + \alpha)}{V_1}$$

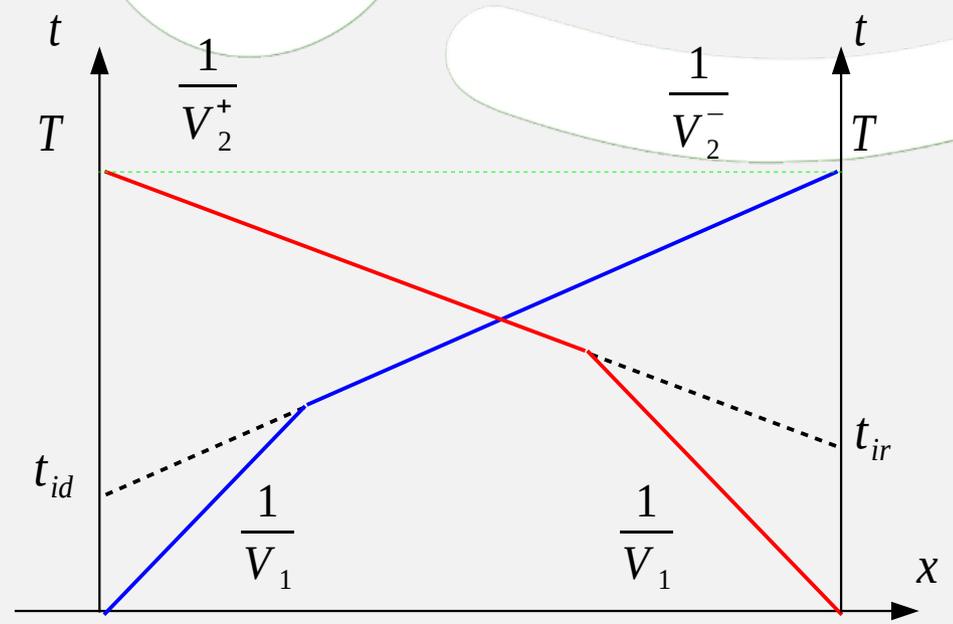
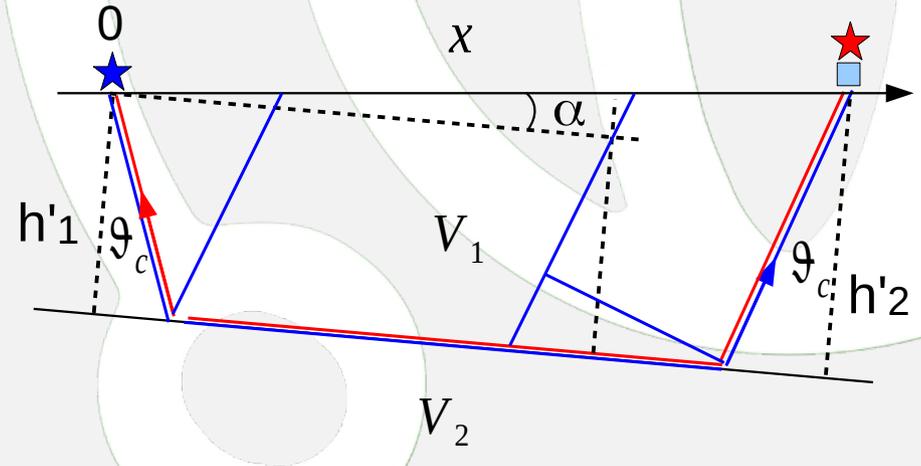
# Dipping-layer case

## Horizontal interface



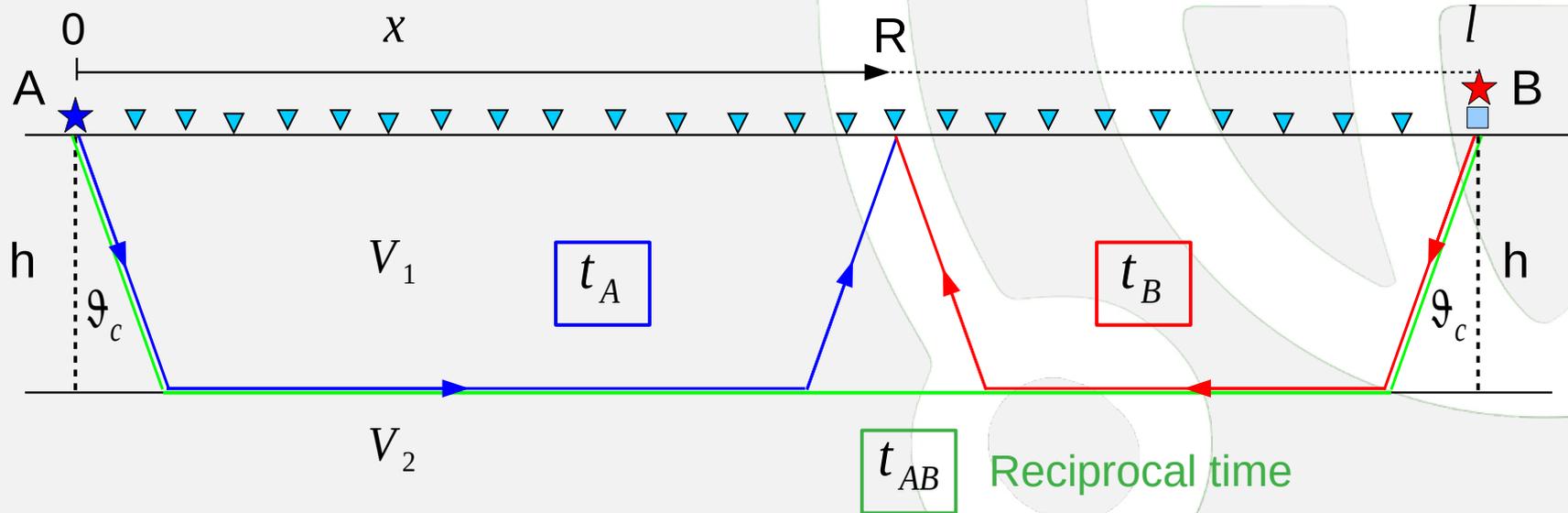
Time-distance graph for forward and reverse shooting

## Dipping interface

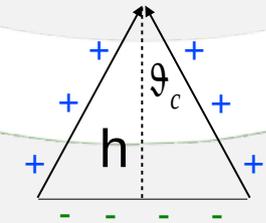


Time-distance graph for forward and reverse shooting

# Reciprocal methods: Plus-Minus



**PLUS time**  $\equiv t_A + t_B - t_{AB} = \frac{2h}{V_1 \cos \vartheta_c} - \frac{2h \tan \vartheta_c}{V_2} = \frac{2h \cos \vartheta_c}{V_1}$



**MINUS time**  $\equiv t_A - t_B - t_{AB}$

$$t_A = t_i + \frac{x}{V_2}$$

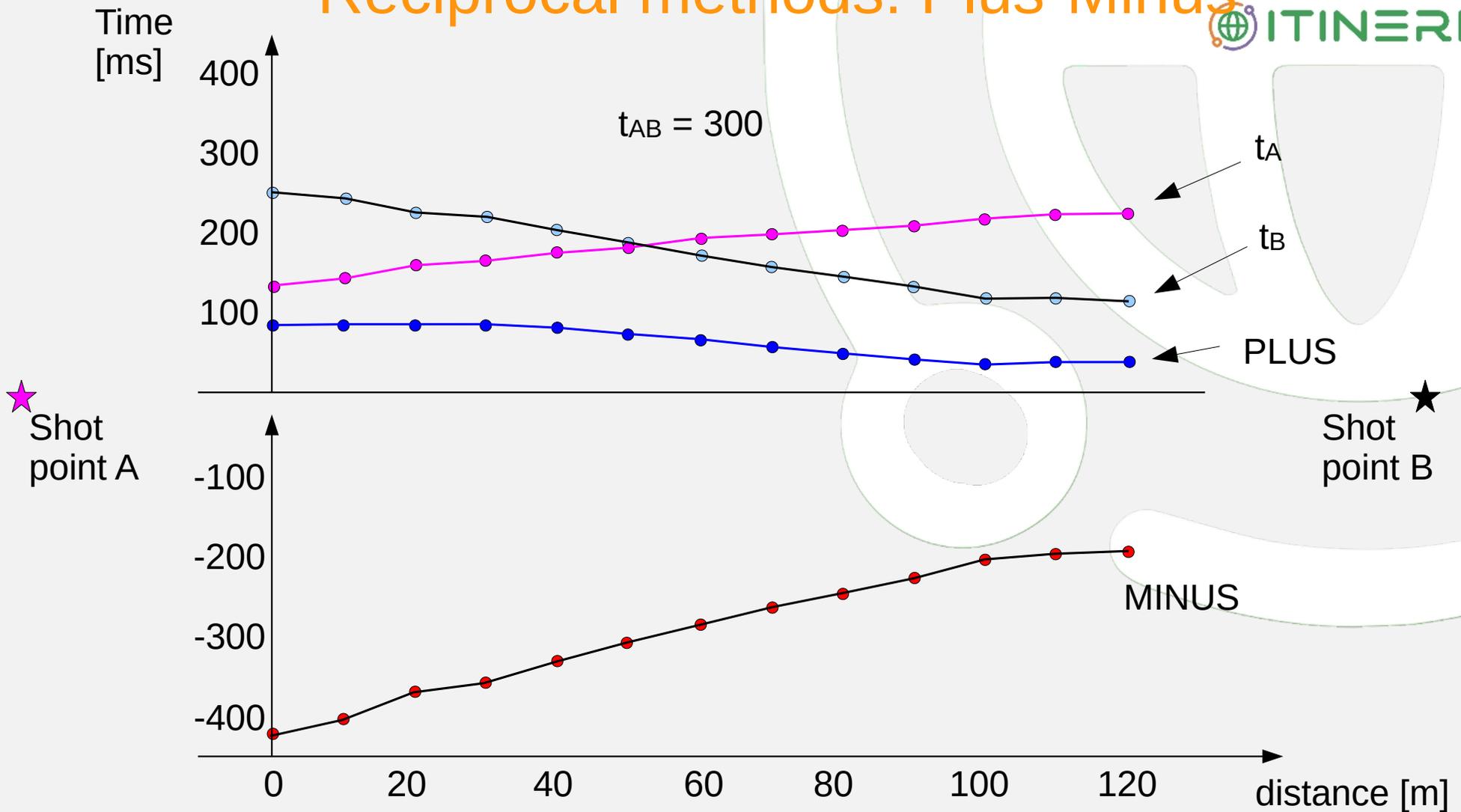
$$t_B = t_i + \frac{l-x}{V_2}$$

and  $t_{AB} = t_i + \frac{l}{V_2}$

**MINUS**  $= -t_i - \frac{2l}{V_2} + \frac{2x}{V_2} \rightarrow$

$$m = \frac{2}{V_2}$$

# Reciprocal methods: Plus-Minus



**Procedure:**  $V_1 \Rightarrow$  from direct arrivals or other measures

$V_2 \Rightarrow$  from MINUS curve

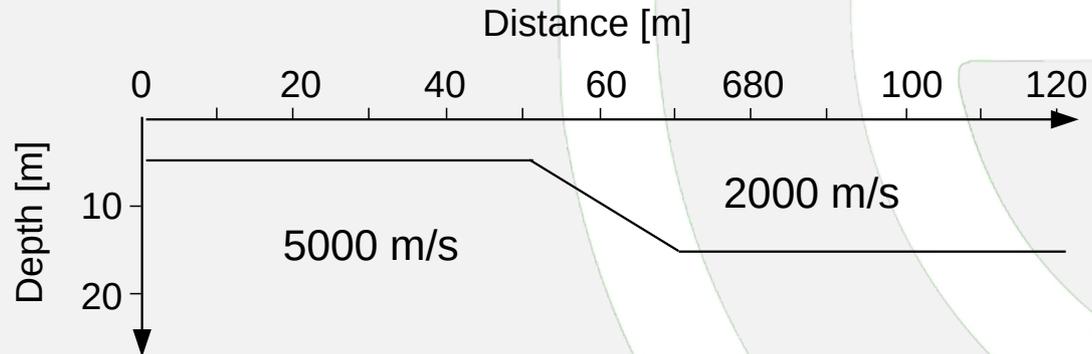
$\cos \vartheta_c \Rightarrow$  through  $V_1$  and  $V_2$

Depth to refractor  $\Rightarrow$  from PLUS curve

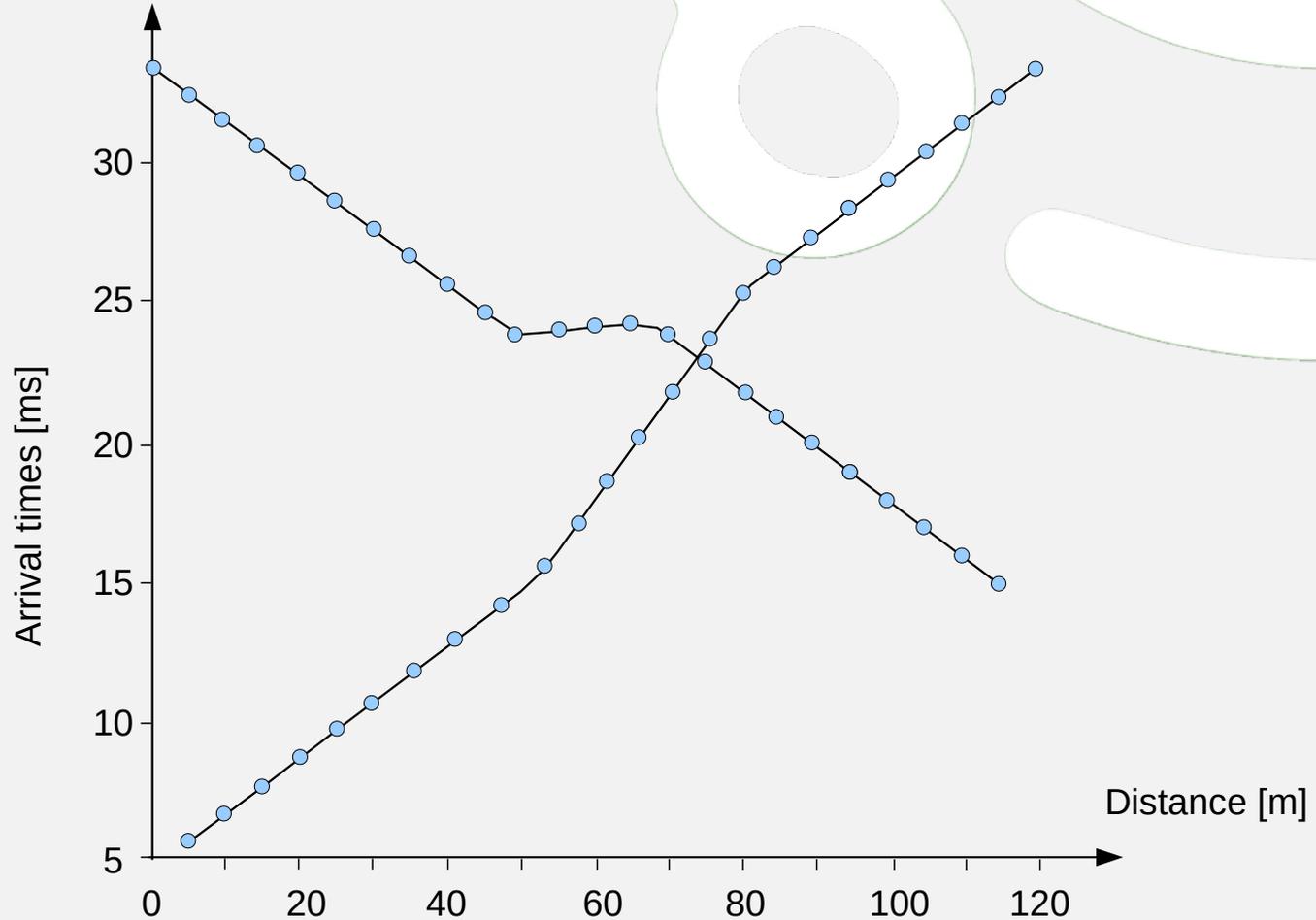
$$D = \frac{\text{PLUS}}{2} \frac{V_1}{\cos \vartheta_c}$$

# Example

Model



Travel times curves

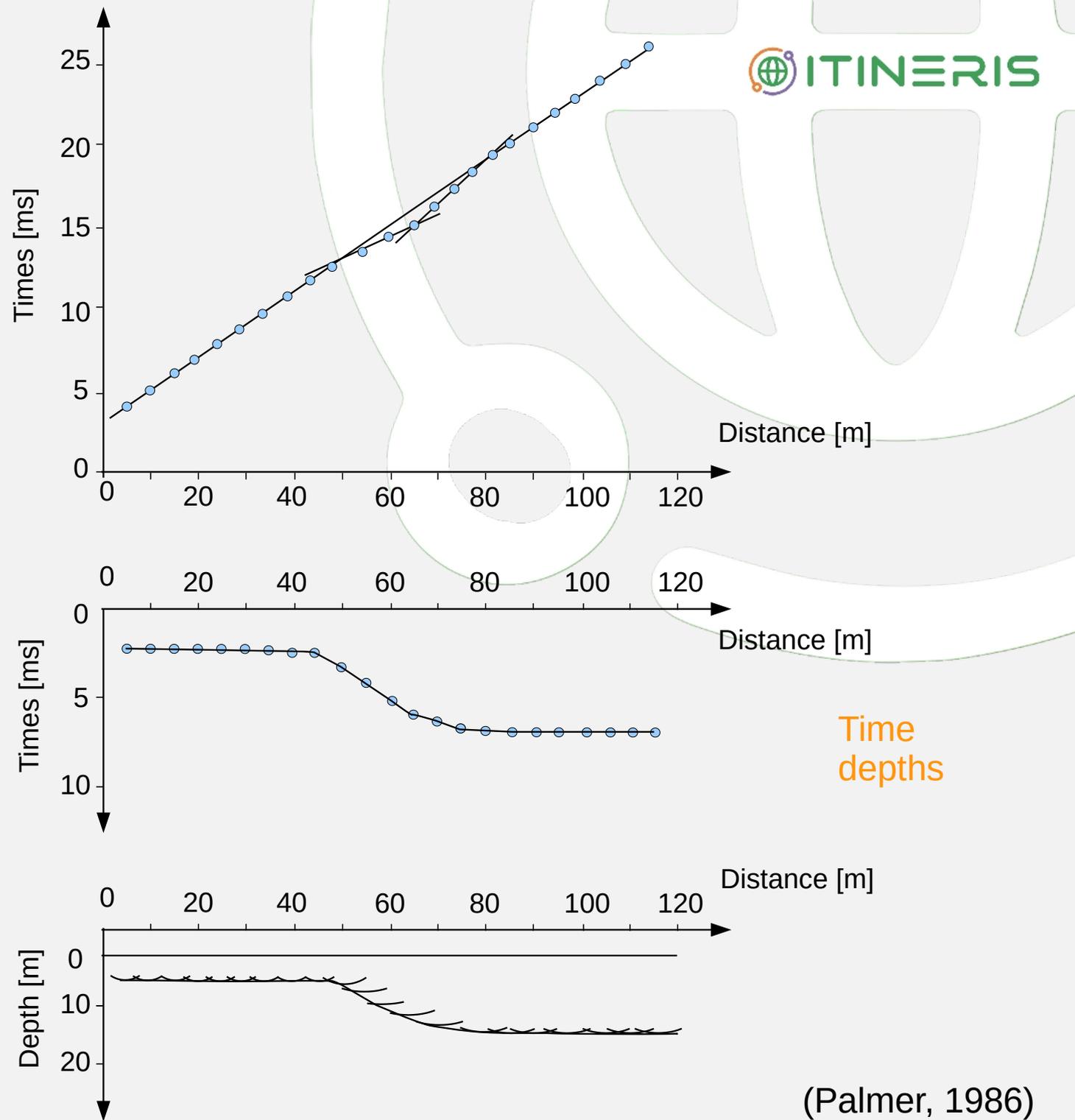


# Example

$$\frac{\text{MINUS}}{2} + \text{const}$$

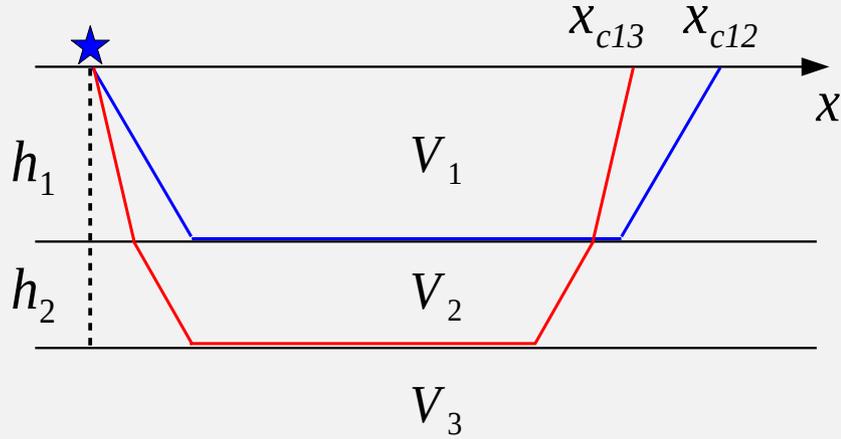
$$\frac{\text{PLUS}}{2}$$

Refractor  
model

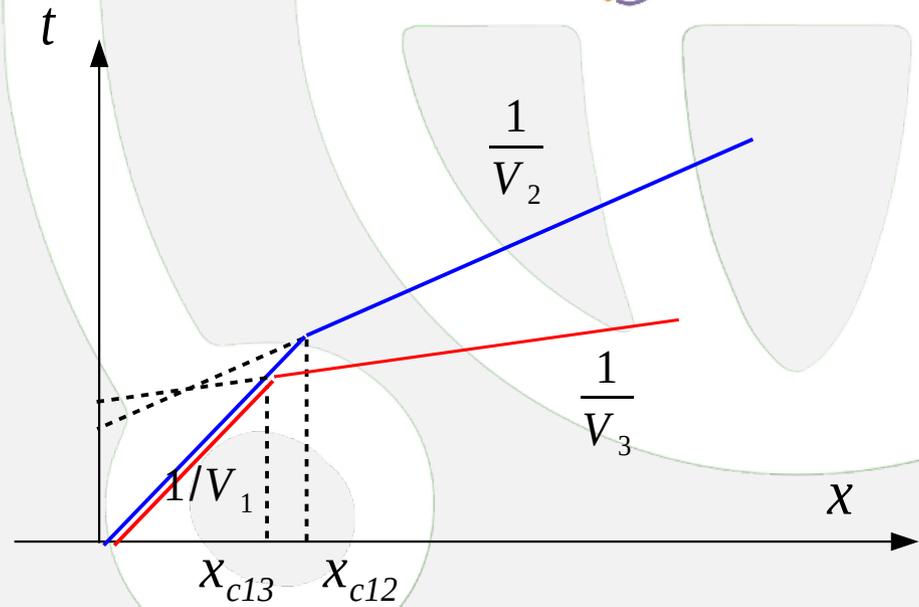


# Hidden-layer problems

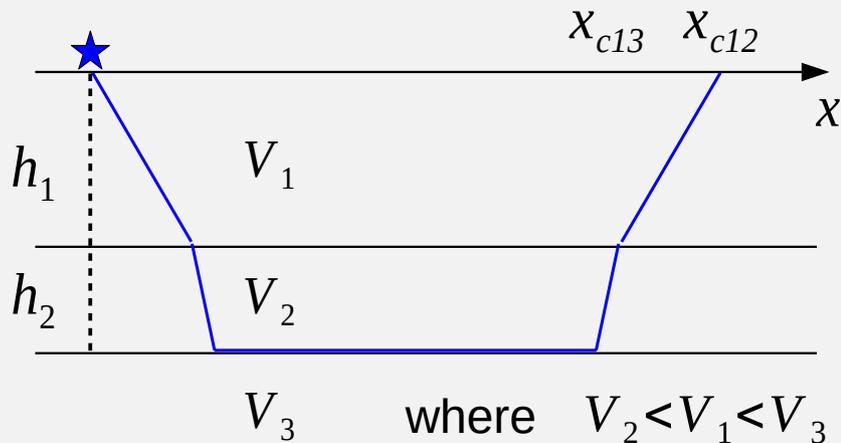
## 1<sup>st</sup> Example



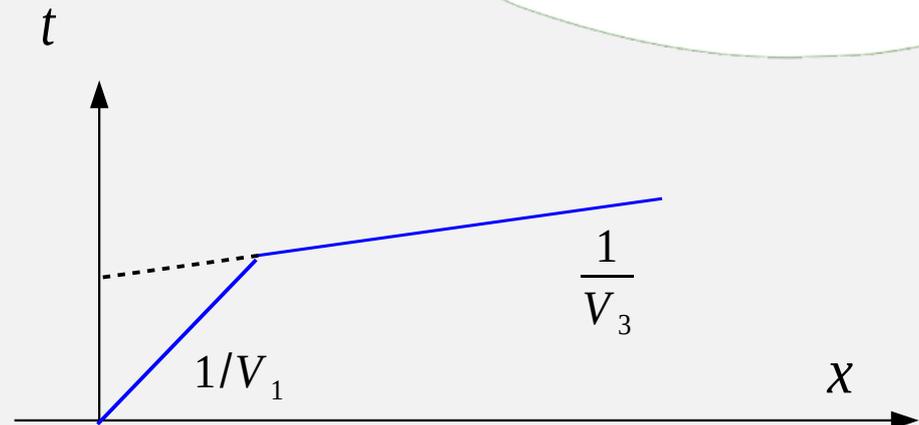
where  $V_2 \ll V_3$  and  $h_2 \ll h_1$



## 2<sup>nd</sup> Example

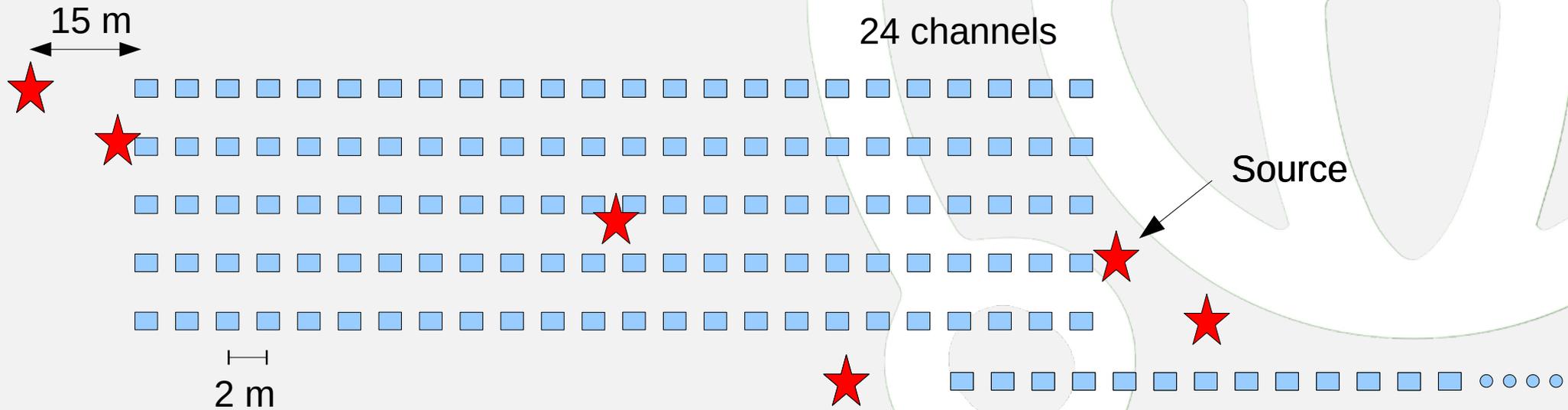


where  $V_2 < V_1 < V_3$



In these situations, the seismic refraction method is not effective since the two upper layers are misinterpreted as a single layer with velocity  $V_1$  and thickness  $h \neq h_1 + h_2$

# Design of a refraction survey: Example



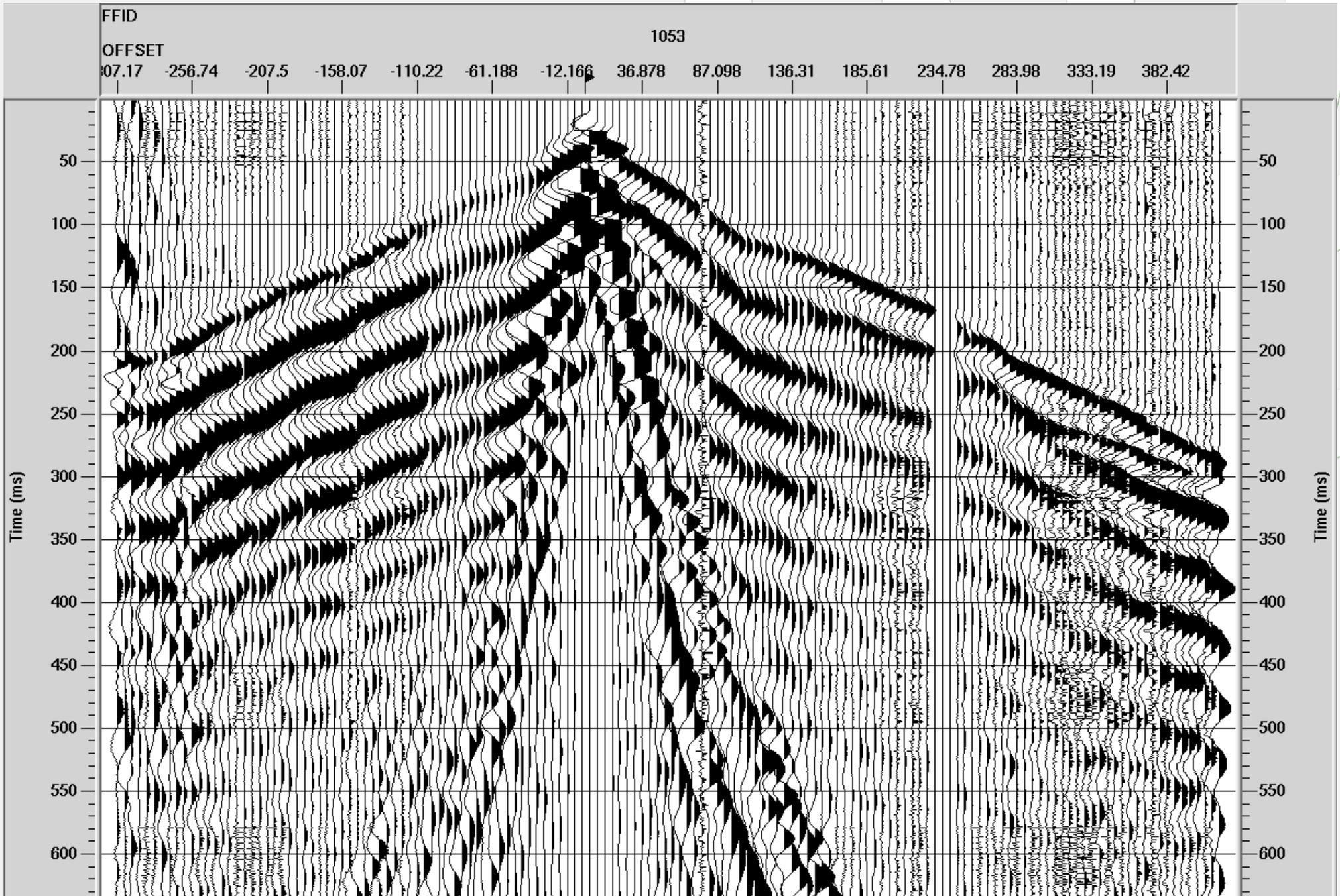
Source: sledge hammer, drop-weight, shot gun, land air-gun, explosives.

Aim: determine thickness and velocity of the weathered layer as well as of shallow layers.

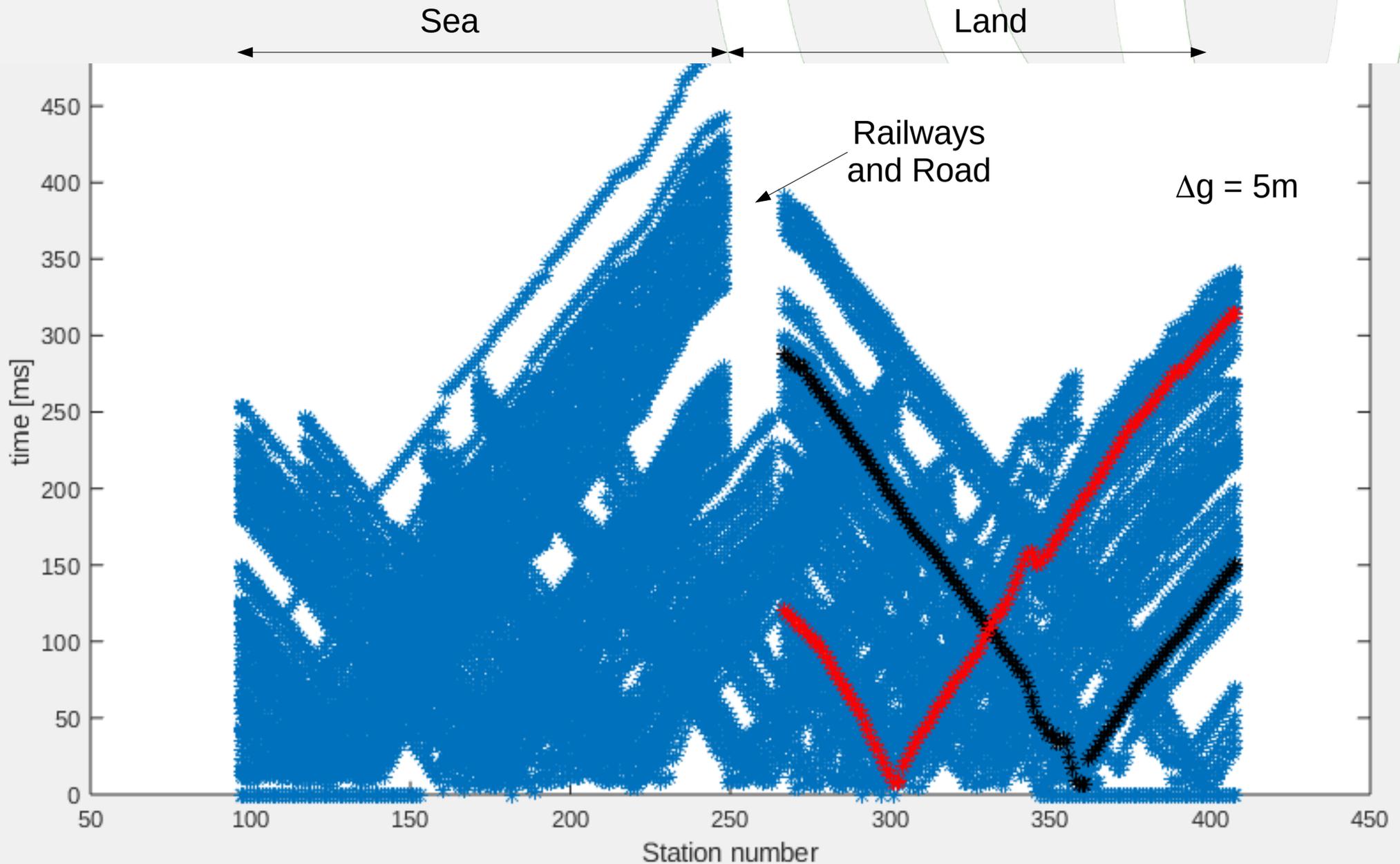
Applications:

- civil engineering
- water prospecting
- determine static corrections for seismic reflection method
- mineral research

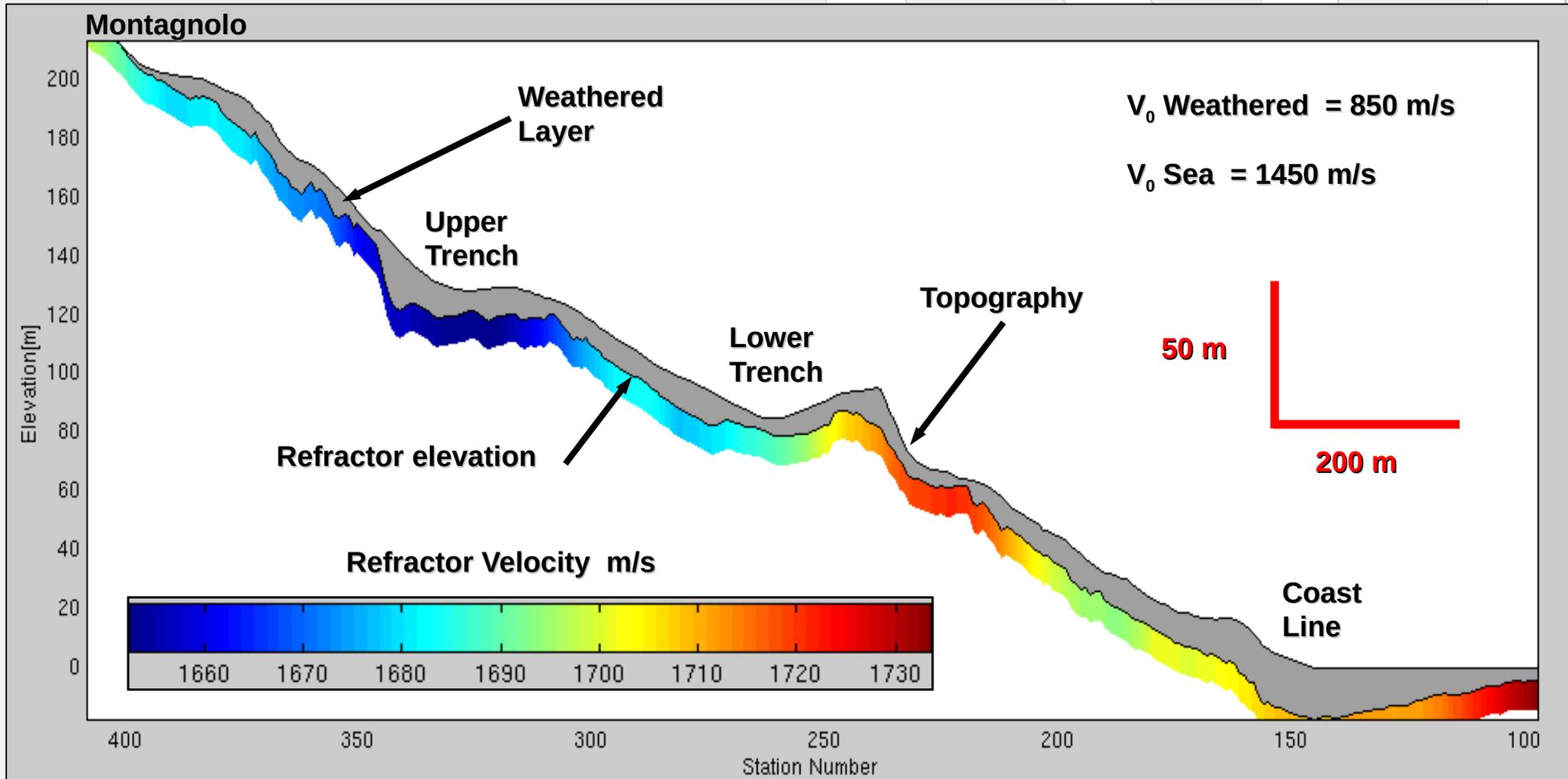
# Example of one layer refractor model estimation: shot of the Ancona survey



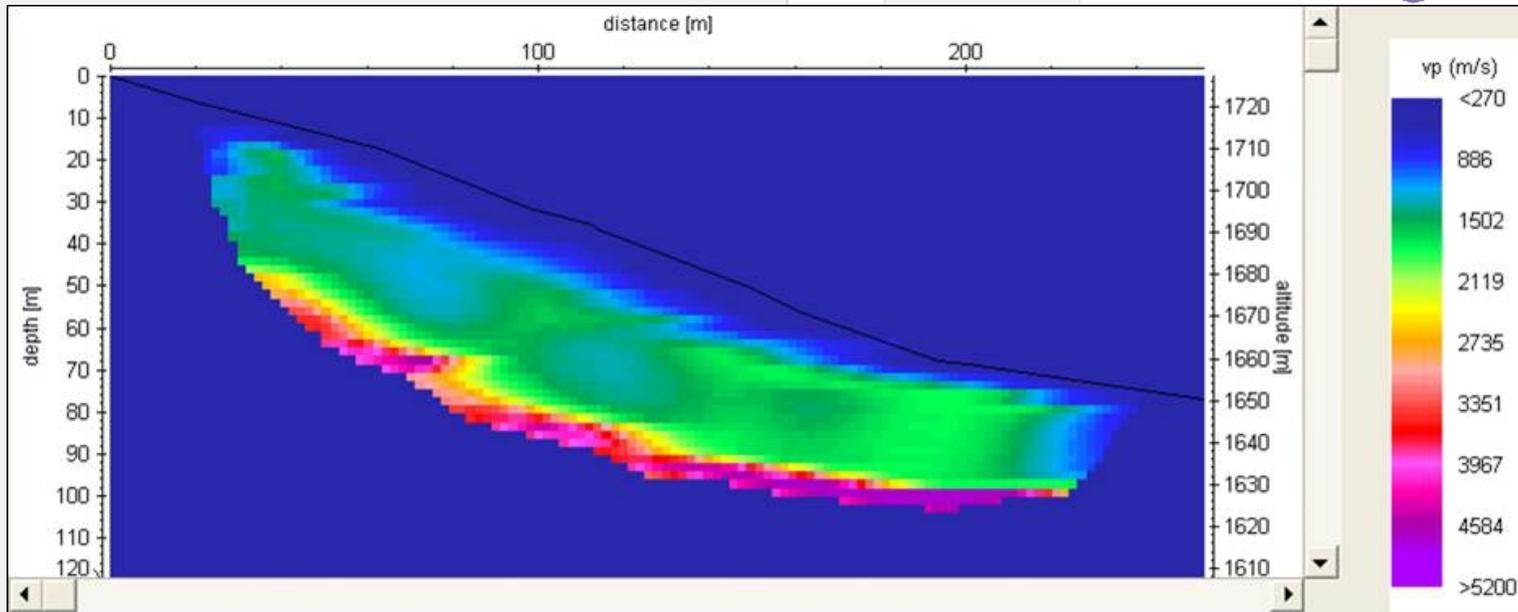
# Example of one layer refractor model: First Break Picks



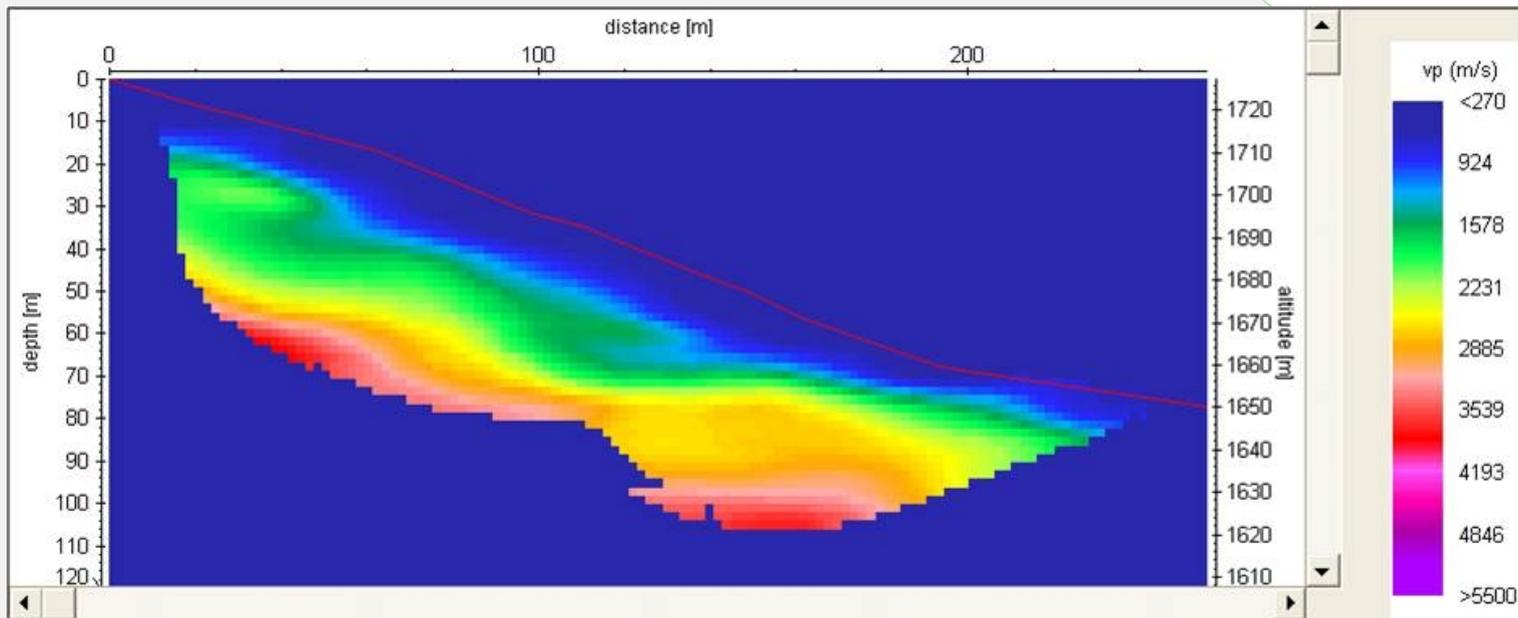
# Example of one layer refractor model estimation: Ancona landslide



# Refraction tomography on a landslide



Refraction tomography (3-layer model)



Refraction tomography (1-layer model with vertical velocity gradient)



# THANKS!

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